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Exuberance, Asymmetric Volatility and Connectedness in Fan Tokens

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Abstract: *In this paper we provide an empirical investigation on some properties of fan tokens of football clubs. Fan tokens are collectible virtual tokens mainly minted on the Socios Blockchain (using the Chiliz cryptocurrency) providing the opportunity to influence club's decisions, unlocking VIP rewards and access to exclusive promotions, games, chat, and a "superfan recognition". Fan tokens are a financial innovation that allows football clubs to retain and, at the same time, monetise their fan base without making any changes to their corporate and ownership structure and without being subject to the legal constraints that the issue of financial products would entail. As fan tokens are immaterial tokens based on a cryptocurrency, in this paper we fill a gap in the literature by studying to what extent several properties of cryptocurrency carry over to fan tokens. In particular, we investigate whether fan tokens exhibit (a) periods of exuberance, (ii) asymmetric volatility response to price changes and (iii) volatility connectedness among themselves and the Bitcoin and Chiliz cryptocurrencies. Our findings suggest that episodes of exuberance characterise the dynamics of fan token prices, that there is some evidence of reverse asymmetric response of volatility to price changes, and that total connectedness among fan tokens is quite sizeable and that most of them exhibit a rich dynamic with multiple reversal in terms of (net) transmitters and (net) receivers of shocks to the entire set of assets.*

Keywords: *fan tokens, asymmetric volatility, spillover effects, bubbles.*

JEL code: *G10, C20, Z23.*

1. Introduction and literature review

Fan tokens are collectible virtual tokens, i.e., they have no material form beyond the digital one, mainly minted on the Socios Blockchain, which provide the owners with access to specific rights. The possession of these tokens gives the power to participate in the life of the club in a way determined by the company itself which, hence, defines its limits and opportunities. They provide the opportunity to influence decisions in proportion to the amount of that team's fan tokens owned, unlocking VIP rewards and access to exclusive promotions, games, chat, and a "superfan recognition" (without any concrete economic benefits though).

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Fan tokens are a financial innovation that allows football clubs to retain and, at the same time, monetise their fan base without making any changes to their corporate and ownership structure and without being subject to the legal constraints that the issue of financial products would entail. In fact, those who buy fan tokens do not buy shares or financial instruments of the sports club but only services and an "influence".

The fan token is issued by a third-party company and distributed on the unregulated market. Sports club agrees to provide "services" to subscribers, while the issuing company finances the sports club: for instance, company A issues fan tokens linked to football team B; whoever buys fan tokens buys from A who in turn finances B. All this takes place through the blockchain tool. Once the initial quota is exhausted in the Fan Token Offering (FTO), the market value of these tokens fluctuates exactly as the market value of shares does. To buy some, you need to find someone who sells them and the higher (lower) the demand for the fan token the more the price will rise (fall).

Notice that price fluctuations do not necessarily follow the game results of the team. For example, if a club offers more interesting, exclusive, valuable advantages, it is less likely that a token holder will want to give them up. If another club, however strong on the pitch, doesn't involve its fans who have bought a token, they might be willing to get rid of it.

The academic literature on fan tokens is scarce. Using a TVP-VAR approach, Ersan, Demir and Assaf (2022) investigate the connectedness among fan tokens and stocks for a sample of listed football clubs. They find that the two asset classes may be considered as independent and that the connectedness is decreasing over time and that the contribution of tokens to stocks (and vice versa) is rather small, around 10%. Demir, Ersan and Popesko (2022) provide empirical evidence that the outcome (wins and losses) in the most prestigious European tournament such as the UEFA Champions League results in abnormal returns in fan tokens while the same does not occur for the results in domestic competitions and in the Europa League. Scharnowski, Scharnowski and Zimmerman (2022) found that fan tokens and stocks are uncorrelated but that they tend to be correlated among themselves and with the cryptocurrency they are defined on. Conversely, Vidal-Tomas (2022) found that fan tokens are uncorrelated with cryptocurrencies implying that they may provide a useful asset for risk diversification purposes and that fan token holders need not worry about the high volatility cryptocurrency markets may exhibit.

As fan tokens are immaterial tokens based on a cryptocurrency, in this paper we fill a gap in the literature by studying to what extent several properties of cryptocurrency carry over to fan tokens. First, using up-to-date test on the presence of multiple bubbles we shall assess whether episodes of exuberance can be identified in the time series of fan tokens prices; second, using asymmetric GARCH models we shall study whether returns volatility exhibit an asymmetric response to price changes, i.e., if higher volatility follows a price fall. Finally, we shall investigate the connectedness among fan tokens and (i) the cryptocurrency they are minted (namely, Chiliz) and (ii) the most popular cryptocurrency, i.e., Bitcoin. The paper is organised as follows. In the next section we describe the dataset, i.e., the specific fan tokens considered and the sample period. Then, we provide a brief but self-contained treatment of the econometric methods used in the empirical analysis. Next, we present the results of the analysis and provide a discussion of the main findings. Final remarks are left to the concluding section.

2. Data and Methodology

2.1. Variables

We considered daily prices and log returns of Fan Tokens of ten football clubs and two cryptocurrencies, Bitcoin (BTC) and Chiliz (CHZ), the latter being an Ethereum token that powers the leading fan token platform Socios.com. The football clubs we considered are AC Milan (ACM), AS Roma (ASR), Juventus (JUV) from Italy, Paris Saint-Germain (PSG) from France, Atlético de Madrid (ATM) and FC Barcelona (BAR) from Spain, Young Boys (YBO from Switzerland, Galatasaray (GAL) and Trabzonspor (TRA) from Turkey, and Apollon Limassol (APL) from Cyprus and the sample period was February 26, 2021-February 15, 2023. Since fan tokens and cryptocurrencies are also quoted on Saturdays and Sunday we have a total of 720 daily observations for each asset. Some descriptive statistics for prices and log returns are reported in Table 1.

On an average, negative log returns for all fan tokens were recorded during this period, even though their magnitude was quite small. Both the standard deviation and the max-min range signal a high volatility in fan tokens and crypto currencies. In particular, we observed a great range of variations in daily returns for fan tokens, with a maximum in daily positive returns for YBO of about 89% and many cases in which daily returns were in the 50%-60% range. Negative daily returns can also be quite sizeable; in the worst case we observed a negative return of about -69% for ATM and many cases of negative returns around minus 50% - minus 60%. The negative skewness for most log returns suggests the presence of a classical asymmetric effect following a shock to returns. High kurtosis indicates that the marginal

Table 1: Summary statistics for price and log returns

Price	*Mean*	*SD*	*Min*	*Max*	*Skewness*	*Kurtosis*
ACM	5.945	2.962	2.223	15.783	.765	2.684
APL	4.848	6.17	.78	29.976	2.075	6.424
ASR	4.873	2.371	1.27	12.91	.967	2.988
ATM	7.876	5.831	2.175	52.264	2.902	18.067
BAR	12.385	10.711	2.64	53.644	1.653	5.171
GAL	5.265	3.463	1.519	20.84	.996	3.264
JUV	7.911	4.346	2.19	26.217	.738	2.722
PSG	15.2	9.373	4.326	56.037	1.226	4.249
TRA	3.268	1.704	1.015	9.319	.796	3.512
YBO	1.431	1.165	.362	5.92	1.787	5.576
BTC	36582.305	14390.705	15787.284	67566.828	.167	1.811
CHZ	.253	.126	.051	.774	1.014	3.671
Log returns	*Mean*	*SD*	*Min*	*Max*	*Skewness*	*Kurtosis*
ACM	-.0022	.0648	-.4639	.362	-.7755	16.3815
APL	-.0022	.0849	-.419	.4612	.4987	8.3501
ASR	-.001	.0636	-.5008	.365	-.5621	16.533
ATM	-.001	.0731	-.6906	.5889	.0869	27.5826
BAR	-.0015	.0633	-.404	.6229	.9151	21.5374
GAL	-.0005	.0687	-.6449	.6042	-.1879	35.9867
JUV	-.0016	.0623	-.5767	.3091	-1.6422	21.5492
PSG	-.0005	.0711	-.6596	.4939	-.5797	24.8014
TRA	-.001	.055	-.4229	.2924	-.7406	18.708
YBO	-.0023	.0787	-.4272	.8979	1.5218	29.3862
BTC	-.0009	.0358	-.1741	.1358	-.3908	5.819
CHZ	.0014	.0821	-.457	.7153	1.3347	16.7115

Table 2: Correlation matrix

	ACM	*APL*	*ASR*	*ATM*	*BAR*	*GAL*	*JUV*	*PSG*	*TRA*	*YBO*	*BTC*	*CHZ*
ACM	1.000											
APL	0.443	1.000										
ASR	0.739	0.475	1.000									
ATM	0.699	0.381	0.678	1.000								
BAR	0.627	0.448	0.624	0.580	1.000							
GAL	0.421	0.347	0.407	0.320	0.418	1.000						
JUV	0.763	0.469	0.745	0.711	0.646	0.401	1.000					
PSG	0.706	0.384	0.608	0.611	0.585	0.353	0.796	1.000				
TRA	0.611	0.382	0.520	0.430	0.500	0.527	0.540	0.506	1.000			
YBO	0.518	0.577	0.477	0.427	0.488	0.333	0.496	0.421	0.395	1.000		
BTC	0.444	0.506	0.440	0.345	0.406	0.317	0.437	0.368	0.319	0.501	1.000	
CHZ	0.457	0.597	0.443	0.388	0.533	0.338	0.451	0.381	0.375	0.638	0.530	1.000

distribution of returns has heavier tails than those of the standard normal one. The correlation matrix (Table 2) shows the substantial positive correlation among fan tokens and between fan tokens and the BitCoin and Chiliz cryptocurrencies too. Finally, prices and log returns for each fan token and the two cryptocurrencies are reported in Figure 1.

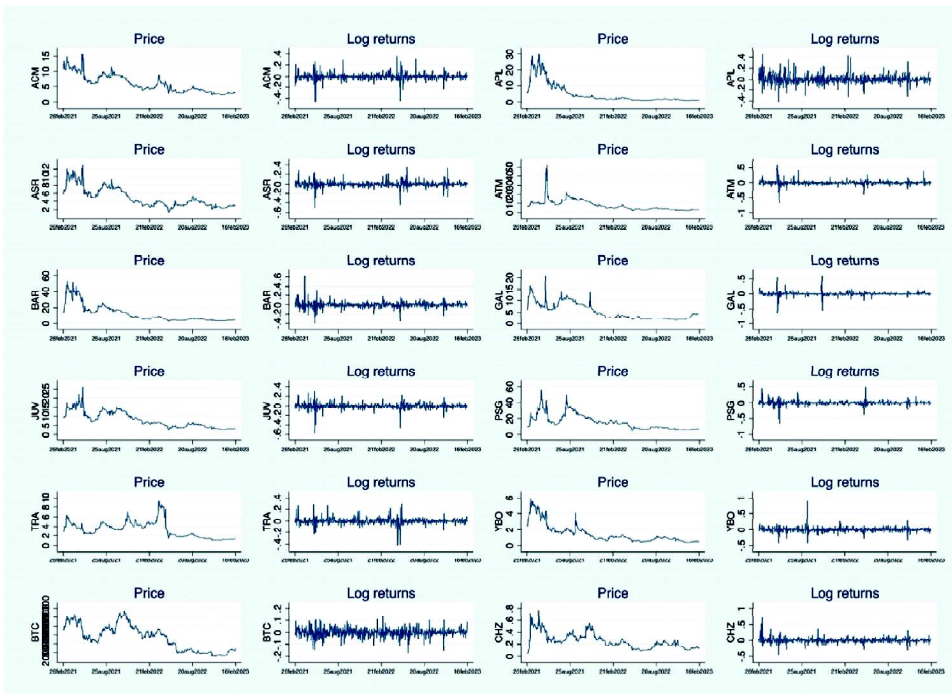

Figure 1: Price and Log Returns of Fan Token and Cryptocurrencies, February 26, 2021 – February 15, 2023

2.2. Econometric models

2.2.1. Bubbles

The statistical investigation on the presence of bubbles follows Phillips, Wu and Yu (2011) and Phillips, Shi and Yu (2015) (PWY and PSY, respectively, hereafter). Briefly, let us start from the general rolling window regression model

$$\Delta y_t = \alpha_{r_1,r_2} + \beta_{r_1,r_2} y_{t-1} + \sum_{i=1}^{k} \psi^{i}_{r_1,r_2} \Delta y_{t-1} + \epsilon_t, \qquad (1)$$

where $\epsilon_t \sim i.i.d.\left(0, \sigma_{r_1,r_2}\right)$ and k is the maximum lag order. The sample used in the estimation window starts from the r_1-th fraction of the sample and ends at the r_2-th fraction of the sample, where $r_2 = r_1 + r_w$ and r_w, the fractional window size of the regression, increases from r_0, the minimal fraction of the sample size, to 1. In this way we may build a sequence of ADF tests, say ADF_0^{r2}, for the null hypothesis of a unit root against the alternative of explosive root. The SADF test is then given by

$$SADF(t_0) = sup_{r_2} ADF_0^{r_2} \tag{2}$$

The GSADF test, which is particularly useful in the detection of multiple bubbles, considers changing the starting point r_1 from 0 to r_2–r_0 and it is given by

$$GSADF(r_0) = sup_{r_2 \in [r_{0,}1], r_1 \in [0, r_2 - r_0 \quad]} ADF_{r_1}^{r_2} \tag{3}$$

The limit distribution of the ASDF and the GSADF tests can be found by simulation, critical values for the null hypothesis of a unit root against the alternative of an explosive root are available in PSY. Rejection of the null hypothesis occurs when the test statistics is greater than the critical value.

2.2.2. Asymmetric volatility

In this study we employed two different measures of asymmetry in volatility. The first one builds on the Threshold GARCH (TGARCH) model proposed by Glosten et al. (1993), which captures the well-known stylised fact of the increase in volatility in response to a fall in the asset price. We also considered a TGARCH-in-mean model (see Engle *et al.* (1987)) which allows for a direct effect of the conditional volatility on expected returns. In both cases, following Avramov et al. (2006), we introduced an AR(1) component in the mean equation to account for the possible presence of noise treaders or, more generally, short-lived frictions of the market. The TGARCH model is given by:

$$r_t = \alpha_0 + \alpha_1 r_{t-1} + \epsilon_t \tag{4}$$

$$h_t = \omega + \alpha \epsilon_{t-1}^2 + \beta\ h_{t-1} + \gamma \epsilon_{t-1}^2\ I(\epsilon_{t-1} < 0) \tag{5}$$

$$\epsilon_t \sim i.i.d.N(0,\ h_t) \tag{6}$$

where r_t stands for the current returns, $I(\cdot)$ is the indicator function equal to 1 whenϵ_{t-1} is negative and zero otherwise, and h_t is the unobserved volatility. The parameters α and β are restricted to be positive while the parameter γ is free, a positive (negative) γ implies an increase (decrease) in volatility after a fall in returns. The TGARCH-in-mean model is specified as follows:

$$r_t = \alpha_0 + \alpha_1 r_{t-1} + \psi\sqrt{h_t} + \epsilon\_t \tag{7}$$

$$h_t = \omega + \alpha\epsilon_{t-1}^2 + \beta\, h_{t-1} + \gamma\epsilon_{t-1}^2\, I(\epsilon_{t-1} < 0) \tag{8}$$

$$\epsilon_t \sim i.i.d.N(0,\ h_t) \tag{9}$$

A positive ψ implies that expected returns will increase with the risk, as proxied by the (square root of) volatility.

An alternative measure of asymmetry, based on quantile autoregressive (QAR) model of Koenker and Xiao (2006), has been proposed by Baur and Dimpfl (2019). Briefly, let us consider a quantile autoregressive model of order 1 for log returns:

$$Q_{r_t}(\tau|r_{t-1}) = \theta_0(\tau) + \theta_1(\tau)r_{t-1} \tag{10}$$

where the τ-th conditional quantile of r_t is modelled as a function of its lagged value r_{t-1}. Baur and Dimpfl (2019) suggest to estimate the QAR(1) model for a set of small and large extreme quantiles, say $\tau = 0.01,\ldots,0.1$ and $\tau = 0.90,\ldots,0.99$, with increments of 0.01, to average the resulting estimates of θ_1 (τ) for the small and large quantiles and then compute the difference between the averages, say δ. The intuition for this indicator of asymmetry is that if estimates of the autoregressive coefficient differ in the tails this would imply an asymmetric effect of negative and positive returns on volatility. In particular, the classical asymmetric volatility effect is obtained when the quantile autocorrelations for small quantiles are positive and the quantile autocorrelations for large quantiles are negative while, on the other hand, an inverted asymmetric effect arises when the quantile autocorrelations for small quantiles are negative and the quantile autocorrelations for large quantiles are positive.

2.2.3. Connectedness

Connectedness is investigated using the spillover indices developed by Diebold and Yilmaz (2009, 2012) to which we refer to for a detailed treatment. Let $x_t = \Phi_1\, x_{t-1} + \ldots + \Phi_p\, x_{t-p} + u_t$ be a $VAR(p)$ model for the $(N\times 1)$ vector of fan token and cryptocurrency volatilities x_t, where $u_t \sim$ i.i.d.(0, Σ), and let A_i for $i = 1,\ldots,$ be the coefficient matrices of the infinite MA representation (with A_0 the identity matrix). The H-step-ahead generalized forecast error variance decomposition matrix has generic element given by

$$\theta_{ij}^{g} = \frac{\sigma_{jj}^{-1}\sum_{h=0}^{H-1}\left(e_i' A_h\, \Sigma e_j\right)^2}{\sum_{h=0}^{H-1}\left(e_i' A_h\, \Sigma A_h'\, e_i\right)} \tag{11}$$

where e_i is the selection vector. Since the row sum of the matrix is not equal to 1, each θ^g_{ij} is normalized as follows

$$\widetilde{\theta^g_{ij}}(H) = \frac{\theta^g_{ij}(H)}{\sum_{j=1}^{N} \theta^g_{ij}(H)} \tag{12}$$

The total spillover is given by

$$S^g(H) = 100 \times \frac{1}{N} \sum_{i,j=1;i\neq j}^{N} \widetilde{\theta^g_{ij}}(H) \tag{13}$$

Directional spillover (from and to) can then be easily computed. The directional volatility spillover received by asset i from all other assets j is

$$S^g_{i\cdot}(H) = 100 \times \frac{1}{N} \sum_{j=1;j\neq i}^{N} \widetilde{\theta^g_{ij}}(H) \tag{14}$$

and the directional volatility spillover transmitted by asset i to all other assets j is

$$S^g_{\cdot i}(H) = 100 \times \frac{1}{N} \sum_{j=1;j\neq i}^{N} \widetilde{\theta^g_{ij}}(H) \tag{15}$$

Finally, net volatility spillovers can be directly obtained as $S^g_i(H) = S^g_{\cdot i}(H) - S^g_{i\cdot}(H)$. Volatilities are computed following Diebold and Yilmaz (2012): let P^H_{it} and P^L_{it} be the maximum and the minimum price for asset i on day t, the daily variance is estimated as $\widetilde{\sigma^2_{it}} = 0.361[ln(P^H_{it}) - ln(P^L_{it})]$ and the annualised daily percent standard deviation (i.e., volatility) is given by $\widehat{\sigma_{it}} = 100\sqrt{365 * \widetilde{\sigma^2_{it}}}$.

3. Results and discussion

To begin our empirical investigation, we consider the tests for the presence of bubbles proposed by PWY and PSY. Table 3 reports the ADF, SADF and GSADF test statistics for the fan tokens and the Chiliz and Bitcoin cryptocurrencies. Asymptotic right-tail critical values at the 5% significance level for the ADF, SADF and GSADF tests are taken from Vasilopoulos, Pavlidis, Spavound and Martinez-Garcia (2020) and are given by 0.02, 1.3693 and 2.1139, respectively. The ADF test is not able to reject the null of a unit root in the autoregressive representation of the time series against the alternative of explosive root. This is in line with theoretical expectations since the ADF test has power against the alternative hypothesis a root less than 1 in absolute value.

Table 3: Testing for episodes of exuberance

	ADF	*SADF*	*GSADF*
ACM	-2.043	-1.286	3.13
APL	-1.968	-1.495	2.168
ASR	-1.68	-1.043	2.482
ATM	-4.254	1.621	3.142
BAR	-1.649	-1.083	3.033
GAL	-1.985	-0.966	4.133
JUV	-1.363	-1.128	2.587
PSG	-2.411	0.045	2.887
TRA	-2.045	-0.747	3.571
YBO	-1.915	-0.5	4.38
BTC	-1.366	-0.466	3.426
CHZ	-2.859	-2.436	3.812

The SADF test indicates the presence of a bubble only for the ATM fan token. However, matters change considerably when we look at the GSADF test, all fan tokens and both cryptocurrencies display episodes of exuberance. Once the null hypothesis has been rejected, we can proceed to plot the sequences of ADF statistics, and related critical values at the 90% and 95%, to possibly identify the relevant episodes of explosive behavior. PSY show that recursive backward window estimation of (ADF) may provide some guidance to date-stamp episodes of explosive behavior. Briefly, if we are interested in establishing whether any observation, say r_2, belongs to a phase of explosive behavior, PSY suggest performing a supADF test fixing the endpoint is fixed at r_2, and then increasing the sample backwards up to a starting point, r_1, between 0 and $r_2 - r_0$. The backward SADF statistic is defined as the supremum of the resulting sequence of ADF test statistics, i.e.,

$$BSADF_{r_2}\,(r_0) = sup_{r_1 \in [0, r_2 - r_0\ ]} ADF_{r_1}^{r_2} \tag{16}$$

The sequence of backward ADF tests, along with the corresponding critical values, is plotted in Figure 2.

Most fan tokens exhibit one episode of exuberance (APL, ATM, GAL, JUV, PSG, YBO) while ACM, ASR, BAR and TRA display at least two such episodes at the 95% critical level. These episodes of explosive behavior are not immediately associated to those occurring at the cryptocurrencies level where, in fact, Bitcoin and Chiliz seem to experience bubbles in different time periods with respect to those occurring to fan tokens and among them.

To conclude, both the fan tokens and the cryptocurrencies considered in the analysis display episodes of explosive behavior during the sample

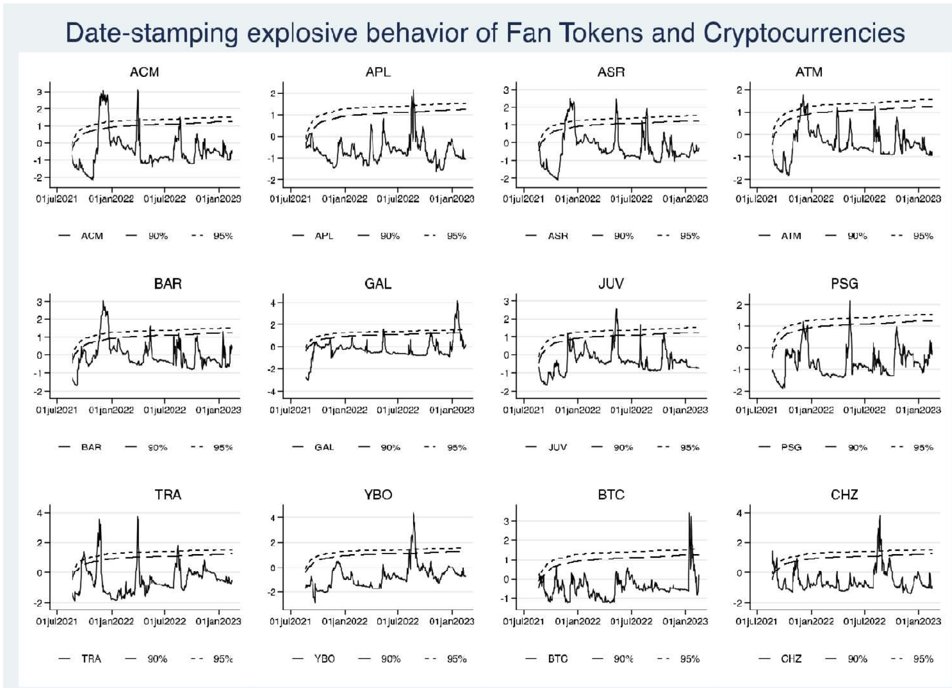

Figure 2: Date-stamping explosive behavior of Fan Tokens, Bitcoin and Chiliz. ADF test: solid line; 90% critical values: dashed line; 95% critical values: dotted line

period under examination. While explosive behavior in the Bitcoin has been documented extensively (see, for instance, Corbet, Lucey and Yarovaya (2018), Geuder, Kinateder and Wagner (2019), and Kyriazis, Papadamou and Corbet (2020) for a comprehensive review), to our knowledge, this is the first empirical evidence on exuberance in fan tokens behaviour.

We now turn to investigate the issue of asymmetric response of fan tokens volatility to price shocks. Table 4 and 5 report the results of the ML estimation of AR(1)-TGARCH and AR(1)-TGARCH-in-mean models, respectively.

First of all, let us consider the AR(1)-TGARCH models. The autoregressive component in the conditional mean equation for fan tokens returns is introduced following Katsiampa (2017) who finds a significant autoregressive coefficient in his comparative analysis of several GARCH models for the volatility of Bitcoin. However, differently from previous findings, we find a significant autoregressive coefficient only for the APL and YBO fan tokens and not for Bitcoin and Chiliz. The parameter driving the asymmetric response of volatility to price shocks, namely γ, is never statistically significant apart for the Bitcoin at the 10% significance level.

Table 4: ML estimation of AR(1)-TGARCH models, robust SE in parenthesis.

	ACM	*APL*	*ASR*	*ATM*	*BAR*	*GAL*	*JUV*	*PSG*	*TRA*	*YBO*	*BTC*	*CHZ*
α_0	0.000	-0.003	-0.001	-0.001	-0.002	-0.003	-0.001	-0.001	-0.002	-0.004*	-0.000	-0.001
	(0.002)	(0.003)	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)	(0.001)	(0.002)
α_1	-0.080	-0.152***	-0.085	0.073	-0.033	0.073	0.030	0.137**	0.042	-0.194**	0.006	-0.051
	(0.065)	(0.043)	(0.060)	(0.074)	(0.059)	(0.091)	(0.054)	(0.066)	(0.063)	(0.080)	(0.042)	(0.050)
α	0.350	0.073*	0.246*	0.090	0.282*	0.378**	0.352**	0.306**	0.158***	0.255*	0.243	0.142**
	(0.213)	(0.038)	(0.144)	(0.103)	(0.145)	(0.156)	(0.170)	(0.144)	(0.058)	(0.153)	(0.150)	(0.060)
γ	-0.132	0.015	0.041	0.189	-0.099	1.504	-0.043	0.396	0.221	0.280	-0.289*	0.131
	(0.182)	(0.045)	(0.150)	(0.175)	(0.139)	(0.929)	(0.180)	(0.287)	(0.176)	(0.409)	(0.153)	(0.105)
β	0.593***	0.870***	0.706***	0.599***	0.736***	0.001	0.597***	0.423***	0.702***	0.421	0.551***	0.638***
	(0.135)	(0.048)	(0.115)	(0.132)	(0.077)	(0.015)	(0.075)	(0.105)	(0.059)	(0.278)	(0.187)	(0.070)
ω	0.001**	0.000*	0.000	0.001**	0.000**	0.001***	0.000***	0.001***	0.000***	0.002*	0.000**	0.001***
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.001)	(0.000)	(0.000)
N	720	720	720	720	720	720	720	720	720	720	720	720

Table 5: ML estimation of AR(1)-TGARCH-in-mean models, robust SE in parenthesis

	ACM	*APL*	*ASR*	*ATM*	*BAR*	*GAL*	*JUV*	*PSG*	*TRA*	*YBO*	*BTC*	*CHZ*
α_0	0.005	-0.003	-0.000	0.008	-0.004	-0.003	0.001	-0.000	-0.002	-0.007**	0.002	-0.002
	(0.004)	(0.005)	(0.002)	(0.042)	(0.003)	(0.002)	(0.002)	(0.003)	(0.002)	(0.003)	(0.004)	(0.003)
ψ	-1.763	-0.052	-0.209	-2.469	0.845	0.111	-0.846	-0.213	-0.027	0.536	-2.326	0.149
	(1.366)	(0.817)	(0.503)	(12.389)	(1.008)	(0.118)	(0.637)	(0.968)	(0.890)	(0.336)	(3.350)	(0.523)
α_1	-0.086	-0.153***	-0.084	0.161	-0.042	0.067	0.034	0.141**	0.043	-0.204**	-0.011	-0.052
	(0.061)	(0.043)	(0.060)	(0.791)	(0.062)	(0.064)	(0.055)	(0.071)	(0.062)	(0.084)	(0.048)	(0.051)
α	0.351*	0.074*	0.245*	0.096	0.278**	0.369***	0.357**	0.306**	0.158***	0.262*	0.243	0.140**
	(0.194)	(0.038)	(0.143)	(0.091)	(0.141)	(0.139)	(0.175)	(0.142)	(0.058)	(0.145)	(0.160)	(0.062)
γ	-0.128	0.015	0.038	0.360	-0.091	1.551	-0.048	0.395	0.221	0.289	-0.290*	0.133
	(0.181)	(0.047)	(0.151)	(3.493)	(0.135)	(0.960)	(0.187)	(0.289)	(0.174)	(0.412)	(0.162)	(0.108)
β	0.584***	0.870***	0.708***	0.306	0.732***	-0.003	0.592***	0.420***	0.701***	0.407*	0.565***	0.641***
	(0.104)	(0.048)	(0.115)	(2.186)	(0.073)	(0.015)	(0.073)	(0.106)	(0.061)	(0.245)	(0.195)	(0.072)
ω	0.001***	0.000*	0.000	0.002	0.000**	0.001***	0.000***	0.001***	0.000***	0.002**	0.000**	0.001***
	(0.000)	(0.000)	(0.000)	(0.005)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.001)	(0.000)	(0.000)
N	720	720	720	720	720	720	720	720	720	720	720	720

This suggests that fan tokens are not characterised by the asymmetric behaviour in volatility which is indeed a well-known stylised fact for asset returns. The GARCH coefficient, namely β, is always significant apart for the GAL and YBO fan tokens, its order of magnitude ranges from 0.423 to 0.87 and it is line with the findings of Katsiampa (2017) for Bitcoin and Liu and Serletis (2019) for Bitcoin, Ethereum and Litecoin. Similarly, the ARCH coefficient is a significant for all fan tokens but the ACM and ATM ones. Overall, a symmetric GARCH model seems to be able to capture the volatility dynamics in fan tokens returns. Next, we consider whether volatility affects the conditional mean of returns by estimating AR(1)-TGARCH-in-mean for fan tokens returns. Results of ML estimation of this model are reported in Table 5. The coefficient capturing the influence of volatility on returns, namely ?, turns out not to be significant thereby excluding the possibility that an ARCH-in-mean model fits returns and volatility dynamics for fan tokens. As in Table 4 for the TGARCH models, the GARCH parameter is statistically significant for all fan tokens but the ATM one, confirming that lagged volatility affects current volatility. The coefficient associated to asymmetric effects is not statistically significant confirming that asymmetry does not characterize volatility dynamics. Overall, our results from estimation of various GARCH models suggest that an autoregressive component does not significantly enter in the conditional mean of fan tokens returns as well as the ARCH-in-mean one and that, on the one hand, the GARCH component is statistically significant while, on the other hand, asymmetric volatility effects are not present for most fan tokens.

Now we consider an alternative measure of asymmetry based on quantile autoregressive (QAR) as proposed by Baur and Dimpfl (2019).

Table 6: Asymmetry indicator from Quantile AutoRegression.

	Lower θ	*Upper* θ	δ
ACM	.098	-0.198	0.296
APL	.012	0.015	-0.003
ASR	-.0196	-0.194	-0.002
ATM	-0.104	-0.024	-0.080
BAR	0.035	0.192	-0.157
GAL	-0.09	0.058	-0.147
JUV	-0.085	0.168	-0.253
PSG	-0.069	0.046	-0.115
TRA	-0.09	0.101	-0.191
YBO	0.164	0.122	0.042
BTC	-0.222	-0.192	-0.030
CHZ	-0.111	0.074	-0.185

Table 6 presents the average of θ estimates for $\tau \in \{0.01, \ldots, 0.1\}$ (lower) and $\tau \in \{0.90, \ldots, 0.99\}$ (upper) quantiles along with the asymmetry indicator δ based on QAR(1) models. Negative estimates for lower quantiles and positive estimates for upper quantiles determine a majority of negative estimates for the difference, δ. These findings can be interpreted as evidence in favor of an inverted asymmetric effect, i.e., positive returns increase the volatility by more than negative returns. These findings from the estimation of QAR(1) models together with those obtained from the estimation of GARCH models suggest that if there is some asymmetry in volatility following a shock in returns, differently from the empirical evidence on stock returns the asymmetry in fan tokens works in an inverted fashion. This is consistent with several explanations. For instance, as pointed out by Baur and Dimpfl (2019), there are theoretical models where trading by uninformed agents will increase volatility whereas the reverse applies for trading by informed investors. Further, Avramov, Chordia and Goyal (2006) argue that such patterns are consistent with several behavioral biases such as the disposition effect, fear of missing out and pump and dump schemes (see Baur and Dimplf 2019 for details).

Finally, we turn to the analysis of connectedness. Table 7 contains the volatility spillovers, all results are obtained using a VAR(4) model.

On the main diagonal we can read the own-variance share of shocks while off-diagonal entries give the interaction among assets. In particular, the entry in row i and column *j* provides an estimate of the contribution to the forecast error variance of asset *i* coming from innovations in asset *j*. For instance, the intersection of the row "ASR" and column "CHZ" is equal to 2.8, this is the contribution to the forecast error variance of the ASR fan token coming from innovations in the cryptocurrency Chiliz. The difference between the "directional from others" and "directional to others" gives the so-called "net volatility spillovers". Asset *j* is a net receiver (transmitter) of shocks when the impact of asset *j* on others is smaller (larger) than the influence of all others on asset *j*.

Gross volatility spillovers from others are similar in magnitude (in the 60%-70% range) while gross volatility spillovers to others are somehow more spread out (in the range 50%-85%). Overall, the magnitude of volatility spillovers is quite large, both to and from. Finally, the total (non-directional) volatility spillover, i.e., the volatility forecast error variance originating from spillovers, which is given by the sum of all directional from others divided by the sum of directional to others (including own), is equal to 73.68%. Total volatility spillover is also quite sizeable. Most fan tokens are net transmitters of shocks, with APL, ASR, TRA and YBO being

Table 7: Volatility spillover table

	ACM	*APL*	*ASR*	*ATM*	*BAR*	*GAL*	*JUV*	*PSG*	*TRA*	*YBO*	*BTC*	*CHZ*	*FROM others*
ACM	25.48	1.90	10.69	12.55	7.06	5.56	10.74	9.21	6.12	3.27	3.67	3.75	74.52
APL	2.89	37.35	2.11	2.61	6.45	6.70	2.88	3.50	4.00	10.96	3.82	16.74	62.65
ASR	13.53	1.45	30.57	11.03	7.12	4.91	11.39	7.47	5.26	2.08	2.40	2.80	69.43
ATM	12.28	1.69	8.53	27.02	8.76	5.64	10.95	9.96	4.60	2.20	4.79	3.60	72.98
BAR	7.28	4.06	5.61	7.96	28.29	5.23	10.94	10.27	4.83	4.72	3.80	7.01	71.71
GAL	5.31	3.11	3.78	6.64	5.28	44.81	4.81	4.21	12.22	3.27	3.34	3.21	55.19
JUV	9.58	2.83	7.77	9.72	10.77	5.19	23.93	12.75	5.37	3.73	4.01	4.35	76.07
PSG	8.51	4.57	4.86	8.31	9.49	4.76	13.23	26.20	6.45	4.17	3.98	5.49	73.80
TRA	8.62	3.39	4.40	5.87	5.14	13.60	5.72	6.53	36.68	3.12	2.16	4.78	63.32
YBO	6.73	9.34	3.69	3.82	4.96	5.87	4.87	4.42	3.44	36.29	4.24	12.34	63.71
BTC	5.11	5.17	3.64	6.37	5.10	7.57	6.35	5.10	2.92	6.19	36.37	10.10	63.63
CHZ	4.37	12.11	2.96	4.06	6.65	5.39	4.85	5.66	4.92	10.69	7.49	30.86	69.14
TO others	84.19	49.60	58.03	78.96	76.78	70.41	86.73	79.07	60.12	54.40	43.69	74.16	68.01
NET connectedness	9.67	-13.05	-11.4	5.98	5.07	15.22	10.66	5.27	-3.2	-9.31	-19.94	5.02	
TO others (including own)	109.68	86.95	88.60	105.98	105.07	115.22	110.66	105.28	96.80	90.69	80.06	105.02	1200.00

the only net receivers of shocks. The Chiliz cryptocurrency is a net transmitter too.

Results in Table 7 provide an aggregate summary of the average volatility spillovers over the whole sample period but they might oversee significant trend and cyclical dynamics in spillovers. To achieve a better understanding of the dynamics in spillovers we estimate the model using a rolling window of 150 observations and plot the time series of the Total Connectedness Index and of the Net Volatility Spillovers, see Figure 3 and 4 respectively.

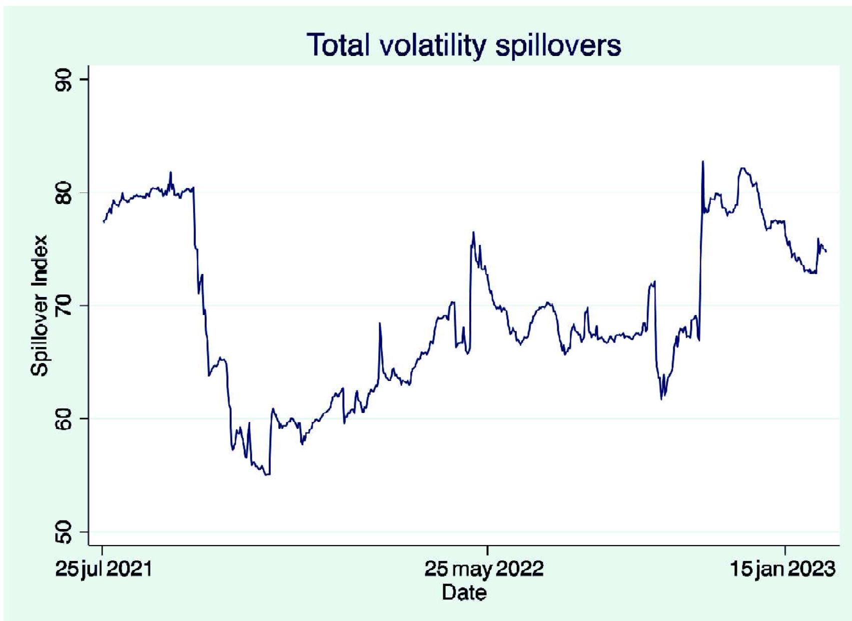

Figure 3: TOTAL dynamic volatility spillovers

Figure 3 on total connectedness reveals that it is higher at the beginning and at the end of the sample period with an episode of high connectedness around the end of the national league and European competitions approximately in May 2022.

Figure 4 on net total connectedness shows the difference between TO and FROM spillovers of each asset considering the entire set of assets under examination. A positive shaded area indicates that, for the specific date, the asset is a net transmitter to others while a negative shaded area indicates that the asset is a net receiver from others. BAR and JUV, and ACM and ATM to a lesser extent, are net transmitters to others, TRA is a net receiver. ASR is first a net transmitter and then a net receiver, the converse occurs for APL, while more erratic behavior can be associated to the remaining fan

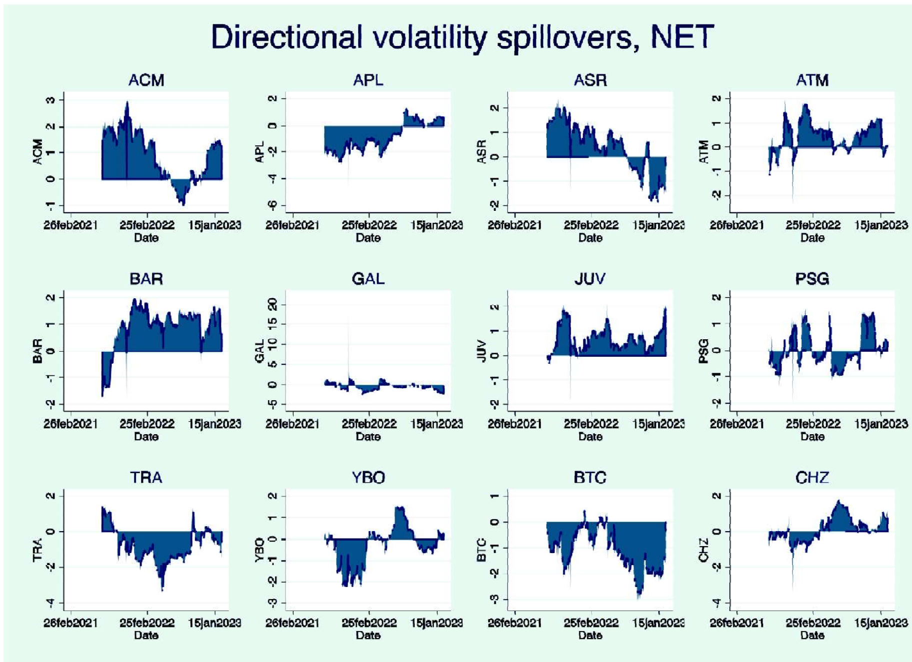

Figure 4: NET dynamic volatility spillovers

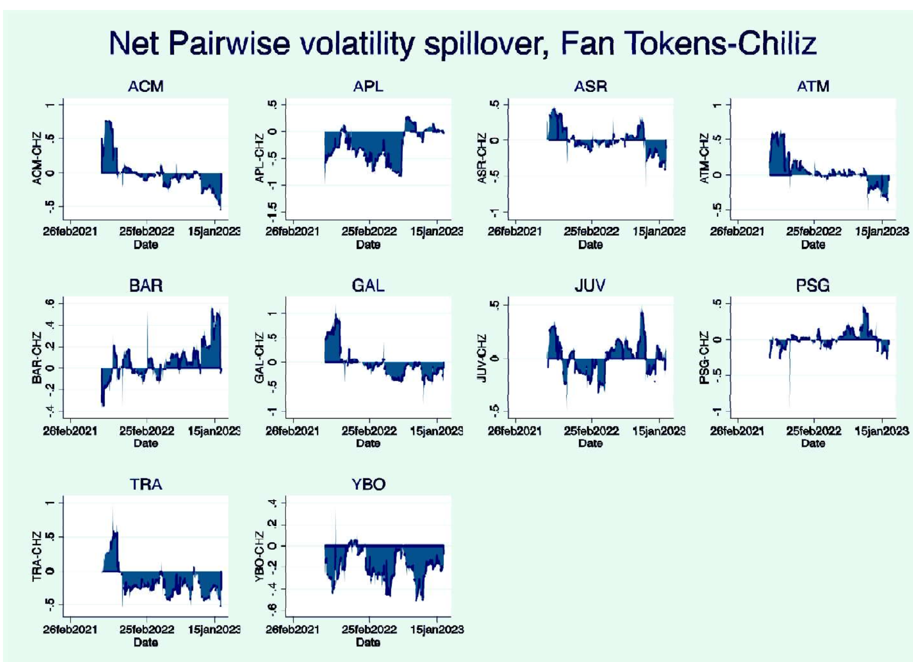

Figure 5: Net pairwise volatility spillover, Fan Tokens-Chiliz

tokens. It seems that most famous and winner clubs such BAR and JUV act as net transmitter while the others follow. Surprisingly, Bitcoin is a net receiver form others and Chiliz starts as a net receiver to become a net transmitter of shocks to the whole set of assets. Next, in Figure 5 we investigate in deeper detail the net pairwise connectedness between fan tokens and Chiliz, the cryptocurrency fan tokens are minted. ACM, ATM, GAL and TRA are net transmitters of shocks to Chiliz at the beginning of the sample period and become net receivers towards the end of the period. ASR, BAR, JUV and PSG switch from net receivers to net transmitters multiple times during the period and finally YBO is almost always a net receiver. Therefore, there is no clear-cut evidence that fan tokens are mainly net receivers or net transmitters to the cryptocurrency they are based upon.

4. Final remarks

In this paper we investigated some empirical properties of fan tokens issued by 10 football clubs and to what extent some empirical regularities of cryptocurrencies carry over to fan tokens. Fan tokens represent a financial innovation that allows football clubs to monetise their fan base without making any changes to their corporate and ownership structure and, therefore, without being subject to the legal constraints that the issue of financial products would otherwise entail. To be precise, those who buy fan tokens do not buy shares or debt or any other financial instruments of the sports club but only services and, say, some "influence". Given that the phenomenon is rather recent, the sample period considered runs from February 2021 to February 2023. The empirical characteristics investigated concerns the presence of episodes of exuberance, the pervasiveness of an asymmetric response of volatility to price changes and the connectedness among fan tokens and two cryptocurrencies, namely Bitcoin which is the first and best known one and Chiliz which is the cryptocurrency fan tokens are based upon. Our findings suggest that fan tokens do indeed exhibit episodes of bubbles or exuberance, further the methodology applies in the analysis has allowed us to date-stamp those episodes. As far as asymmetry in volatility after a price shock, results are mixed in the sense that the kind of asymmetry typically found in the empirical literature on volatility models is not confirmed but, conversely, there is evidence of some reverse asymmetric effect which can be interpreted as the result of the presence of uninformed traders (which may indeed be the case given the nature of fan tokens where the fact that it is a financial instrument is often overlooked by fan tokens owners). Finally, we provide interesting evidence of volatility connectedness among fan tokens. The total connectedness index is quite huge and much greater than the values usually found in the empirical literature on connectedness among stocks, bonds, Forex and commodities. Also, "from" and "to" directional connectedness are sizeable. "Net" and "net pairwise" directional spillovers suggest that some fan tokens are net transmitters of shocks to the whole system, while for most of them there are several reversals from net transmitters to net receivers and vice versa.

A possible direction of future research would be to extend this analysis to fan tokens of national teams or in other sports, such as basketball or baseball. It would also be interesting, but it is of course beyond the scope of this paper, to design a survey among fan tokens holders to analyze the individual motivation behind the choice to hold an immaterial asset which does not guarantee any monetary reward and the relation of this investment choice to the holder's socio-demographic characteristics, including her/his financial literacy and the time spent in personal finance decisions.

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