*Asian Journal of Statistical Sciences*
Vol. 2, No-1, (2022), pp. 01-35

ARF INDIA
Academic Open Access Publishing
Journal homepage link:
http://arfjournals.com/ajss

# Statistics and Mathematics of General Control Function Optimization for Continuous Cover Forestry, with a Swedish Case Study based on Picea abies

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**ARTICLE HISTORY**
Compiled August 18, 2022

Received 27 March 2022; Accepted 15 June 2022

**ABSTRACT**
Continuous cover forests contain large numbers of spatially distributed trees of different sizes. The growth of a particular tree is a function of the properties of that tree and the neighbor trees, since they compete for light, water and nutrients. Such a dynamical system is highly nonlinear and multidimensional. In this paper, a particular tree is instantly harvested if a control function based on two local state variables, S and Q, is satisfied, where S represents the size of the particular tree and Q represents the level of local competition. The control function has two parameters. An explicit nonlinear present value function, representing the total value of all forestry activities over time, is defined. This is based on the parameters in the control function, now treated as variables, and six new parameters. Explicit functions for the optimal values of the two parameters in the control function are determined via optimization of the present value function. Two equilibria are obtained, where one is a unique local maximum and the other is a saddle point. An equation is determined that defines the region where the solution is a unique local maximum. Then, a case study with a continuous cover Picea abies forest, in southern Sweden, is presented. A new growth function is estimated and used in the simulations. The following procedure is repeated for five alternative levels of the interest rate: The total present value of all forest management activities in the forest, during 300 years, is determined for 1000 complete system simulations. In each system simulation, different random combinations of control function parameters are used and the total present value of all harvest activities is determined. Then, the parameters of the present value function are estimated via multivariate regression analysis. All parameters are determined with high precision and high absolute t-values. The present value function fits the data very well. Then, the optimal control function parameters and the optimal present values are analytically determined for alternative interest rates. The optimal solutions found within the relevant regions are shown to be unique maxima and the solutions that are saddle points are located far outside the relevant regions.

## 1. Introduction

Continuous cover forests, CCF, can be found in most parts of our world. Such forests may over time be partly harvested. All the time, however, living trees grow there. In typical cases, these trees also give seeds and new seedlings. In this way, the forest has

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a continuous tree cover. This paper is focusing on optimization of the decision rules to be used in continuous cover forest management.

In earlier research on optimal CCF management, several interesting results have been reported. However, the level of analysis has not made it possible to investigate optimal harvest decisions for each tree. The reason is that, when we optimize the decisions to be made with respect to a particular tree, we have to know the exact spatial structure of the forest. We have to know if the particular tree is surrounded by several other trees or if it has a lot of free space. Usually, which can be proved, it is rational to let a tree without competitors continue to grow longer, than a tree that has several competitors.

Pukkala *et. al* (2010), optimized the steady-state structure and associated management of uneven-sized Scots pine and Norway spruce stands, in Finland. This was done based on the tree diameter distribution and partial harvesting with fixed time intervals. In such a study, it was of course not possible to determine the optimal treatment for each tree, based on tree properties and local competition. Nevertheless, quite interesting results could be derived. For instance, it was found that, in most cases, uneven-sized management was more profitable than even-aged management, with simultaneous harvests of all trees. Similar results were also found and reported by Tahvonen et al (2010). They applied a very general growth model and could study changes of tree size distributions over time. However, the individual tree properties, local competition and associated optimal decisions were not covered. R¨am¨o and Tahvonen (2014) studied the CCF problems with more detailed models. Several interesting numerical results followed. Still, however, the decisions were not optimized for each individual tree.

Thanks to new growth functions for individual trees, and thanks to spatially explicit numerical simulations, we can now optimize decisions at a more detailed level. In order to manage CCF forests, it is of fundamental importance to understand and to be able to predict how the trees grow in uneven- aged forest stands. Fagerberg et al (2021) and (2022) are recent and detailed investigations of this topic. Several alternative growth models for individual trees are estimated, compared and tested. One of these models, developed and reported in Lohmander (2017), is one such example. That model was extended by Hatami et al (2018) to handle competition in multi species forest stands in the Caspian forests. The model has also been extended and the new parameters have been determined, to handle competition between trees in spruce forests in southern Sweden, as reported and evaluated by Fagerberg et al (2021) and (2022). This particular version of the model is also used in the analysis presented in this paper.

Continuous cover forests contain large numbers of spatially distributed trees of different sizes. The growth of a particular tree is a function of the properties of that tree and the neighbor trees, since they compete for light, water and nutrients. Such a dynamical system is highly nonlinear and multidimensional. Lohmander (2018), (2019a) and (2019b) shows how dynamic management of such systems can be optimized, via optimal control functions, dynamically applied to each tree. Lohmander (2021) applies a related method to optimize the management of a dynamic multi species system with animals.

In this paper, the ambition is to really optimize the decisions in CCF forests, for each tree, based on detailed information about tree properties and local competition. A general mathematical approach will be used and concrete numerical analysis, based on CCF forest management decisions, empirical data and estimated functions from Fagerberg et al (2022).

## 2. Materials and methods

In this analysis, a particular tree is instantly harvested if a control function based on two local state variables, S and Q, is satisfied, where S represents the size of the particular tree and Q represents the level of local competition. The control function has two parameters. An explicit nonlinear present value function, representing the total value of all forestry activities over time, is defined. In typical cases, this represents the expected present value of forestry, per hectare, in the region. Of course, if the expected present value of forestry should be numerically specified and calculated, it is necessary to have access to a large number of particular parameters, representing growth functions, interest rates, price functions, cost functions, etc.
In this first phase of the analysis, we can however leave the empirical details. It is sufficient to investigate the problem in the following general form: The value function is based on the parameters in the control function, now treated as variables, and six new parameters. Then, explicit functions for the optimal values of the two parameters in the control function are determined via optimization of the present value function. A particular tree is instantly harvested if a control function, C(.), based on two local state variables, S and Q, is satisfied, where S represents the size of the particular tree and Q represents the level of local competition. The control boundary, B, has two parameters (x and y).

$$B = x + yQ \qquad x > 0,\ y > 0 \tag{1}$$

$$C(S,Q) = S - B(Q; x, y) \tag{2}$$

$$C(S,Q) = S - (x + yQ) \tag{3}$$

$$C(S,Q) = S - x - yQ \tag{4}$$

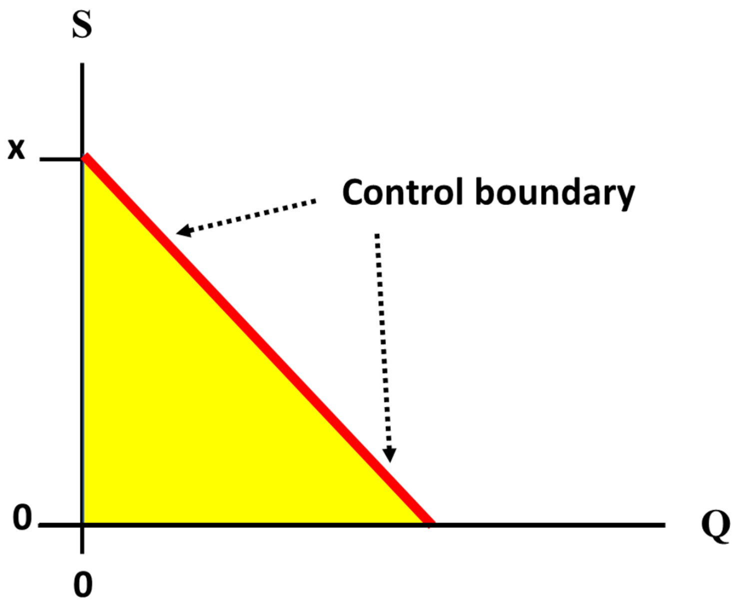


**Figure 1**
Illustration of the control function C(S,Q) via the control boundary: A particular tree should instantly be harvested if a control function based on two local state variables, S and Q, is satisfied. S represents the size of the particular tree and Q represents the level of local competition. The control boundary, B, has two parameters, x and y. y is the slope, $\frac{dS}{dQ}$, of the control boundary. In case the local state, (S, Q), is found in the yellow sector, below and to the left of the control boundary, then the particular tree should not be instantly harvested. In case the local state is found in the white sector, above and to the right of the control boundary, then the particular tree should be instantly harvested. If the state is located exactly on the control boundary, then both decisions can be considered optimal.

$$\begin{Bmatrix} S > x + yQ \\ S = x + yQ \\ S < x + yQ \end{Bmatrix} \Rightarrow \begin{Bmatrix} C(S,Q) > 0 \\ C(S,Q) = 0 \\ C(S,Q) < 0 \end{Bmatrix} \Rightarrow \begin{Bmatrix} Harvest \\ Harvest\ or\ Wait \\ Wait \end{Bmatrix} \tag{5}$$

Special case and optimal resource distribution interpretation:

We may define S, the size of the tree, as the diameter, D, at "breast height", 1.3 meters above ground. Then, we may interpret B as the diameter limit, DL.

If D> DL, then the tree should be instantly harvested.

If D< DL, then we should wait. The tree should not yet be harvested. It should continue to grow until it reaches DL.

If D = DL, then we may harvest the tree or wait longer. With a constantly growing tree, this special case however occurs only during a time interval of length zero.

DL is a decreasing function of local competition.

This is reasonable, since growth resources such as nutrients, light and water are locally constrained. The particular tree negatively affects the growth of the competitors and the competitors negatively affect the growth of the particular tree. In order to avoid these negative competition effects, the trees with more local competition should be harvested when the diameters are smaller than the DL determined with less competition.

The present value function:

An explicit nonlinear present value function, (6), representing the total value of all forestry activities over time, is suggested.

$$f(x, y) \tag{6}$$

This is based on the parameters in the control function, x and y, now treated as variables, and six new parameters.

$$f(x, y) = k + ax - bx^{1.5} + cy - gy^2 - hxy \tag{7}$$

$$(a, b, c, g, h > 0) \tag{8}$$

The functional form is motivated the following way: If there are no exogenous disturbances, such as competition, f(.) is a strictly concave function of x. Then, since y does not influence the decisions, the relevant function can be simplified to (9).

$$f_1(x) = f(x, y)|_{y=0} = k + ax - bx^{1.5} \tag{9}$$

$$\frac{df_1(x)}{dx} = a - 1.5bx^{0.5} \tag{10}$$

$$\frac{d^2 f_1(x)}{dx^2} = -0.75bx^{-0.5} \tag{11}$$

$$(x > 0) \Rightarrow \frac{d^2 f_1(x)}{dx^2} < 0 \tag{12}$$

Hence, if there are no exogenous disturbances, such as competition, then f(.) is a strictly concave function of x, for strictly positive values of x. The optimal value of x is unique and maximizes f. The first order optimum condition is:

$$\frac{df_1(x)}{dx} = a - 1.5bx^{0.5} = 0 \tag{13}$$

As a result, we get:

$$1.5bx^{0.5} = a \tag{14}$$

$$x^{0.5} = \frac{2a}{3b} \tag{15}$$

$$0 < x = \frac{4a^2}{9b^2} < \infty \tag{16}$$

This optimal value is an explicit function of a and b. It is strictly greater than 0 and strictly less than infinity. When there is competition, however, we also have to determine the value of y. This value is in general different from zero. The optimal value of y is unique and dependent on x. In fact, x and y should then be simultaneously optimized. The equations (17) – (19) show this fact. The first order optimum condition with respect to y is (17).

$$\frac{df(x,y)}{dy} = c - 2gy - hx = 0 \tag{17}$$

The second order maximum condition (18) is always satisfied:

$$\frac{d^2 f(x,y)}{dy^2} = -2g < 0 \tag{18}$$

The optimal value of y is unique and dependent on x:

$$\left(\frac{df(x,y)}{dy} = 0\right) \Rightarrow \left(y = \frac{c - hx}{2g}\right) \tag{19}$$

In order to get a perspective on the objective function, we may study the optimal value of x as a function of an exogenously determined value of y. Equations (20) to (28) show that the optimal value of x is a strictly decreasing function of y. We may consider the objective function to be a mountain, with a mountain ridge, which is parallel to an arrow with a direction such that dx/dy < 0. This is found from the following equations. A 3 - dimensional map of the objective function, with such a mountain ridge, is shown in Figure 2.

$$\frac{df(x,y)}{dx} = a - 1.5bx^{0.5} - hy = 0 \tag{20}$$

$$a - hy = 1.5bx^{0.5} \tag{21}$$

$$x^{0.5} = \frac{a - hy}{1.5b} \tag{22}$$

$$(x > 0) \Rightarrow (a - hy > 0) \tag{23}$$

$$\sqrt{x} = \frac{2(a - hy)}{3b} \tag{24}$$

$$x = \frac{4(a-hy)^2}{9b^2} \tag{25}$$

$$x = \left(\frac{4}{9b^2}\right)(a^2 - 2ahy + h^2y^2) \tag{26}$$

$$\frac{dx}{dy} = \left(\frac{4}{9b^2}\right)(-2ah + 2h^2y) \tag{27}$$

$$\frac{dx}{dy} = \left(\frac{4}{9b^2}\right)(-2h)(a-hy) < 0 \tag{28}$$

$$(>0) \quad (<0) \quad (>0)$$

We may also determine the sign of (28) via total differentiation of the first order optimum condition. This is done via the equations (29) to (37)

$$\frac{df(x,y)}{dx} = a - 1.5bx^{0.5} - hy = 0 \tag{29}$$

$$d\left(\frac{df(x,y)}{dx}\right) = \frac{d^2f}{dx^2}dx + \frac{d^2f}{dxdy}dy = 0 \tag{30}$$

$$\frac{d^2f}{dx^2}dx = -\frac{d^2f}{dxdy}dy \tag{31}$$

$$\frac{dx}{dy} = \frac{\left(\frac{d^2f}{dxdy}\right)}{\left(\frac{d^2f}{dx^2}\right)} \tag{32}$$

$$\frac{df(x,y)}{dx} = a - 1.5bx^{0.5} - hy = 0 \tag{33}$$

$$\frac{d^2f}{dx^2} = -0.75bx^{-0.5} < 0 \tag{34}$$

$$\frac{d^2f}{dxdy} = -h < 0 \tag{35}$$

$$\frac{dx}{dy} = \frac{(h)}{(-0.75bx^{-0.5})} \tag{36}$$

$$\frac{dx}{dy} = \frac{-4h\sqrt{x}}{3b} < 0 \tag{37}$$

**Alternative functional forms**

The following functional form, (38), was found to have reasonable qualitative properties

$$f(x,y) = k + ax - bx^{1.5} + cy - gy^2 - hxy \tag{38}$$

However, there are other functional forms that also give reasonable qualitative results. Equation (39) is one such example:

$$f(x,y) = k + ax - bx^2 + cy - gy^2 - hxy \tag{39}$$

Regression analyses based on empirical data however showed that the functional form (38) fits the empirical data better than the alternative function (39). Hence, the alternative function was not selected.

**The case of only one free variable**

If x is a free variable and there is no competition, we may select y = 0 without influencing the decisions.

$$\frac{df_1(x)}{dx} = a - 1.5bx^{0.5} = 0 \tag{40}$$

Then, the first order optimum condition gives the following solution:

$$\left(\frac{df_1(x)}{dx} = 0\right) \Rightarrow \left(\sqrt{x} = \frac{2a}{3b}\right) \tag{41}$$

The optimal value of x is strictly positive:

$$x = \frac{4a^2}{9b^2} > 0 \tag{42}$$

This solution is a unique maximum, since:

$$\frac{d^2 f_1(x)}{dx^2} = \frac{-3b}{4\sqrt{x}} \tag{43}$$

$$(x > 0) \Rightarrow \frac{d^2 f_1(x)}{dx^2} < 0 \tag{44}$$

**The case of two free variables**

If x and y are free variables, then x and y should be determined from the two first order optimum conditions found in (45).

$$\begin{cases} \frac{df}{dx} = a - 1.5b\sqrt{x} - hy = 0 \\ \\ \frac{df}{dy} = c - hx - 2gy = 0 \end{cases} \tag{45}$$

From the first order conditions, we get:

$$\left(\frac{df}{dy} = 0\right) \Rightarrow \left(y = \frac{c - hx}{2g}\right) \tag{46}$$

Substitution gives (47).

$$\left(\frac{df}{dx} = 0 \wedge \frac{df}{dy} = 0\right) \Rightarrow \left(a - 1.5\sqrt{x} - h\left(\frac{c - hx}{2g}\right) = 0\right) \tag{47}$$

$$a - 1.5b\sqrt{x} - \frac{ch}{2g} + \frac{h^2 x}{2g} = 0 \tag{48}$$

$$\frac{h^2}{2g}x - \frac{3bg}{2g}\sqrt{x} + \frac{2ag - ch}{h^2} = 0 \tag{49}$$

$$(g \neq 0) \Rightarrow (h^2 x - 3bg\sqrt{x} + (2ag - ch) = 0) \tag{50}$$

Via extended substitution, we get (51).

$$(z = \sqrt{x}) \Rightarrow (h^2 z^2 - 3bgz + (2ag - ch) = 0) \tag{51}$$

Now, it becomes possible to determine the optimal solution via the famous Quadratic Equation.

$$(h \neq 0) \Rightarrow \left(z^2 - \frac{3bg}{h^2} + \frac{2ag - ch}{h^2} = 0\right) \tag{52}$$

$$p = -\frac{3bg}{h^2} \wedge q = \frac{2ag - ch}{h^2} \tag{53}$$

$$z^2 + pz + q = 0 \tag{54}$$

$$z = -\frac{p}{2} \pm \sqrt{\left(\frac{-p}{2}\right)^2 - q} \tag{55}$$

$$z = \frac{3bg}{2h^2} \pm \sqrt{\left(\frac{3bg}{2h^2}\right)^2 + \frac{ch - 2ag}{h^2}} \tag{56}$$

From now on, we restrict the attention to the case with two real solutions. That is also what we get with empirically relevant parameters.

$$z = \frac{3bg \pm \sqrt{9b^2g^2 + 4ch^3 - 8agh^2}}{2h^2} \tag{57}$$

There are to solutions, explicitly presented in (58) and (59).

$$z_1 = \frac{3bg - \sqrt{9b^2g^2 + 4ch^3 - 8agh^2}}{2h^2} \tag{58}$$

$$z_2 = \frac{3bg + \sqrt{9b^2g^2 + 4ch^3 - 8agh^2}}{2h^2} \tag{59}$$

The two different z solutions to the first order optimum conditions can be transformed to corresponding x and y values, as seen in (60) and (61). We have to investigate if one of these is a maximum. It is also interesting to determine the properties of the other solution.

$$(x_1, y_1) = \left((z_1)^2, \frac{c - h(z_1)^2}{2g}\right) \tag{60}$$

$$(x_2, y_2) = \left((z_2)^2, \frac{c - h(z_2)^2}{2g}\right) \tag{61}$$

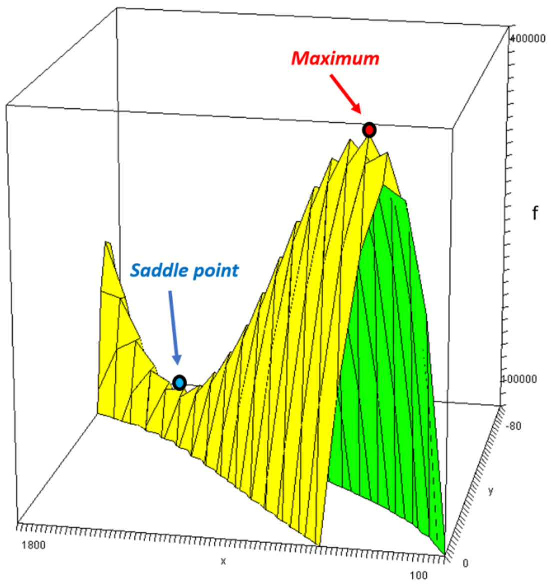

**Figure 2**

The graph shows the objective function "mountain", f, when r = 1%. Notice that the "mountain ridge" is parallel in the (x,y) plane, to an arrow with a direction $\frac{dx}{dy} < 0$. Obviously, there are two stationary points. One of these is a unique local maximum. The other is a saddle point.

**A more detailed analysis of the nature of the equilibria**

Two equilibria are obtained. One is a local maximum and the other is a saddle point.

We already know the first order optimum conditions:

$$\begin{cases} \frac{df}{dx} = a - \frac{3b}{2}\sqrt{x} - hy = 0 \\ \\ \frac{df}{dy} = c - hx - 2gy = 0 \end{cases} \tag{62}$$

The second order derivatives give us:

$$\left[D\right] = \begin{bmatrix} \frac{d^2f}{dx^2} & \frac{d^2f}{dxdy} \\ \\ \frac{d^2f}{dydx} & \frac{d^2f}{dy^2} \end{bmatrix} = \begin{bmatrix} -\frac{3b}{4\sqrt{x}} & -h \\ \\ -h & -2g \end{bmatrix} \tag{63}$$

**Second order conditions of a unique local maximum:**

Two strict inequalities, (64) and (65), should be satisfied, if the solution is a unique local maximum.

The first inequality, (64), is satisfied everywhere, for x > 0 and all y.

$$\left|D_1\right| = \left|\frac{d^2f}{dx^2}\right| = \left|-\frac{3b}{4\sqrt{x}} < 0\right| \tag{64}$$

The second inequality, (65), is not satisfied for all values of x. The limiting value of x will be determined from the inequality.

$$\left|D_2\right| = \begin{vmatrix} \frac{d^2f}{dx^2} & \frac{d^2f}{dxdy} \\ \\ \frac{d^2f}{dydx} & \frac{d^2f}{dy^2} \end{vmatrix} = \begin{vmatrix} -\frac{3b}{4\sqrt{x}} & -h \\ \\ -h & -2g \end{vmatrix} = \frac{3bg}{2\sqrt{x}} - h^2 > 0 \tag{65}$$

$$\frac{3bg}{2\sqrt{x}} - h^2 > 0 \tag{66}$$

$$3bg > 2h^2\sqrt{x} \tag{67}$$

$$2h^2\sqrt{x} < 3bg \tag{68}$$

$$\sqrt{x} < \frac{3bg}{2h^2} \tag{69}$$

$$x < \frac{9b^2g^2}{4h^4} \tag{70}$$

The inequality (70) gives the limiting value of x. Below that limit, we have a maximum. Above the limit, we have a saddle point. Compare Figure 3.

**General Observations**

It has been found that two equilibria exist, where one is a unique local maximum and the other is a saddle point. An equation is determined that defines the region where the solution is a unique local maximum.

## 3. Result

The general mathematical findings developed in the earlier section have been applied to a CCF case study. Below, numerical results from this analysis, with a continuous cover Picea abies forest in southern Sweden, is presented. An individual tree growth function, which is an extended version of Lohmander (2017), developed in Fagerberg at al (2021) and tested in Fagerberg et al (2022), was used in the simulations.
A spatially and dynamically explicit simulation model was created. This is found in the Appendix. This made it possible to follow the development of individual trees, during 300 years. The initial positions and properties of the trees were collected from a real forest area. In the simulation model, during each period of five years, the growth of each tree was determined as a function of the tree properties and the competition from neighbor trees. In a particular period, a tree was harvested if the control function gave that instruction. Compare Figure 1 and equation (5). The control function was based on information about the individual tree and the local competition. If the tree was not harvested, it continued to grow until the next period. Each period, new seedlings started to grow in random locations.
The following procedure was repeated for five alternative levels of the interest rate: The total present value of all forest management activities in the forest, during 300 years, was determined for 1000 complete forest system simulations. In each simulation, different random combinations of control function parameters were used and the total present value of all harvest activities was determined. In this process, cost functions, other functions and parameters were obtained from Fagerberg (2021). Then, the parameters of the present value function were estimated via multivariate regression analysis. All parameters were determined with high precision and high absolute t-values. The estimated present value function was found to fit the data very well. Then, the optimal control function parameters and the optimal present values were analytically determined for alternative interest rates. The optimal solutions found within the relevant regions are shown to be unique maxima and the solutions that are saddle points are located far outside the relevant regions.

**The definition of competition in the control function and control boundary:**

The competition was defined as the total basal area of competing trees, square meters per hectare, within a circle with radius 10 meters. The subject tree was in the center of the circle. With that definition of competition, all parameters of relevance to the determination of the optimal control function could be determined with high precision and high absolute t-values. Compare the values reported in Tables 1, 2 and 3.

It could be practically convenient and perhaps cost efficient with a simpler definition of competition. Time could perhaps be saved if we did not have to investigate the basal area of the competitors. Let us investigate how the results change if we temporarily redefine competition and only consider information about the "closest and biggest neighbor tree". Hence, we temporarily redefined competition the following way: We only considered the competition from the neighbor tree with the largest value of the ratio (Tree diameter) / (Distance to Subject tree). For each subject tree, all neighbor trees were investigated. The local competition of relevance to a particular subject tree, was then temporarily redefined as the maximum value of this ratio. In this test, we also selected the rate of interest 2%. With the redefined competition, the p-values of the parameters c, g and h were approximately 0.116, 0.352 and 0.182 respectively. Normally, in statistical analyses, p-values should, at least, be below 0.05 in order to be considered significant. The redefined competition gave much higher p-values than the original definition. Thus, the estimated parameter values of redefined competition were very unreliable. Furthermore, the estimated signs of the parameters differed from the corresponding signs reported in Table 1., in two of the three cases. The p-values of the parameters in Table 1. are extremely low. Hence, we have no reason to believe in the signs and/or the parameter values, determined with the redefined competition. Observation concerning the definition of competition: The redefined competition did not give satisfactory t-values and p-values of the estimated parameters. This is understandable, since if we only consider the competition from the biggest and/or closest competitor, we do not consider all of the other trees in the local area, that also contribute to the total level of competition.

Table 1.

t-values and p-values in five different regression analyses, with different rates of interest in the objective function. In the table, the rows "k, x, $x^{1.5}$, y, $y^2$ and xy", correspond to the parameters "k, a, -b, c, -g and -h". The follows from the functional form of the objective function, namely

$$f(x,y) = k + ax - bx^{1.5} + cy - gy^2 - hxy$$

| r% | 1 | 1,5 | 2 | 2,5 | 3 |
|---|---|---|---|---|---|
| | | | t- values in the regression analyses | | |
| k | -8,97041 | -6,11472 | -1,14039 | 6,363739 | 16,74447 |
| x | 27,66237 | 34,24437 | 41,18366 | 45,60594 | 46,64495 |
| $x^{1.5}$ | -30,5576 | -38,3753 | -46,6355 | -51,7893 | -52,7345 |
| y | 32,01628 | 35,58006 | 38,24762 | 37,00359 | 32,11392 |
| $y^2$ | -25,4535 | -27,9865 | -28,9649 | -26,5497 | -21,8865 |
| xy | -53,3308 | -62,2809 | -69,5523 | -69,3717 | -62,1029 |
| | | | p- values in the regression analyses | | |
| k | 1,44E-18 | 1,39E-09 | 0,2544 | 3E-10 | 1,25E-55 |
| x | 2,3E-125 | 2,2E-170 | 4,1E-217 | 6,4E-246 | 1,5E-252 |
| $x^{1.5}$ | 3,9E-145 | 2,2E-198 | 1,7E-252 | 1,5E-284 | 2,8E-290 |
| y | 3,9E-155 | 1,8E-179 | 1,6E-197 | 3,9E-189 | 8,4E-156 |
| $y^2$ | 1,9E-110 | 1,4E-127 | 3,1E-134 | 8E-118 | 5,6E-87 |
| xy | 7,2E-294 | 0 | 0 | 0 | 0 |

**Determination of the optimal control function parameter values:**

Now, the optimal control function parameters, and as a consequence, the optimal present values, are analytically determined for alternative interest rates. In this process, we use the parameter estimates from the regressions. The optimal solutions found within the relevant regions are shown to be unique maxima and the solutions that are saddle points are located far outside the relevant regions. The regression analysis gave the parameter values reported in Table 2.

Table 2.

Estimated parameter values in five different regression analyses, with different rates of interest in the objective function. In the table, the rows "k, x, $x^{1.5}$, y, $y^2$ and xy", correspond to the signed parameters "k, a, -b, c, -g and -h". This follows from the functional form of the objective function, namely

$$f(x, y) = k + ax - bx^{1.5} + cy - gy^2 - hxy$$

Compare Table 3.

| r% | 1 | 1,5 | 2 | 2,5 | 3 |
|---|---|---|---|---|---|
| k | -235946,7178 | -78658,858127 | -8370,1564967 | 30756,29305 | 58298,89751 |
| x | 5501,829188 | 3555,596824 | 2616,173465 | 2056,220151 | 1642,834135 |
| $x^{1.5}$ | -202,9194332 | -137,408258 | -105,7591839 | -86,51339767 | -71,63220229 |
| y | 31104,53869 | 18364,61462 | 12331,92025 | 8681,028154 | 6064,113391 |
| $y^2$ | -1255,699477 | -769,3098941 | -522,3284433 | -366,2971758 | -255,8380047 |
| xy | -114,2172637 | -74,32288849 | -54,44982093 | -41,549852 | -31,51469255 |

From the results reported in Table 2., the parameter values of the objective function could be determined. These are reported in Table 3.

Table 3.

Estimated parameter values with different rates of interest in the objective function,

$$f(x,y) = k + ax - bx^{1.5} + cy - gy^2 - hxy$$

Compare Table 2.

| r% | 1 | 1,5 | 2 | 2,5 | 3 |
|---|---|---|---|---|---|
| a | 5501,829188 | 3555,596824 | 2616,173465 | 2056,220151 | 1642,834135 |
| b | 202,9194332 | 137,408258 | 105,7591839 | 86,51339767 | 71,63220229 |
| c | 31104,53869 | 18364,61462 | 12331,92025 | 8681,028154 | 6064,113391 |
| g | 1255,699477 | 769,3098941 | 522,3284433 | 366,2971758 | 255,8380047 |
| h | 114,2172637 | 74,32288849 | 54,44982093 | 41,549852 | 31,51469255 |

**Optimal results when x and y are free variables:**

Now, we can determine the empirically relevant optimal control function parameters x and y. We remember that the first order optimum conditions are:

$$\begin{cases} \frac{df}{dx} = a - \frac{3b}{2}\sqrt{x} - hy = 0 \\ \\ \frac{df}{dy} = c - hx - 2gy = 0 \end{cases} \tag{71}$$

As we saw in the earlier section, there are two solutions to this equation system, namely:

$$(x_1, y_1) = \left( (z_1)^2, \frac{c - h(z_1)^2}{2g} \right) \tag{72}$$

$$(x_2, y_2) = \left( (z_2)^2, \frac{c - h(z_2)^2}{2g} \right) \tag{73}$$

Hence, we need the alternative values of z, reported in equations (74) and (75).

$$z_1 = \frac{3bg - \sqrt{9b^2g^2 + 4ch^3 - 8agh^2}}{2h^2} \tag{74}$$

$$z_2 = \frac{3bg + \sqrt{9b^2g^2 + 4ch^3 - 8agh^2}}{2h^2} \tag{75}$$

When we calculate these values, we may use the following steps:

$$z = -\frac{p}{2} \pm \sqrt{\left(\frac{-p}{2}\right)^2 - q} \tag{76}$$

$$p = -\frac{3bg}{h^2} \wedge q = \frac{2ag - ch}{h^2} \tag{77}$$

The numerical values of relevance to equations (74) – (77) are reported in Table 4.

Table 4.

These values will be used to derive the results in Table 5.

| r% | 1 | 1,5 | 2 | 2,5 | 3 |
|---|---|---|---|---|---|
| p | -58,59588331 | -57,41035693 | -55,89720722 | -55,06799078 | -55,35648026 |
| q | 786,8266608 | 743,280392 | 695,3401909 | 663,6273957 | 653,9528201 |
| Discriminant | 71,54272451 | 80,70687872 | 85,78425297 | 94,49350648 | 112,1321566 |
| $z_1$ | 20,83964842 | 19,72147774 | 18,68662472 | 17,81321834 | 17,08899291 |
| $z_2$ | 37,75623489 | 37,68887919 | 37,2105825 | 37,25477244 | 38,26748735 |

The optimal values of x, y and the objective function, reported in Table 5., are determined based on the values reported in Table 4. Table 5. also includes information about the saddle point and the value of x, "x_ lim for maximum", below which the objective function is strictly concave and a unique local maximum may be found.

Table 5.

Optimization results when x and y are free variables: The maximum, the saddle point and the highest value of x that is consistent with a maximum, namely "x_ lim for maximum". $(x_1, y_1)$ is the optimal solution which maximizes f(x,y). $(x_2, y_2)$ is a saddle point, which does not maximize f(x,y). f(.) is a strictly concave function, which is consistent with a maximum, for x < "x_ lim for maximum". Units: x: mm, f: SEK/ha

| r% | 1 | 1,5 | 2 | 2,5 | 3 |
|---|---|---|---|---|---|
| x_lim for maximum | 858,3693853 | 823,9872707 | 781,1244439 | 758,1209022 | 766,0849767 |
| $x_1$ | 434,2909462 | 388,9366842 | 349,1899435 | 317,3107476 | 292,0336788 |
| $y_1$ | -7,36600802 | -6,851779285 | -6,395793422 | -6,146903042 | -6,13520695 |
| $x_2$ | 1425,533273 | 1420,451615 | 1384,62745 | 1387,91807 | 1464,400588 |
| $y_2$ | -52,44725093 | -56,67901388 | -60,36507994 | -66,86751287 | -78,34258428 |
| $f(x_1, y_1)$ | 385063,3506 | 286382,7313 | 236439,9027 | 208055,4769 | 190206,8231 |
| $f(x_2, y_2)$ | 139478,344 | 87127,99343 | 68381,80349 | 49121,6605 | 20095,36724 |

Observations:

The numerical results found in Table 5. clearly show that the solution to the first order optimum conditions in the empirically relevant region is a maximum. The other solution, the saddle point, is found far outside the relevant region. This is also illustrated in Figure 3.

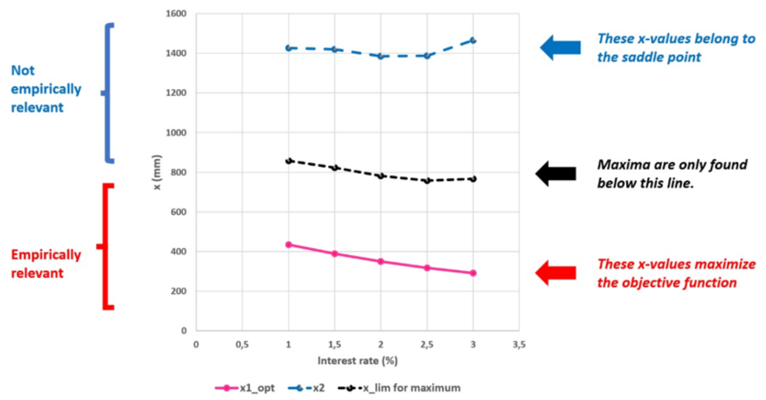


Figure 3.

The graphs illustrate the relationship between the two x- solutions. The x-value that gives a maximum is smaller than the x-value that gives a saddle point. The maximum is the empirically relevant solution. The graph is constructed from the data found in Table 5.

Optimal results when only x is a free variable:

We remember that, if we do not consider competition when the harvest decisions are taken, we may let y = 0. Then, the optimal value of x is found in equation (78).

$$x = \frac{4a^2}{9b^2} > 0 \tag{78}$$

That solution is a unique maximum, which follows from equation (79).

$$\frac{d^2 f_1(x)}{dx^2} = \frac{-3b}{4\sqrt{x}} < 0 \tag{79}$$

In Table 6., we find the optimal solutions for different rates of interest. We also find how much the optimal solution changes if we first ignore competition and let y = 0 when we select x, and then start to consider competition in the optimal way.

Table 6.

Optimization results when x is a free variable and y = 0: x(y=0) is the optimal value of x, which maximizes f(x,y), in case y = 0. "$x_1 - x(y = 0)$" says how much higher the optimal x-value should be, in case we also introduce competition as a component in the control function, and let y be different from zero. $f_1(x, y = 0)$ gives the optimal objective function value in case we do not consider competition in the decisions. "$f(x_1, y_1) - f1(.)$" shows how much we gain if we first only consider the size of the subject tree, x, in the control function, and then start to also consider the competition, via optimal values of y, different from zero. Units: x: mm, f: SEK/ha.

| r% | 1 | 1,5 | 2 | 2,5 | 3 |
|---|---|---|---|---|---|
| x(y = 0) | 326,7265347 | 297,5889722 | 271,965849 | 251,0671928 | 233,7697494 |
| $x_1 - x(y = 0)$ | 107,5644115 | 91,34771205 | 77,22409448 | 66,24355478 | 58,26392943 |
| $f_1(x, y = 0)$ | 363251,1438 | 274043,2767 | 228799,7893 | 202839,4334 | 186313,8722 |
| $f(x_1, y_1) - f_1(.)$ | 21812,20684 | 12339,45466 | 7640,11335 | 5216,043419 | 3892,950946 |

Figure 4. illustrates that the objective function is strictly concave in the neighborhood of the maximum.

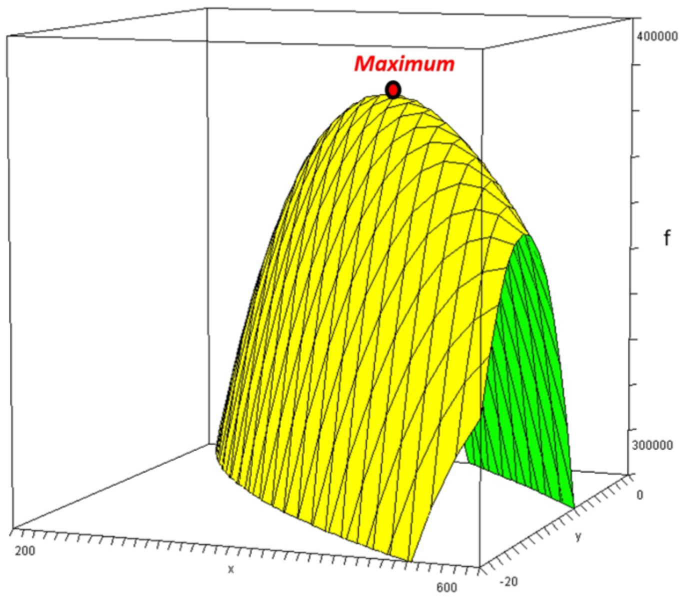

Figure 4.

The objective function in the neighborhood of the maximum, if the rate of interest is 1%. This graph is based on the parameter and variable values found in Table 3. and in Table 5. Units: x: mm, f: SEK/ha.

In Figure 5., we observe that the present value of forestry is strongly affected by the rate of interest in the capital market. If the rate of interest is very low, the present value if very high, and vice versa. Furthermore, the shape of the function is affected by changes in the rate of interest.

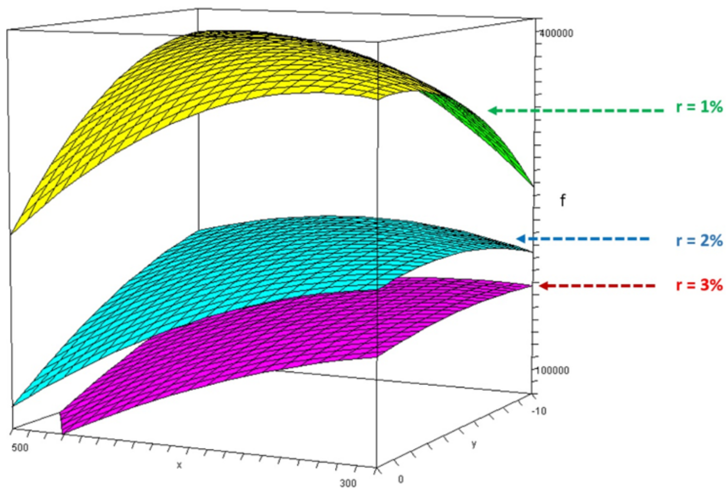


Figure 5.

The objective function in the neighborhood of the maximum, for different rates of interest. This graph is based on the parameter and variable values found in Table 3. The rate of interest is 1% (upper surface), 2% (middle surface) or 3% (bottom surface). Units: x: mm, f: SEK/ha.

Figure 6 shows the optimal solutions as balls in 3D space. In the Figures 7, 8 and 9, the corresponding information is illustrated in three 2D graphs.

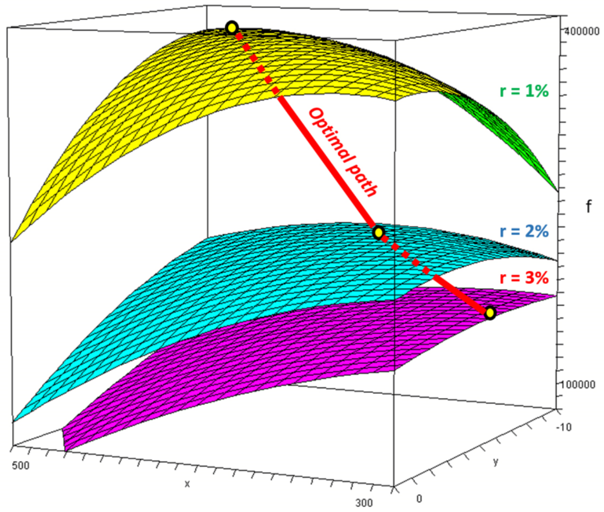


Figure 6.

The objective function in the neighborhood of the maximum, for different rates of interest. This graph is based on the parameter and variable values found in Table 3. and in Table 5. The rate of interest is 1% (upper surface), 2% (middle surface) or 3% (bottom surface). The optimal solutions, for different rates of interest, are shown in the graph, in the form of balls. These may be considered as functions of the rate of interest. The graph also illustrates that, with optimal values of x and y, the objective function value is a decreasing function of the rate of interest. Units: x: mm, f: SEK/ha.

In Figure 7., we observe that x is a strictly decreasing function of the rate of interest. This means that the optimal limit diameter of a tree is a decreasing function of the rate of interest. Such results should be expected, since they were also found in earlier studies, such as Pukkala et al (2010), Tahvonen et al (2010) and Rämö & Tahvonen (2014).

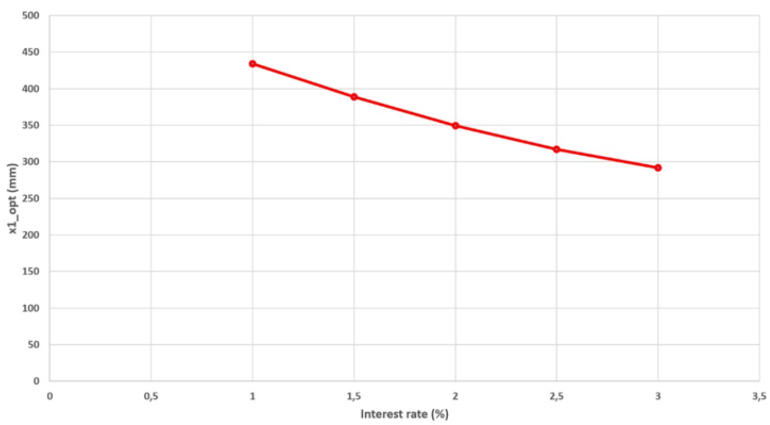

Figure 7.

The x value that maximizes the objective function, as a function of the interest rate, when we also consider competition and select the optimal value of y. A linear regression estimation of this function is found in (80).

$$x^* \approx 499 - 71.2r$$
$$(t \approx 50.1)(t \approx -15.2) \tag{80}$$

In Figure 8., we see that the optimal value of y, is negative. This is consistent with Figure 1 and equation (5). Trees that have more local competition should be harvested when they are smaller, than trees that have less local competition. Furthermore, y is an increasing function of the rate of interest. This means that the degree to which we should consider local competition when harvest decisions are taken, is reduced if the rate of interest increases. Mostly, such results have not been possible to find earlier studies, since the level of spatial detail in the analyses usually has been too low. Similar results have however been reported by Lohmander (2018) and (2019b).

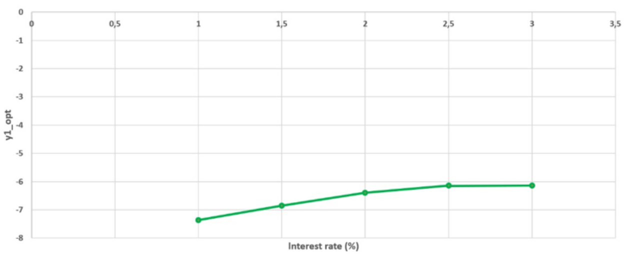


Figure 8.

The y value that maximizes the objective function, as a function of the interest rate, when we also consider and select the optimal values of x. A linear regression estimation of this function is found in (81).

$$y^* \approx -7.85 + 0.633r$$
$$(t \approx -30.8)(t \approx 5.27) \tag{81}$$

In Figure 9., we see that the optimal present value of forestry is a strictly decreasing and strictly convex function of the rate of interest. This type of results should be expected. They are also found in earlier studies, such as Pukkala et al (2010), Tahvonen et al (2010) and R¨am¨o & Tahvonen (2014).

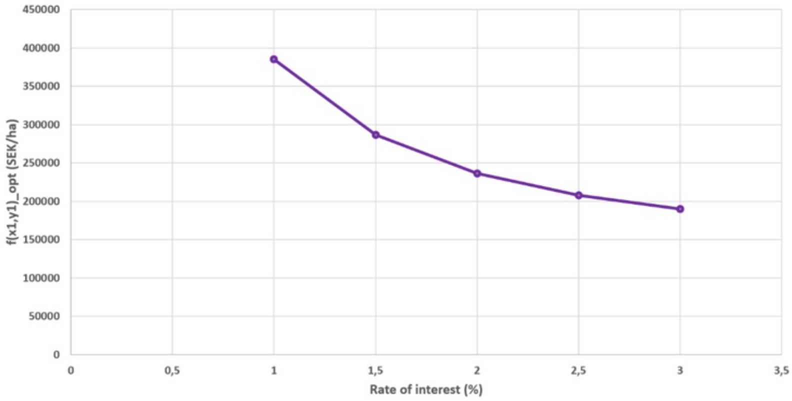


Figure 9.

The maximum objective function value as a function of the rate of interest, when we select optimal values of x and y. A regression estimation of this function is found in

$$f(x^*, y^*) \approx 90917 + 293641r^{-1}$$
$$(t \approx 65.9)(t \approx 133.5) \quad (82)$$

Figure 10. shows that the optimal value of x is a strictly decreasing function of the rate of interest, also if we do not take the local competition into account in the harvest decisions. The values of x are however different from the optimal values of x reported in Figure 7. The reason is that, in Figure 7., we also take local competition into account in the optimal way.

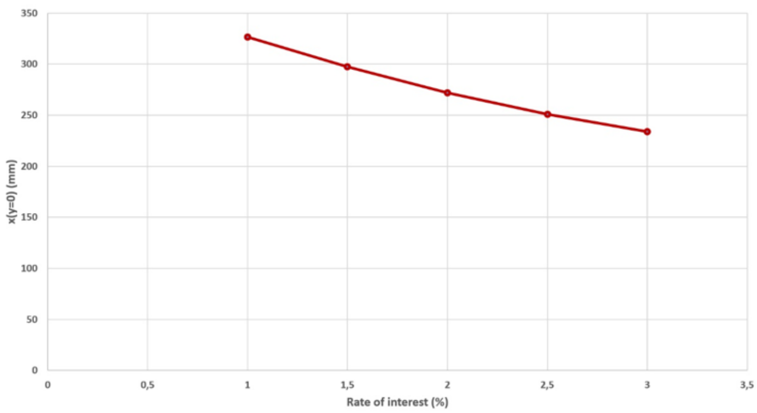


Figure 10.

The x value that maximizes the objective function as a function of the interest rate, if we do not consider competition and let y = 0. A linear regression estimation of this function is found in (83)

$$x^* \approx 369 - 46.5r$$
$$(t \approx 62.7)(t \approx -16.8) \tag{83}$$

If we do not take local competition into account when harvest decisions are taken, in the optimal way, then the present value of forestry follows the function in Figure 11. These function values are lower than the optimal present values of forestry, based on optimal values of x and y, as reported in Figure 9.

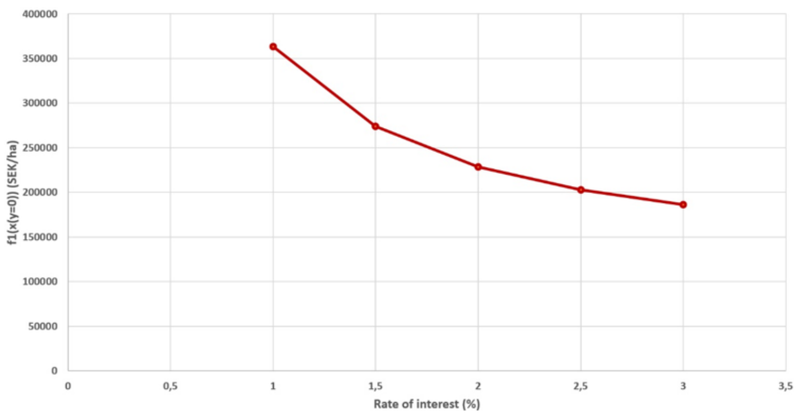


Figure 11.

The maximum objective function value as a function of the rate of interest, when we select optimal values of x, but we do not consider competition and let y = 0. A regression estimation of this function is found in (84)

$$f(x^*, y = 0) \approx 96529 + 266415r^{-1}$$
$$(t \approx 100.2)(t \approx 173.5) \tag{84}$$

Figure 12. illustrates how much the optimal value of x increases, if we first ignore local competition and optimize x, and then start to consider local competition in the optimal way.

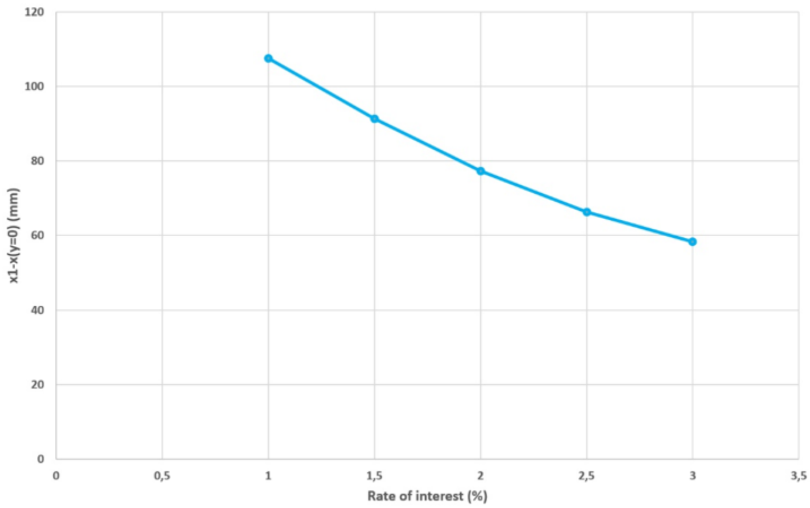


Figure 12.

The graph shows how much the optimal values of x, determined for y = 0, increase when we also start to consider competition in the control function, and optimize y as a free variable. Hence, this graph shows the optimal x values reported in Figure 7 minus the optimal x values reported in Figure 10. A linear regression estimation of this difference is found in (85).

$$\Delta x^* \approx 130 - 24.7r \\ (t \approx 31.9)(t \approx -12.9) \tag{85}$$

Figure 13. shows how much higher the optimal present value of forestry becomes if we consider local competition when harvest decisions are taken, compared to what the optimal present value would be if we just ignored local competition. One example: If the rate of interest is 2%, this difference is approximately 7500 SEK/ha. Practical interpretation: If the present value of the costs of investigating the local competition levels, and to use that information when harvest decisions are taken, is lower than 7500 SEK/ha, it is rational to undertake such actions. If the present value of these costs is higher, then it is rational to ignore competition when harvest decisions are taken.

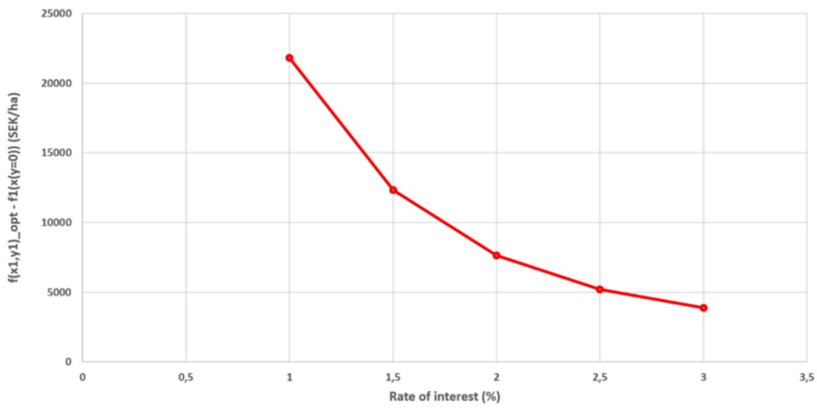


Figure 13.

The graph shows how much the maximum present value improves, if we first do not consider competition at all, and then start to consider competition in the optimal way. (In the first case, we optimize x, with the constraint y = 0. Then, we optimize x and y simultaneously.) Hence, this graph shows the function values in Figure 9 minus the function values in Figure 11. A regression estimation of this difference is found in (86). (It is important to be aware that the costs of collecting and using information about local competition in harvesting decisions, have not been considered when this graph has been constructed.)

$$f^*(x_1, y_1) - f_1^*(x)|_{y=0} \approx -5611 + 27226r^{-1}$$
$$(t \approx -13.1)(t \approx 39.8) \qquad (86)$$

## 4. Discussion

The general approach to continuous cover optimal forest management developed here, can of course be applied in every forest region of the world. Uneven aged forests, suitable for CCF management, are common in large parts of the world, for instance in tropical rain forests and in remote areas of Canada and Russia. With the new approach, it is possible to obtain higher expected present values from forestry than before, since the harvest decisions now can be adapted to conditions of relevance that were not earlier possible to consider. In this paper, we have in seen that it is important to consider the local competition situation, which may differ strongly. We have also seen how the harvest decisions optimally should be affected by this information, and exactly how the optimal harvest control function should be determined from empirical data. For these reasons, cooperation initiatives and projects of this nature, in different parts of the world, are strongly recommended.

For the Taiga area, representing large parts of the Nordic countries, even more specific knowledge has been obtained. From the earlier studies of continuous cover forestry, by Pukkala et al (2010), Tahvonen et al (2010) and R¨am¨o & Tahvonen (2014), we know that CCF often gives higher present values than rotation forestry based on periodic harvesting of all trees. This is very important information, since rotation forestry is the most common forestry method in the Nordic countries. CCF also has several environmental advantages compared to rotation forestry. So, already based on the earlier studies, it would be reasonable to adjust forestry methods, and to start using CCF more often.

Now, with the new findings in this paper, we know that the economic results of CCF can be even more improved, if we also take information about the local competition into account when harvest decisions are taken. This has been clearly shown in Figure 13 and equation (86).

As a consequence, we have even stronger reasons than before, to increase the areas of CCF forestry in the Nordic countries.

## 5. Conclusions

A control function based on the size of the tree and the local competition can be used to optimize continuous cover forestry.

The mathematical principles and statistical methods of determination of the optimal parameters of such control functions have been presented.

Numerical values of optimal control function parameters of relevance to a particular case study forest with the species picea Abies have been determined.

Generalized versions of the method can be used to optimize management of multi species forests, also with stochastic prices and adaptive decisions, as shown by Lohmander (2018), (2019a) and (2019b).

The readers are encouraged to use the methodology to optimize management of continuous cover forestry in all regions of the world.

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**Appendix**

## SIMULATION SOFTWARE

The computer code is developed in the programming language QB64.

```
Rem
Rem CCF_SIM_OPT_211110_1316_r2
Rem Peter Lohmander SIMULATION & OPTIMIZATION & SOFTWARE
Rem Nils Fagerberg PARAMETERS AND FUNCTIONS
Rem
```

```
Rem Explanations:
Rem Ts = Tree species.
Rem Dbh = Diameter (mm) at breast height, 1.3 m above ground, at t=0.
Rem Th0 = Tree height (m) at t=0.
Rem Lc = Lowest part of the crown (m) at t=0.
Rem x = Distance in direction east from origo in the coordinate system (m).
```

Rem y = Distance in direction north from origo in the coordinate system (m).
Rem xing = x for ingrowth (m).
Rem ying = y for ingrowth (m).
Rem Disc = Discounting factor in continuous time.
Rem BA = Basal area (m2).
Rem dBAdt = Time derivative of basal area (m2/year).
Rem Diam = Diameter (mm) at breast height, 1.3 m above ground.
Rem Harv = Harvest decision (0 = no, 1 = yes).
Rem Vol = Tree volume (m3).
Rem Height = Tree height (m).
Rem CR = Crown ratio = Crown length divided by tree height.
Rem Alive = Alive (0 = no, 1 = yes)
Rem t = Time (Five year period). In the model, 0 =< t =<60.
Rem r = Rate of interest, in continuous time.

Dim Ts(2000), Dbh(2000), Th0(2000), Lc(2000), x(2000), y(2000), Disc(61)
Dim BA(2000, 61), dBAdt(2000, 61), Diam(2000, 61), Harv(2000, 61)
Dim Vol(2000, 61), Height(2000, 61), CR(2000, 61), ALIVE(2000, 61)
Dim xing(100, 61), ying(100, 61)

“Open ”C:\Users\Peter\OneDrive\Desktop\CCF opt\CCF_opt\FOREST_data.txt”
For Input As #1
Open ”C:\Users\Peter\OneDrive\Desktop\CCF opt\CCF_opt#FOREST_OUT_r2.txt”
For Output As #2”

Screen _NewImage(1800, 1000, 256)

Input ”Default parameters? (Yes = 1, No = 0)”, NewPar
r = 0.02
DL0a = 250
DL0b = 450
DLCOMPa = -10
DLCOMPb = 0
NSIM = 1000
seed = 1
Randomize seed

If NewPar > 0.5 Then GoTo 2

Input ”r = ”, r
Input ”DL0a = ”, DL0a

Input "DL0b = ", DL0b
Input "DLCOMPa = ", DLCOMPa
Input "DLCOMPb = ", DLCOMPb
Input "NSIM = ", NSIM
Input "Random seed = ", seed
Randomize seed
2 Rem

Rem Some Parameters
PI = 3.141593
Rem IngPer = Ingrowth per five year period in a 60m*60m area.
Rem We assume ingrowth of 6 trees per ha per year. During five years, we get 30 trees.
Rem However, in the experimental area of 60 m * 60 m, we get 30 * 0.36 = approximately 10 trees
Rem per five year period. This variable is called IngPer.
IngPer = 10

Rem Discounting factors in the middle of periods
Rem r = 0.03
For t = 0 To 61
year = 5 * (t + 0.5)
Disc(t) = Exp(-r * year)
Next t

Rem Initial values of matrixes
For i = 1 To 2000
Ts(i) = 0
Dbh(i) = 0
Th0(i) = 0
Lc(i) = 0
x(i) = 0
y(i) = 0
Next i

For t = 0 To 61
For i = 1 To 2000
BA(i, t) = 0
dBAdt(i, t) = 0
Diam(i, t) = 0
Harv(i, t) = 0
Vol(i, t) = 0

```
Height(i, t) = 0
CR(i, t) = 0
ALIVE(i, t) = 0
Next i
For i = 1 To 100
xing(i, t) = 0
ying(i, t) = 0
Next i
Next t

Rem Here, the initial conditions are imported from the indata file.

Trees = 0
i = 0

10 Rem Initial forest conditions
Input #1, Site
If Site = 999 Then GoTo 20
Stand = Site
i = i + 1
Input #1, Tn
Input #1, Ts(i)
Input #1, Dbh(i)
Input #1, Th0(i)
Input #1, Lc(i)
Input #1, x(i)
Input #1, y(i)
GoTo 10
20 Rem
Trees = i

Rem CR values
For i = 1 To 2000
For t = 0 To 61
CR(i, t) = 0.85
Next t
Next i

Print ""
Print " Site = "; Stand;
Print #2, ""
```

```
Print #2, " Site = "; Stand;
Print " The number of trees = "; Trees
Print #2, " The number of trees = "; Trees


Print " Number of simulations = "; NSIM;
Print #2, " Number of simulations = "; NSIM;
Print " Rate of interest = ", r;
Print #2, " Rate of interest = ", r;
Print " Random seed = "; seed
Print #2, " Random seed = "; seed


Print " DL0a = ", DL0a;
Print " DL0b = ", DL0b
Print " DLCOMPa = ", DLCOMPa;
Print " DLCOMPb = ", DLCOMPb
Print #2, " DL0a = ", DL0a;
Print #2, " DL0b = ", DL0b
Print #2, " DLCOMPa = ", DLCOMPa;
Print #2, " DLCOMPb = ", DLCOMPb
Print ""
Print #2, ""

Rem Simulation section ***************************
Print " SIM STAND kR kDL0 kDLCOMP Obj"
Print #2, " SIM STAND kR kDL0 kDLCOMP Obj"


Obj_best = 0
DL0_best = 0
DLCOMP_best = 0

For SIMULATION = 1 To NSIM
DL0 = DL0a + Rnd * (DL0b - DL0a)
DLCOMP = DLCOMPa + Rnd * (DLCOMPb - DLCOMPa)


Rem Initial conditions
For i = 1 To 2000
Diam(i, 0) = Dbh(i)
lifestart = 0
If Dbh(i) > 0.1 Then lifestart = 1
For t = 0 To 61
ALIVE(i, t) = lifestart
Next t
```

```
BA(i, 0) = PI * Dbh(i) * Dbh(i) / 4000000
Next i

   ObjLoc = 0

   TMAX = 59
For t = 0 To TMAX

   Rem Ingrowth in random locations

   For i = 1 To IngPer
Rem Here, we simulate coordinates with a uniform probability density function
Rem covering the 60 m * 60 m experimental area.
xev = 60 * Rnd
yev = 60 * Rnd
xing(i, t) = xev
ying(i, t) = yev
Next i

   Rem Here, the ingrowth trees are placed in the forest coordinate system.
Rem They are also given the initial sizes.
iing0 = Trees
For iing = 1 To IngPer
i = iing0 + IngPer * t + iing
x(i) = xing(iing, t)
y(i) = ying(iing, t)
Diam(i, t) = 60
For t2 = t To 61
ALIVE(i, t2) = 1
Next t2
BA(i, t) = PI * Diam(i, t) * Diam(i, t) / 4000000
Next iing

   Rem Tree property calculations
Rem The already known basal area is used to determine diameter, height and
volume.
For i = 1 To 2000
Diam(i, t) = 0
Dcm = 0
Height(i, t) = 0
```

Hm = 0
Vol(i, t) = 0
If ALIVE(i, t) < 1 Then GoTo
Diam(i, t) = 2 * ((BA(i, t) / PI)$^{0.5}$) * 1000
Dcm = Diam(i, t) / 10
Height(i, t) = Dcm $^{3}$ / (10 * (0.9402 + 0.1317 * Dcm) $^{3}$) + 1.3
Hm = Height(i, t)
Vol(i, t) = (10$^{-1.06019}$ * Dcm $^{2.04239}$* (Dcm + 20)$^{-0.54292}$* Hm $^{2.80843}$* (Hm - 1.3)$^{-1.52110}$) / 1000

30 Rem
Next i

Rem Harvest section
For i = 1 To 2000
Harv(i, t) = 0

If ALIVE(i, t) < 1 Then GoTo 200

Rem Calculation of competition index for tree i.
COMP = 0
For j = 1 To 2000
If ALIVE(j, t)< 1 Then GoTo 180
If i = j Then GoTo 180
dist = ((x(i) - x(j)) $^{2}$ + (y(i) - y(j)) $^{2}$)$^{0.5}$
If dist > 10 Then GoTo 180
Rem COMP = COMP + BA(j, t) * (BA(j, t) / BA(i, t))$^{0.325}$ * EXP(-1 * (dist / 4.918)$^{2}$)
Rem Note that the COMP sum now equals the competition basal area per hectare.
COMP = COMP + BA(j, t)
180 Rem
Next j
COMP = COMP * 10000 / (PI * 10 * 10)
Rem end of calculation of competition index for tree i based on the area within the 60m*60m area.

Rem Calculation of the net price of tree i.
PRICE = 670
COST = 1150 * (0.0105 + 0.0458 * Vol(i, t)) + 750 * Vol(i, t) / (8.7 + 10 * Vol(i, t))
NetPrice = PRICE - COST
Rem End of calculation of net price of tree i.
Dlim = DL0 + DLCOMP * COMP

If Diam(i, t) > Dlim And ALIVE(i, t) = 1 Then Harv(i, t) = 1
If Harv(i, t) < 1 Then GoTo 200

Rem If the tree is harvested, the present value of the tree is added to the objective function.
ObjLoc = ObjLoc + Disc(t) * NetPrice * Vol(i, t) * Harv(i, t)

Rem If a tree is harvested, it is instantly defined as not alive.
For tt = t To 61
ALIVE(i, tt) = 0
Next tt
BA(i, t) = 0
Diam(i, t) = 0
Height(i, t) = 0
Vol(i, t) = 0
200 Rem
Next i

Rem The basal areas of not harvested trees continue to grow.
For i = 1 To 2000
dBAdt(i, t) = 0
COMP = 0
If ALIVE(i, t) ¡ 1 Then GoTo 400
For j = 1 To 2000
If ALIVE(j, t)<1 Then GoTo 300
If Diam(j, t)< 60 Then GoTo 300
If i = j Then GoTo 300
dist = ((x(i) - x(j)) $^{2}$ + (y(i) - y(j)) $^{2}$) $^{0.5}$
If dist > 10 Then GoTo 300
COMP = COMP + BA(j, t) * (BA(j, t) / BA(i, t))$^{0.325}$ * Exp(-1 * (dist / 4.918)$^{2}$)
300 Rem
Next j
prelgrowth = CR(i, t) * BA(i, t) $^{0.5}$* (154.4 - 212.2 * BA(i, t) - 142.7 * COMP$^{0.357}$)
prelgrowth = prelgrowth / 10000
dBAdt(i, t) = 0
If prelgrowth > 0 Then dBAdt(i, t) = prelgrowth
BA(i, t + 1) = BA(i, t) + 5 * dBAdt(i, t)
Diam(i, t + 1) = 2 * ((BA(i, t) / PI) $^{0.5}$* 1000
400 Rem
Next i
Next t

```
kr = 1000 * r
kDL0 = 1000 * DL0
kDLCOMP = 1000 * DLCOMP
OBJperHA = ObjLoc / 0.36
Print Using "# # # # # # # #"; SIMULATION, Stand, kr, kDL0, kDL-
COMP, OBJperHA
Print #2, Using "# # # # # # # #"; SIMULATION, Stand, kr, kDL0;
kDLCOMP; OBJperHA

If ObjLoc> Obj_best Then DL0_best = DL0
If ObjLoc > Obj_best Then DLCOMP_best = DLCOMP
If ObjLoc > Obj_best Then Obj_best = ObjLoc

Next SIMULATION

PVperHA_best = Obj_best / 0.36
Print ""
Print "Optimal Solutions:"
Print #2, ""
Print #2, "Optimal Solutions:"
Print "The optimal value of DL0 = "; DL0_best; " mm"
Print "The optimal value of DLCOMP = "; DLCOMP_best
Print "The optimal local objective function value = "; Obj_best
Print "The optimal E(PV) = "; PVperHA_best; " SEK/ha"
Print #2, "The optimal value of DL0 = "; DL0_best; " mm"
Print #2, "The optimal value of DLCOMP = "; DLCOMP_best
Print #2, "The optimal local objective function value = "; Obj_best
Print #2, "The optimal E(PV) = "; PVperHA_best; " SEK/ha"

Close #1
Close #2
End
```