PROGRESS IN ELECTRICAL ENGINEERING
Vol. 1, No. 1, 2020, 77-105

URL : www.arfjournals.com

ON ANALYTICAL APPROACH TO MODEL MANUFACTURING OF AN ENHANCED SWING DIFFERENTIAL COLPITTS OSCILLATOR IN A HETEROSTRUCTURE WITH ACCOUNT STRESS BETWEEN LAYERS

E.L. Pankratov

[1]*Nizhny Novgorod State University, 23 Gagarin avenue, Nizhny Novgorod, 603950, Russia*
[2]*Nizhny Novgorod State Technical University, 24 Minin Street, Nizhny Novgorod, 603950, Russia*
E-mail: elp2004@mail.ru

Received: 15 April 2020 / Revised: 23 April 2020 / Accepted: 29 June 2020 / Published: 25 August 2020

Abstract: We introduce an analytical approach to model manufacturing of an enhanced swing differential Colpitts oscillator to optimize this process for increasing of density of field-effect heterotransistors framework the considered circuit with decreasing of their dimensions. The approach gives a possibility to take into account spatial and at the same time temporal variations of parameters of technological process. At the same time the approach gives a possibility to take into account nonlinearity of physical processes. The approach gives a possibility to formulate recommendations for optimization of technological processes.

Keywords: analytical approach for modelling, enhanced swing differential Colpitts oscillator, optimization of manufacturing, increasing of density of elements.

INTRODUCTION

Currently density of elements of integrated circuits and their performance intensively increasing. Simultaneously with increasing of the density of the elements of integrated circuit their dimensions decreases. [1-11] One way to decrease dimensions of these elements of these integrated circuit is manufacturing of these elements in thin-film heterostructures [11-15]. An alternative approach to decrease dimensions of the elements of integrated circuits is using laser and microwave types annealing [16-18]. Using these types of annealing leads to generation inhomogeneous distribution of temperature. Due to Arrhenius law the inhomogeneity of the diffusion coefficient and other parameters of process. The inhomogeneity gives us possibility to decrease dimensions of elements of integrated circuits. Changing of properties of electronic materials could be obtain by using radiation processing of these materials [21,23].

In this paper we consider enhanced swing differential Colpitts oscillator based on field-effect transistors described in Ref. [24] (see Fig.1). We assume, that the considered element has been manufactured in heterostructure from Fig. 1. The heterostructure consist of a substrate and an epitaxial layer. The epitaxial layer includes into itself several sections manufactured by using another materials. The sections have been doped by diffusion or ion implantation to generation into these sections required type of conductivity (*n* or *p*). Framework this paper we analyzed redistribution of dopant during annealing of dopant and/ or radiation defects to formulate conditions for decreasing of dimensions of the enhanced swing differential Colpitts oscillator.

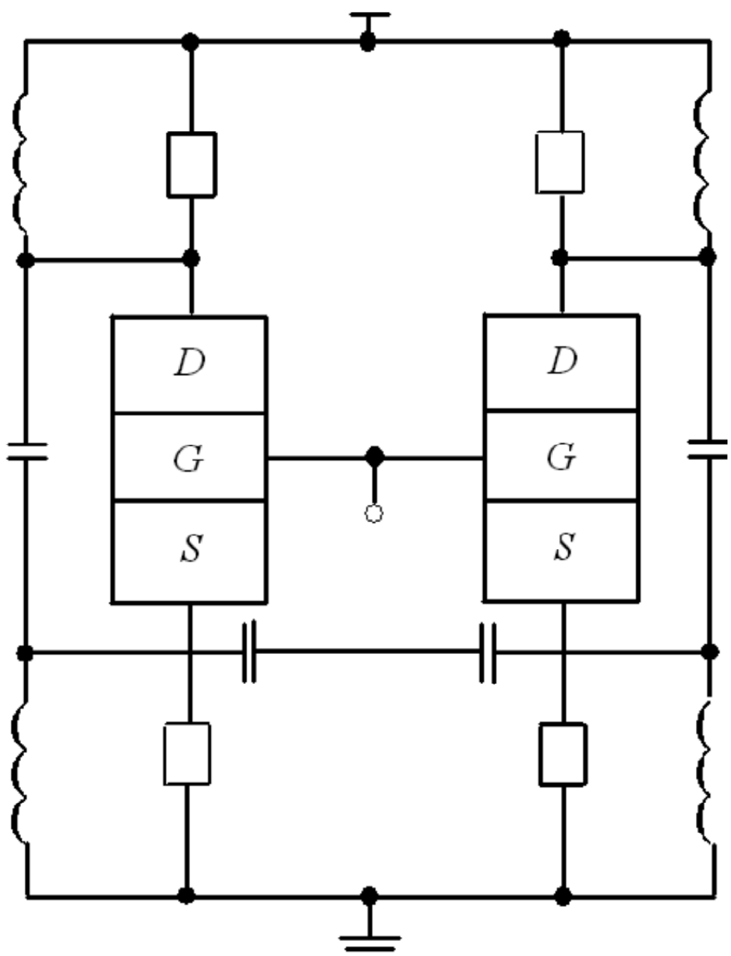

Figure 1*a*. Considered of the oscillator [24]

METHOD OF SOLUTION

We determine spatio-temporal distribution of concentration of dopant by solving the following boundary problem [1,19-23]

$$\frac{\partial C(x,y,z,t)}{\partial t}=\frac{\partial}{\partial x}\left[D\frac{\partial C(x,y,z,t)}{\partial x}\right]+\frac{\partial}{\partial y}\left[D\frac{\partial C(x,y,z,t)}{\partial y}\right]+\frac{\partial}{\partial z}\left[D\frac{\partial C(x,y,z,t)}{\partial z}\right]+$$

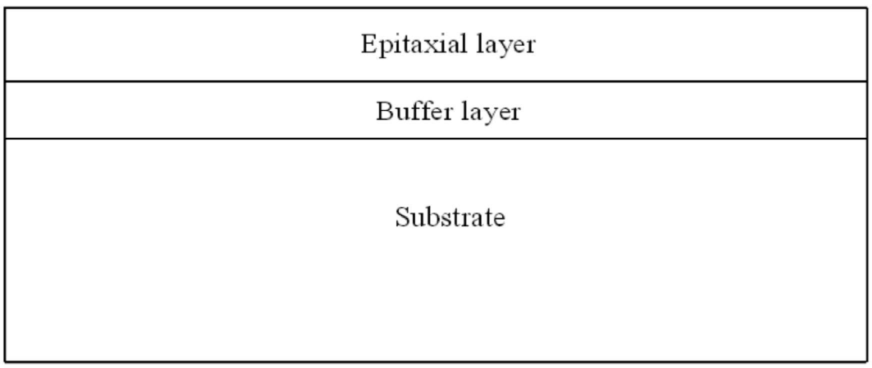

Figure 1*b*: Considered heterostructure with three layers (view from side)

$$+\Omega\frac{\partial}{\partial x}\left[\frac{D_S}{kT}\nabla_S\mu_1(x,y,z,t)\int_0^{L_z}C(x,y,W,t)\,dW\right]+$$

$$+\Omega\frac{\partial}{\partial y}\left[\frac{D_S}{kT}\nabla_S\mu_1(x,y,z,t)\int_0^{L_z}C(x,y,W,t)\,dW\right] \qquad (1)$$

Boundary and initial conditions for Eq. (1) are

$$\left.\frac{\partial C(x,y,z,t)}{\partial x}\right|_{x=0}=0,\ \left.\frac{\partial C(x,y,z,t)}{\partial x}\right|_{x=L_x}=0,\ \left.\frac{\partial C(x,y,z,t)}{\partial y}\right|_{y=0}=0,\ C(x,y,z,0)=f_C(x,y,z),$$

$$\left.\frac{\partial C(x,y,z,t)}{\partial y}\right|_{x=L_y}=0,\ \left.\frac{\partial C(x,y,z,t)}{\partial z}\right|_{z=0}=0,\ \left.\frac{\partial C(x,y,z,t)}{\partial z}\right|_{x=L_z}=0.$$

Function $C(x,y,z,t)$ describes distribution of concentration of dopant in space and time; parameter W describes the dopant's atomic volume; symbol $\tilde{N}_s$ describes the surficial gradient of considered concentration; function $\int_0^{L_z}C(x,y,z,t)\,dz$ describes the dopant concentration on interface between layers of the heterostructure for the case, when interface between layers of heterostructure is perpendicular to the Z-axis; function $m_1(x,y,z,t)$ describes the chemical potential, which was generated due to mechanical stress between layers of heterostructure; parameters D and D_S describes coefficients of volumetric and surficial diffusions. Diffusions coefficients of dopants will be changed with changing of materials of heterostructure; speed of cooling and heating of materials during annealing; distribution of dopant and radiation defects (after radiation processing) concentrations in space and time. Framework this paper we consider the following approximations of diffusions coefficients of dopant [21-23]

$$D_C = D_L(x,y,z,T)\left[1+\xi\frac{C^\gamma(x,y,z,t)}{P^\gamma(x,y,z,T)}\right]\left[1+\varsigma_1\frac{V(x,y,z,t)}{V^*}+\varsigma_2\frac{V^2(x,y,z,t)}{(V^*)^2}\right],$$

$$D_S = D_{SL}(x,y,z,T)\left[1+\xi_S\frac{C^\gamma(x,y,z,t)}{P^\gamma(x,y,z,T)}\right]\left[1+\varsigma_1\frac{V(x,y,z,t)}{V^*}+\varsigma_2\frac{V^2(x,y,z,t)}{(V^*)^2}\right]. \quad (2)$$

Functions $D_L(x,y,z,T)$ and $D_{LS}(x,y,z,T)$ describe variations with variation of spatial coordinate x, y and z (heterostruicture has several layers with different properties) and variation of temperature T (framework the Arrhenius law); function $P(x,y,z,T)$ describes dependence of solubility limit of dopant on spatial coordinates and time; value of parameter g is integer (usually $\gamma \in [1,3]$ [21]) and changing with changing material of heterostructure; function $V(x,y,z,t)$ describes dependence of radiation vacancies (with equilibrium distribution V^*) concentration on spatial coordinates and time. Dependence of diffusion coefficient of dopant on considered concentrations was described in [21]. Dependences of point radiation defects concentration on spatial coordinates and time have been considered as solution of equations bellow [19,22,23]

$$\frac{\partial I(x,y,z,t)}{\partial t} = \frac{\partial}{\partial x}\left[D_I(x,y,z,T)\frac{\partial I(x,y,z,t)}{\partial x}\right]+\frac{\partial}{\partial y}\left[D_I(x,y,z,T)\frac{\partial I(x,y,z,t)}{\partial y}\right]+$$

$$+\frac{\partial}{\partial z}\left[D_I(x,y,z,T)\frac{\partial I(x,y,z,t)}{\partial z}\right]-k_{I,I}(x,y,z,T)\,I^2(x,y,z,t)-k_{I,V}(x,y,z,T)\times$$

$$\times I(x,y,z,t)V(x,y,z,t)+\Omega\frac{\partial}{\partial x}\left[\frac{D_{IS}}{kT}\nabla_S\mu\,(x,y,z,t)\int_0^{L_z}I(x,y,W,t)\,dW\right]+$$

$$+\Omega\frac{\partial}{\partial y}\left[\frac{D_{IS}}{kT}\nabla_S\mu\left(x,y,z,t\right)\int_0^{L_z}I\left(x,y,W,t\right)dW\right] \quad (3)$$

$$\frac{\partial V\left(x,y,z,t\right)}{\partial t}=\frac{\partial}{\partial x}\left[D_V\left(x,y,z,T\right)\frac{\partial V\left(x,y,z,t\right)}{\partial x}\right]+\frac{\partial}{\partial y}\left[D_V\left(x,y,z,T\right)\frac{\partial V\left(x,y,z,t\right)}{\partial y}\right]+$$

$$+\frac{\partial}{\partial z}\left[D_V\left(x,y,z,T\right)\frac{\partial V\left(x,y,z,t\right)}{\partial z}\right]-k_{V,V}\left(x,y,z,T\right)V^2\left(x,y,z,t\right)-k_{I,V}\left(x,y,z,T\right)\times$$

$$\times I\left(x,y,z,t\right)V\left(x,y,z,t\right)+\Omega\frac{\partial}{\partial x}\left[\frac{D_{VS}}{kT}\nabla_S\mu\left(x,y,z,t\right)\int_0^{L_z}V\left(x,y,W,t\right)dW\right]+$$

$$+\Omega\frac{\partial}{\partial y}\left[\frac{D_{VS}}{kT}\nabla_S\mu\left(x,y,z,t\right)\int_0^{L_z}V\left(x,y,W,t\right)dW\right]$$

Initial and boundary conditions for above equations are

$$\left.\frac{\partial I\left(x,y,z,t\right)}{\partial x}\right|_{x=0}=0,\ \left.\frac{\partial I\left(x,y,z,t\right)}{\partial x}\right|_{x=L_x}=0,\ \left.\frac{\partial I\left(x,y,z,t\right)}{\partial y}\right|_{y=0}=0,$$

$$\left.\frac{\partial I\left(x,y,z,t\right)}{\partial y}\right|_{y=L_y}=0,\ \left.\frac{\partial I\left(x,y,z,t\right)}{\partial z}\right|_{z=0}=0,\ \left.\frac{\partial I\left(x,y,z,t\right)}{\partial z}\right|_{z=L_z}=0,$$

$$\left.\frac{\partial V\left(x,y,z,t\right)}{\partial x}\right|_{x=0}=0,\ \left.\frac{\partial V\left(x,y,z,t\right)}{\partial x}\right|_{x=L_x}=0,\ \left.\frac{\partial V\left(x,y,z,t\right)}{\partial y}\right|_{y=0}=0, \quad (4)$$

$$\left.\frac{\partial V\left(x,y,z,t\right)}{\partial y}\right|_{y=L_y}=0,\ \left.\frac{\partial V\left(x,y,z,t\right)}{\partial z}\right|_{z=0}=0,\ \left.\frac{\partial V\left(x,y,z,t\right)}{\partial z}\right|_{z=L_z}=0$$

$$I\,(x,y,z,0)=f_I(x,y,z),\ V\,(x,y,z,0)=f_V(x,y,z).$$

Function $I\,(x,y,z,t)$ describes dependences of radiation interstitials concentration (with the equilibrium distribution I^*) on spatial coordinates and time; functions $D_I(x,y,z,T)$, $D_V(x,y,z,T)$, $D_{IS}(x,y,z,T)$ and $D_{VS}(x,y,z,T)$ describes dependences of surficial and volumetric diffusions coefficients interstitials and vacancies on spatial coordinates and time, respectively; terms $I^2(x,y,z,t)$ and $V^2(x,y,z,t)$ describe accounting generation of diinterstitials and divacancies (these terms were described in details in [23] and appropriate references in this book); functions $k_{I,V}(x,y,z,T)$, $k_{I,I}(x,y,z,T)$ and $k_{V,V}(x,y,z,T)$ describe dependences of recombination parameters of point radiation defects generation of their complexes on spatial coordinates and time. Distributions of concentratios of diinterstitials F_I (x,y,z,t) and divacancies F_V (x,y,z,t) in space and time we determine as solution of the following equations [19,22,23]

$$\frac{\partial \Phi_I(x,y,z,t)}{\partial t}=\frac{\partial}{\partial x}\left[D_{\Phi_I}(x,y,z,T)\frac{\partial \Phi_I(x,y,z,t)}{\partial x}\right]+\frac{\partial}{\partial y}\left[D_{\Phi_I}(x,y,z,T)\frac{\partial \Phi_I(x,y,z,t)}{\partial y}\right]+$$
$$+\frac{\partial}{\partial z}\left[D_{\Phi_I}(x,y,z,T)\frac{\partial \Phi_I(x,y,z,t)}{\partial z}\right]+\Omega\frac{\partial}{\partial x}\left[\frac{D_{\Phi_I S}}{kT}\nabla_S\mu_1(x,y,z,t)\int_0^{L_z}\Phi_I(x,y,W,t)\,dW\right]+$$
$$+\Omega\frac{\partial}{\partial y}\left[\frac{D_{\Phi_I S}}{kT}\nabla_S\mu_1(x,y,z,t)\int_0^{L_z}\Phi_I(x,y,W,t)\,dW\right]+k_{I,I}(x,y,z,T)I^2(x,y,z,t)+$$
$$+k_I(x,y,z,T)I(x,y,z,t) \quad (5)$$

$$\frac{\partial \Phi_V(x,y,z,t)}{\partial t}=\frac{\partial}{\partial x}\left[D_{\Phi_V}(x,y,z,T)\frac{\partial \Phi_V(x,y,z,t)}{\partial x}\right]+\frac{\partial}{\partial y}\left[D_{\Phi_V}(x,y,z,T)\frac{\partial \Phi_V(x,y,z,t)}{\partial y}\right]+$$
$$+\frac{\partial}{\partial z}\left[D_{\Phi_V}(x,y,z,T)\frac{\partial \Phi_V(x,y,z,t)}{\partial z}\right]+\Omega\frac{\partial}{\partial x}\left[\frac{D_{\Phi_V S}}{kT}\nabla_S\mu_1(x,y,z,t)\int_0^{L_z}\Phi_V(x,y,W,t)\,dW\right]+$$
$$+\Omega\frac{\partial}{\partial y}\left[\frac{D_{\Phi_V S}}{kT}\nabla_S\mu_1(x,y,z,t)\int_0^{L_z}\Phi_V(x,y,W,t)\,dW\right]+k_{V,V}(x,y,z,T)V^2(x,y,z,t)+$$
$$+k_V(x,y,z,T)V(x,y,z,t)$$

Initial and boundary conditions for the considered concentrations are

$$\left.\frac{\partial \Phi_I(x,y,z,t)}{\partial x}\right|_{x=0}=0,\ \left.\frac{\partial \Phi_I(x,y,z,t)}{\partial x}\right|_{x=L_x}=0,\ \left.\frac{\partial \Phi_I(x,y,z,t)}{\partial y}\right|_{y=0}=0,$$

$$\left.\frac{\partial \Phi_I(x,y,z,t)}{\partial y}\right|_{y=L_y}=0,\ \left.\frac{\partial \Phi_I(x,y,z,t)}{\partial z}\right|_{z=0}=0,\ \left.\frac{\partial \Phi_I(x,y,z,t)}{\partial z}\right|_{z=L_z}=0,$$

$$\left.\frac{\partial \Phi_V(x,y,z,t)}{\partial x}\right|_{x=0}=0,\ \left.\frac{\partial \Phi_V(x,y,z,t)}{\partial x}\right|_{x=L_x}=0,\ \left.\frac{\partial \Phi_V(x,y,z,t)}{\partial y}\right|_{y=0}=0, \quad (6)$$

$$\left.\frac{\partial \Phi_V(x,y,z,t)}{\partial y}\right|_{y=L_y}=0,\ \left.\frac{\partial \Phi_V(x,y,z,t)}{\partial z}\right|_{z=0}=0,\ \left.\frac{\partial \Phi_V(x,y,z,t)}{\partial z}\right|_{z=L_z}=0,$$

$$F_I(x,y,z,0)=f_{FI}(x,y,z),\ F_V(x,y,z,0)=f_{FV}(x,y,z).$$

Functions $D_{FI}(x,y,z,T)$, $D_{FV}(x,y,z,T)$, $D_{FIS}(x,y,z,T)$ and $D_{FVS}(x,y,z,T)$ describe dependences of surficial and volumetric diffusion coefficients of considered complexes of radiation defects on spatial coordinates and temperature; functions $k_I(x,y,z,T)$ and $k_V(x,y,z,T)$ describe dependences of decay parameters of the considered complexes also on spatial coordinates and temperature. Chemical potential μ_1 in Eqs.(1), (3) and (5) was determined by using the following relation [19]

$$m_1=E(z)\,\mathrm{W}s_{ij}[u_{ij}(x,y,z,t)+u_{ji}(x,y,z,t)]/2. \tag{7}$$

Here function $E(z)$ describes dependences of the Young modulus on coordinate z, which is perpendicular to interface between layers of heterostructure; σ_{ij} is the tensor of stress; tensor $u_{ij}=\frac{1}{2}\left(\frac{\partial u_i}{\partial x_j}+\frac{\partial u_j}{\partial x_i}\right)$ describes deformation in materials of heterostructure due to miss-match induced stress; functions u_i, u_j are equal to components $u_x(x,y,z,t)$, $u_y(x,y,z,t)$ and $u_z(x,y,z,t)$ of the displacement vector $\vec{u}(x,y,z,t)$; x_i, x_j are equal to spatial coordinates x, y, z. The above Eq. (7) could be considered in the following form

$$\mu(x,y,z,t)=\left[\frac{\partial u_i(x,y,z,t)}{\partial x_j}+\frac{\partial u_j(x,y,z,t)}{\partial x_i}\right]\left\{\frac{1}{2}\left[\frac{\partial u_i(x,y,z,t)}{\partial x_j}+\frac{\partial u_j(x,y,z,t)}{\partial x_i}\right]-\right.$$

$$\left.-\varepsilon_0\delta_{ij}+\frac{\sigma(z)\delta_{ij}}{1-2\sigma(z)}\left[\frac{\partial u_k(x,y,z,t)}{\partial x_k}-3\varepsilon_0\right]-K(z)\beta(z)\left[T(x,y,z,t)-T_0\right]\delta_{ij}\right\}\frac{\Omega}{2}E(z).$$

Here parameter s describes coefficient of Poisson; relation $e_0=(a_s-a_{EL})/a_{EL}$ describes the mismatch parameter; parameters a_s, a_{EL} are equal to lattice distances in the considered epitaxial layer and the substrate; parameter K describes modulus of uniform compression; parameter β describes thermal expansion coefficient; parameter T_r describes the equilibrium distribution of temperature, which coincide (framework our paper) with room temperature. We calculate distributions of components displacement vector in space and time by solving the following system of equations [25]

$$\begin{cases} \rho(z)\dfrac{\partial^2 u_x(x,y,z,t)}{\partial t^2} = \dfrac{\partial \sigma_{xx}(x,y,z,t)}{\partial x} + \dfrac{\partial \sigma_{xy}(x,y,z,t)}{\partial y} + \dfrac{\partial \sigma_{xz}(x,y,z,t)}{\partial z} \\ \rho(z)\dfrac{\partial^2 u_y(x,y,z,t)}{\partial t^2} = \dfrac{\partial \sigma_{yx}(x,y,z,t)}{\partial x} + \dfrac{\partial \sigma_{yy}(x,y,z,t)}{\partial y} + \dfrac{\partial \sigma_{yz}(x,y,z,t)}{\partial z} \\ \rho(z)\dfrac{\partial^2 u_z(x,y,z,t)}{\partial t^2} = \dfrac{\partial \sigma_{zx}(x,y,z,t)}{\partial x} + \dfrac{\partial \sigma_{zy}(x,y,z,t)}{\partial y} + \dfrac{\partial \sigma_{zz}(x,y,z,t)}{\partial z} \end{cases}$$

Here $\sigma_{ij} = \dfrac{E(z)}{2[1+\sigma(z)]}\left[\dfrac{\partial u_i(x,y,z,t)}{\partial x_j} + \dfrac{\partial u_j(x,y,z,t)}{\partial x_i} - \dfrac{\delta_{ij}}{3}\dfrac{\partial u_k(x,y,z,t)}{\partial x_k}\right] + K(z)\delta_{ij} \times$ $\times \dfrac{\partial u_k(x,y,z,t)}{\partial x_k} - \beta(z)K(z)[T(x,y,z,t) - T_r]$, function r (z) describes dependences of the density of heterostructure's materials on coordinate z; parameter d_{ij} is the Kronecker symbol. Accounting of the last relation for tensor s_{ij} leads to transformation of systems of equations for components displacement vector to the following form

$$\rho(z)\frac{\partial^2 u_x(x,y,z,t)}{\partial t^2} = \left\{K(z) + \frac{5E(z)}{6[1+\sigma(z)]}\right\}\frac{\partial^2 u_x(x,y,z,t)}{\partial x^2} + \left\{K(z) - \frac{E(z)}{3[1+\sigma(z)]}\right\} \times$$
$$\times \frac{\partial^2 u_y(x,y,z,t)}{\partial x \partial y} + \frac{E(z)}{2[1+\sigma(z)]}\left[\frac{\partial^2 u_y(x,y,z,t)}{\partial y^2} + \frac{\partial^2 u_z(x,y,z,t)}{\partial z^2}\right] + \left[K(z) + \frac{E(z)}{3[1+\sigma(z)]}\right] \times$$
$$\times \frac{\partial^2 u_z(x,y,z,t)}{\partial x \partial z} - K(z)\beta(z)\frac{\partial T(x,y,z,t)}{\partial x}$$

$$\rho(z)\frac{\partial^2 u_y(x,y,z,t)}{\partial t^2} = \frac{E(z)}{2[1+\sigma(z)]}\left[\frac{\partial^2 u_y(x,y,z,t)}{\partial x^2} + \frac{\partial^2 u_x(x,y,z,t)}{\partial x \partial y}\right] - \frac{\partial T(x,y,z,t)}{\partial y} \times$$
$$\times K(z)\beta(z) + \frac{\partial}{\partial z}\left\{\frac{E(z)}{2[1+\sigma(z)]}\left[\frac{\partial u_y(x,y,z,t)}{\partial z} + \frac{\partial u_z(x,y,z,t)}{\partial y}\right]\right\} + \frac{\partial^2 u_y(x,y,z,t)}{\partial y^2} \times \qquad (8)$$
$$\times \left\{\frac{5E(z)}{12[1+\sigma(z)]} + K(z)\right\} + \left\{K(z) - \frac{E(z)}{6[1+\sigma(z)]}\right\}\frac{\partial^2 u_y(x,y,z,t)}{\partial y \partial z} + K(z)\frac{\partial^2 u_y(x,y,z,t)}{\partial x \partial y}$$

$$\rho(z)\frac{\partial^2 u_z(x,y,z,t)}{\partial t^2}=\frac{E(z)}{2[1+\sigma(z)]}\left[\frac{\partial^2 u_z(x,y,z,t)}{\partial x^2}+\frac{\partial^2 u_z(x,y,z,t)}{\partial y^2}+\frac{\partial^2 u_x(x,y,z,t)}{\partial x\partial z}+\right.$$

$$\left.+\frac{\partial^2 u_y(x,y,z,t)}{\partial y\partial z}\right]+\frac{\partial}{\partial z}\left\{K(z)\left[\frac{\partial u_x(x,y,z,t)}{\partial x}+\frac{\partial u_y(x,y,z,t)}{\partial y}+\frac{\partial u_x(x,y,z,t)}{\partial z}\right]\right\}+$$

$$+\frac{1}{6}\frac{\partial}{\partial z}\left\{\frac{E(z)}{1+\sigma(z)}\left[6\frac{\partial u_z(x,y,z,t)}{\partial z}-\frac{\partial u_x(x,y,z,t)}{\partial x}-\frac{\partial u_y(x,y,z,t)}{\partial y}-\frac{\partial u_z(x,y,z,t)}{\partial z}\right]\right\}-$$

$$-K(z)\beta(z)\frac{\partial T(x,y,z,t)}{\partial z}.$$

Conditions for the above components have the form

$$\frac{\partial \vec{u}(0,y,z,t)}{\partial x}=0;\ \frac{\partial \vec{u}(L_x,y,z,t)}{\partial x}=0;\ \frac{\partial \vec{u}(x,0,z,t)}{\partial y}=0;\ \frac{\partial \vec{u}(x,L_y,z,t)}{\partial y}=0;$$

$$\frac{\partial \vec{u}(x,y,0,t)}{\partial z}=0;\ \frac{\partial \vec{u}(x,y,L_z,t)}{\partial z}=0;\ \vec{u}(x,y,z,0)=\vec{u}_0;\ \vec{u}(x,y,z,\infty)=\vec{u}_0.$$

Now we solve Eqs. (1), (3) and (5) by standard method of averaging of function corrections [26]. First of all it is attracted an interest transformation of the considered equations to the form, which will be take account recently considered initial distributions of concentrations of dopant and radiation defects

$$\frac{\partial C(x,y,z,t)}{\partial t}=\frac{\partial}{\partial x}\left[D\frac{\partial C(x,y,z,t)}{\partial x}\right]+\frac{\partial}{\partial y}\left[D\frac{\partial C(x,y,z,t)}{\partial y}\right]+\frac{\partial}{\partial z}\left[D\frac{\partial C(x,y,z,t)}{\partial z}\right]+ \quad (1a)$$

$$+f_C(x,y,z)\delta(t)+\Omega\frac{\partial}{\partial x}\left[\frac{D_S}{kT}\nabla_S\mu(x,y,z,t)\int_0^{L_z}C(x,y,W,t)dW\right]+$$

$$+\Omega\frac{\partial}{\partial y}\left[\frac{D_S}{kT}\nabla_S\mu(x,y,z,t)\int_0^{L_z}C(x,y,W,t)dW\right]$$

$$\frac{\partial I(x,y,z,t)}{\partial t}=\frac{\partial}{\partial x}\left[D_I(x,y,z,T)\frac{\partial I(x,y,z,t)}{\partial x}\right]+\frac{\partial}{\partial y}\left[D_I(x,y,z,T)\frac{\partial I(x,y,z,t)}{\partial y}\right]+$$

$$+\frac{\partial}{\partial z}\left[D_I(x,y,z,T)\frac{\partial I(x,y,z,t)}{\partial z}\right]+\Omega\frac{\partial}{\partial x}\left[\frac{D_{IS}}{kT}\nabla_S\mu_1(x,y,z,t)\int_0^{L_z}I(x,y,W,t)dW\right]+$$

$$+\Omega\frac{\partial}{\partial y}\left[\frac{D_{IS}}{kT}\nabla_S\mu_1(x,y,z,t)\int_0^{L_z}I(x,y,W,t)\,dW\right]-k_{I,I}(x,y,z,T)I^2(x,y,z,t)-$$

$$-k_{I,V}(x,y,z,T)I(x,y,z,t)V(x,y,z,t)+f_I(x,y,z)\delta(t) \qquad (3a)$$

$$\frac{\partial V(x,y,z,t)}{\partial t}=\frac{\partial}{\partial x}\left[D_V(x,y,z,T)\frac{\partial V(x,y,z,t)}{\partial x}\right]+\frac{\partial}{\partial y}\left[D_V(x,y,z,T)\frac{\partial V(x,y,z,t)}{\partial y}\right]+$$

$$+\frac{\partial}{\partial z}\left[D_V(x,y,z,T)\frac{\partial V(x,y,z,t)}{\partial z}\right]+\Omega\frac{\partial}{\partial x}\left[\frac{D_{VS}}{kT}\nabla_S\mu_1(x,y,z,t)\int_0^{L_z}V(x,y,W,t)\,dW\right]+$$

$$+\Omega\frac{\partial}{\partial y}\left[\frac{D_{VS}}{kT}\nabla_S\mu_1(x,y,z,t)\int_0^{L_z}V(x,y,W,t)\,dW\right]-k_{V,V}(x,y,z,T)V^2(x,y,z,t)-$$

$$-k_{I,V}(x,y,z,T)I(x,y,z,t)V(x,y,z,t)+f_V(x,y,z)\delta(t)$$

$$\frac{\partial \Phi_I(x,y,z,t)}{\partial t}=\frac{\partial}{\partial x}\left[D_{\Phi_I}(x,y,z,T)\frac{\partial \Phi_I(x,y,z,t)}{\partial x}\right]+\frac{\partial}{\partial y}\left[D_{\Phi_I}(x,y,z,T)\frac{\partial \Phi_I(x,y,z,t)}{\partial y}\right]+$$

$$+\frac{\partial}{\partial z}\left[D_{\Phi_I}(x,y,z,T)\frac{\partial \Phi_I(x,y,z,t)}{\partial z}\right]+\Omega\frac{\partial}{\partial x}\left[\frac{D_{\Phi_I S}}{kT}\nabla_S\mu_1(x,y,z,t)\int_0^{L_z}\Phi_I(x,y,W,t)\,dW\right]+$$

$$+\Omega\frac{\partial}{\partial y}\left[\frac{D_{\Phi_I S}}{kT}\nabla_S\mu_1(x,y,z,t)\int_0^{L_z}\Phi_I(x,y,W,t)\,dW\right]+k_I(x,y,z,T)I(x,y,z,t)+$$

$$+k_{I,I}(x,y,z,T)I^2(x,y,z,t)+f_{\Phi_I}(x,y,z)\delta(t) \qquad (5a)$$

$$\frac{\partial \Phi_V(x,y,z,t)}{\partial t}=\frac{\partial}{\partial x}\left[D_{\Phi_V}(x,y,z,T)\frac{\partial \Phi_V(x,y,z,t)}{\partial x}\right]+\frac{\partial}{\partial y}\left[D_{\Phi_V}(x,y,z,T)\frac{\partial \Phi_V(x,y,z,t)}{\partial y}\right]+$$

$$+\frac{\partial}{\partial z}\left[D_{\Phi_V}(x,y,z,T)\frac{\partial \Phi_V(x,y,z,t)}{\partial z}\right]+\Omega\frac{\partial}{\partial x}\left[\frac{D_{\Phi_V S}}{kT}\nabla_S\mu_1(x,y,z,t)\int_0^{L_z}\Phi_V(x,y,W,t)\,dW\right]+$$

$$+\Omega\frac{\partial}{\partial y}\left[\frac{D_{\Phi_V S}}{kT}\nabla_S\mu_1(x,y,z,t)\int_0^{L_z}\Phi_V(x,y,W,t)\,dW\right]+k_V(x,y,z,T)V(x,y,z,t)+$$

$$+k_{V,V}(x,y,z,T)V^2(x,y,z,t)+f_{\Phi_V}(x,y,z)\delta(t).$$

Now we will replace considered concentrations in right sides of Eqs. (1*a*), (3*a*) and (5*a*) on their average values α_{1r}, which not yet known. After that we obtain equations to determine the first-order approximations of concentrations of dopant and radiation defects

$$\frac{\partial C_1(x,y,z,t)}{\partial t}=\alpha_{1C}\Omega\frac{\partial}{\partial x}\left[z\frac{D_S}{kT}\nabla_S\mu_1(x,y,z,t)\right]+\alpha_{1C}\Omega\frac{\partial}{\partial y}\left[z\frac{D_S}{kT}\nabla_S\mu_1(x,y,z,t)\right]+$$

$$+f_C(x,y,z)\delta(t) \quad (1b)$$

$$\frac{\partial I_1(x,y,z,t)}{\partial t}=\alpha_{1I}z\Omega\frac{\partial}{\partial x}\left[\frac{D_{IS}}{kT}\nabla_S\mu(x,y,z,t)\right]+\alpha_{1I}\Omega\frac{\partial}{\partial y}\left[z\frac{D_{IS}}{kT}\nabla_S\mu(x,y,z,t)\right]+$$

$$+f_I(x,y,z)\delta(t)-\alpha_{1I}^2k_{I,I}(x,y,z,T)-\alpha_{1I}\alpha_{1V}k_{I,V}(x,y,z,T) \quad (3b)$$

$$\frac{\partial V_1(x,y,z,t)}{\partial t}=\alpha_{1V}z\Omega\frac{\partial}{\partial x}\left[\frac{D_{VS}}{kT}\nabla_S\mu_1(x,y,z,t)\right]+\alpha_{1V}\Omega\frac{\partial}{\partial y}\left[z\frac{D_{VS}}{kT}\nabla_S\mu_1(x,y,z,t)\right]+$$

$$+f_V(x,y,z)\delta(t)-\alpha_{1V}^2k_{V,V}(x,y,z,T)-\alpha_{1I}\alpha_{1V}k_{I,V}(x,y,z,T)$$

$$\frac{\partial \Phi_{1I}(x,y,z,t)}{\partial t}=\alpha_{1\Phi_I}z\Omega\frac{\partial}{\partial x}\left[\frac{D_{\Phi_I S}}{kT}\nabla_S\mu_1(x,y,z,t)\right]+\alpha_{1\Phi_I}z\Omega\frac{\partial}{\partial y}\left[\frac{D_{\Phi_I S}}{kT}\nabla_S\mu_1(x,y,z,t)\right]+$$

$$+f_{\Phi_I}(x,y,z)\delta(t)+k_I(x,y,z,T)I(x,y,z,t)+k_{I,I}(x,y,z,T)I^2(x,y,z,t) \quad (5b)$$

$$\frac{\partial \Phi_{1V}(x,y,z,t)}{\partial t}=\alpha_{1\Phi_V}z\Omega\frac{\partial}{\partial x}\left[\frac{D_{\Phi_V S}}{kT}\nabla_S\mu_1(x,y,z,t)\right]+\alpha_{1\Phi_V}z\Omega\frac{\partial}{\partial y}\left[\frac{D_{\Phi_V S}}{kT}\nabla_S\mu_1(x,y,z,t)\right]+$$

$$+f_{\Phi_V}(x,y,z)\delta(t)+k_V(x,y,z,T)V(x,y,z,t)+k_{V,V}(x,y,z,T)V^2(x,y,z,t).$$

To obtain relations to calculate of the required concentrations one shall integrate of the above relations on time. The integration leads to the following results

$$C_1(x,y,z,t)=\alpha_{1C}\Omega\frac{\partial}{\partial x}\int_0^t D_{SL}(x,y,z,T)\frac{z}{kT}\left[1+\varsigma_1\frac{V(x,y,z,\tau)}{V^*}+\varsigma_2\frac{V^2(x,y,z,\tau)}{(V^*)^2}\right]\times$$

$$\times\nabla_S\mu_1(x,y,z,\tau)\left[1+\frac{\xi_S\alpha_{1C}^\gamma}{P^\gamma(x,y,z,T)}\right]d\tau\Bigg\}+\alpha_{1C}\frac{\partial}{\partial y}\int_0^t D_{SL}(x,y,z,T)\left[1+\frac{\xi_S\alpha_{1C}^\gamma}{P^\gamma(x,y,z,T)}\right]+$$

$$\times \Omega \nabla_S \mu_1(x,y,z,\tau) \frac{z}{kT}\left[1+\varsigma_1 \frac{V(x,y,z,\tau)}{V^*}+\varsigma_2 \frac{V^2(x,y,z,\tau)}{(V^*)^2}\right] d\tau + f_C(x,y,z) \quad (1c)$$

$$I_1(x,y,z,t)=\alpha_{1I} z\Omega \frac{\partial}{\partial x}\int_0^t \frac{D_{IS}}{kT}\nabla_S \mu_1(x,y,z,\tau)\,d\tau+\alpha_{1I} z\Omega \frac{\partial}{\partial y}\int_0^t \frac{D_{IS}}{kT}\nabla_S \mu_1(x,y,z,\tau)\,d\tau+$$

$$+f_I(x,y,z)-\alpha_{1I}^2\int_0^t k_{I,I}(x,y,z,T)\,d\tau-\alpha_{1I}\alpha_{1V}\int_0^t k_{I,V}(x,y,z,T)\,d\tau \quad (3c)$$

$$V_1(x,y,z,t)=\alpha_{1V} z\Omega \frac{\partial}{\partial x}\int_0^t \frac{D_{IS}}{kT}\nabla_S \mu_1(x,y,z,\tau)\,d\tau+\alpha_{1V} z\Omega \frac{\partial}{\partial y}\int_0^t \frac{D_{IS}}{kT}\nabla_S \mu_1(x,y,z,\tau)\,d\tau+$$

$$+f_V(x,y,z)-\alpha_{1V}^2\int_0^t k_{V,V}(x,y,z,T)\,d\tau-\alpha_{1I}\alpha_{1V}\int_0^t k_{I,V}(x,y,z,T)\,d\tau$$

$$\Phi_{1I}(x,y,z,t)=\alpha_{1\Phi_I} z\Omega \frac{\partial}{\partial x}\int_0^t \frac{D_{\Phi_I S}}{kT}\nabla_S \mu_1(x,y,z,\tau)\,d\tau+\Omega \frac{\partial}{\partial x}\int_0^t \frac{D_{\Phi_I S}}{kT}\nabla_S \mu_1(x,y,z,\tau)\,d\tau\times$$

$$\times\alpha_{1\Phi_I} z+f_{\Phi_I}(x,y,z)++\int_0^t k_I(x,y,z,T)I(x,y,z,\tau)\,d\tau+\int_0^t k_{I,I}(x,y,z,T)I^2(x,y,z,\tau)\,d\tau \quad (5c)$$

$$\Phi_{1V}(x,y,z,t)=\alpha_{1\Phi_V} z\Omega \frac{\partial}{\partial x}\int_0^t \frac{D_{\Phi_V S}}{kT}\nabla_S \mu_1(x,y,z,\tau)\,d\tau+\Omega \frac{\partial}{\partial x}\int_0^t \frac{D_{\Phi_V S}}{kT}\nabla_S \mu_1(x,y,z,\tau)\,d\tau\times$$

$$\times\alpha_{1\Phi_V} z+f_{\Phi_V}(x,y,z)+\int_0^t k_V(x,y,z,T)V(x,y,z,\tau)\,d\tau+\int_0^t k_{V,V}(x,y,z,T)V^2(x,y,z,\tau)\,d\tau.$$

We calculate required average values of the considered approximations of the above concentrations by using standard relation [26]

$$\alpha_{1\rho}=\frac{1}{\Theta L_x L_y L_z}\int_0^\Theta\int_0^{L_x}\int_0^{L_y}\int_0^{L_z}\rho_1(x,y,z,t)\,d z\,d y\,d x\,d t. \quad (9)$$

Now it is necessary to substitute relations (1*c*), (3*c*), (5*c*) into the relation. After that we obtain the required relations to calculate recently considered average values a_{1r}. The relations could be written as

$$\alpha_{1C}=\frac{1}{L_x L_y L_z}\int_0^{L_x}\int_0^{L_y}\int_0^{L_z} f_C(x,y,z)\,dz\,dy\,dx,\quad \alpha_{1I}=\sqrt{\frac{(a_3+A)^2}{4a_4^2}-4\left(B+\frac{\Theta a_3 B+\Theta^2 L_x L_y L_z a_1}{a_4}\right)}-$$

$$-\frac{a_3+A}{4a_4},\quad \alpha_{1V}=\frac{1}{S_{IV00}}\left[\frac{\Theta}{\alpha_{1I}}\int_0^{L_x}\int_0^{L_y}\int_0^{L_z} f_I(x,y,z)\,dz\,dy\,dx-\alpha_{1I}S_{II00}-\Theta L_x L_y L_z\right].$$

Here $S_{\rho\rho ij}=\int\limits_0^{\Theta}(\Theta-t)\int\limits_0^{L_x}\int\limits_0^{L_y}\int\limits_0^{L_z}k_{\rho,\rho}(x,y,z,T)I_1^i(x,y,z,t)V_1^j(x,y,z,t)d\,z\,d\,y\,d\,x\,d\,t$, $a_4=S_{II00}\times$ $\times\left(S_{IV00}^2-S_{II00}S_{VV00}\right)$, $a_3=S_{IV00}S_{II00}+S_{IV00}^2-S_{II00}S_{VV00}$, $a_2=\int\limits_0^{L_x}\int\limits_0^{L_y}\int\limits_0^{L_z}f_V(x,y,z)d\,z\,d\,y\,d\,x\times$ $\times S_{IV00}S_{IV00}^2+S_{IV00}\Theta L_x^2L_y^2L_z^2+2\,S_{VV00}S_{II00}\int\limits_0^{L_x}\int\limits_0^{L_y}\int\limits_0^{L_z}f_I(x,y,z)d\,z\,d\,y\,d\,x-\Theta\,L_x^2L_y^2L_z^2S_{VV00}-$ $-S_{IV00}^2\int\limits_0^{L_x}\int\limits_0^{L_y}\int\limits_0^{L_z}f_I(x,y,z)d\,z\,d\,y\,d\,x$, $a_1=S_{IV00}\int\limits_0^{L_x}\int\limits_0^{L_y}\int\limits_0^{L_z}f_I(x,y,z)d\,z\,d\,y\,d\,x$, $a_0=S_{VV00}\times$ $\times\left[\int\limits_0^{L_x}\int\limits_0^{L_y}\int\limits_0^{L_z}f_I(x,y,z)d\,z\,d\,y\,d\,x\right]^2$, $A=\sqrt{8y+\Theta^2\frac{a_3^2}{a_4^2}-4\Theta\frac{a_2}{a_4}}$, $B=\frac{\Theta\,a_2}{6a_4}+\sqrt[3]{\sqrt{q^2+p^3}-q}-$ $-\sqrt[3]{\sqrt{q^2+p^3}+q}$, $q=\frac{\Theta^3a_2}{24\,a_4^2}\left(4a_0-\Theta\,L_xL_yL_z\frac{a_1a_3}{a_4}\right)-\Theta^2\frac{a_0}{8a_4^2}\left(4\,\Theta\,a_2-\Theta^2\frac{a_3^2}{a_4}\right)-$ $-\frac{\Theta^3a_2^3}{54a_4^3}-L_x^2L_y^2L_z^2\frac{\Theta^4a_1^2}{8a_4^2}$, $p=\Theta^2\frac{4a_0a_4-\Theta\,L_xL_yL_za_1a_3}{12a_4^2}-\frac{\Theta\,a_2}{18a_4}$,

$$\alpha_{1\Phi_I}=\frac{R_{I1}}{\Theta L_xL_yL_z}+\frac{S_{II20}}{\Theta L_xL_yL_z}+\frac{1}{L_xL_yL_z}\int\limits_0^{L_x}\int\limits_0^{L_y}\int\limits_0^{L_z}f_{\Phi_I}(x,y,z)\,d\,z\,d\,y\,d\,x$$

$$\alpha_{1\Phi_V}=\frac{R_{V1}}{\Theta L_xL_yL_z}+\frac{S_{VV20}}{\Theta L_xL_yL_z}+\frac{1}{L_xL_yL_z}\int\limits_0^{L_x}\int\limits_0^{L_y}\int\limits_0^{L_z}f_{\Phi_V}(x,y,z)\,d\,z\,d\,y\,d\,x.$$

Here $R_{\rho i}=\int\limits_0^{\Theta}(\Theta-t)\int\limits_0^{L_x}\int\limits_0^{L_y}\int\limits_0^{L_z}k_I(x,y,z,T)I_1^i(x,y,z,t)\,d\,z\,d\,y\,d\,x\,d\,t$.

We calculate the second-order approximation and approximations with higher orders of dopant and radiation defects concentrations by using standard averaging of function corrections [26]. Using this procedure for calculation of the *n*-th order approximations of radiation defects and dopant concentrations leads to necessity of replacement of considered concentrations on the sum $\alpha_{nr}+\rho_{n-1}(x,y,z,t)$ in right sides of the Eqs. (1*c*), (3*c*), (5*c*). After the replacement we obtain the following equations

$$\frac{\partial C_2(x,y,z,t)}{\partial t}=\frac{\partial}{\partial x}\left(\left\{1+\xi\frac{[\alpha_{2C}+C_1(x,y,z,t)]^{\gamma}}{P^{\gamma}(x,y,z,T)}\right\}\left[1+\varsigma_1\frac{V(x,y,z,t)}{V^*}+\varsigma_2\frac{V^2(x,y,z,t)}{(V^*)^2}\right]\times\right.$$

$$\times D_L(x,y,z,T)\frac{\partial C_1(x,y,z,t)}{\partial x}\Bigg)+\frac{\partial}{\partial y}\left(\left[1+\varsigma_1\frac{V(x,y,z,t)}{V^*}+\varsigma_2\frac{V^2(x,y,z,t)}{(V^*)^2}\right]\frac{\partial C_1(x,y,z,t)}{\partial y}\times\right.$$

$$\left.\times D_L(x,y,z,T)\left\{1+\xi\frac{[\alpha_{2C}+C_1(x,y,z,t)]^\gamma}{P^\gamma(x,y,z,T)}\right\}\right)+\frac{\partial}{\partial z}\left(\left[1+\varsigma_1\frac{V(x,y,z,t)}{V^*}+\varsigma_2\frac{V^2(x,y,z,t)}{(V^*)^2}\right]\times\right.$$

$$\left.\times D_L(x,y,z,T)\frac{\partial C_1(x,y,z,t)}{\partial z}\left\{1+\xi\frac{[\alpha_{2C}+C_1(x,y,z,t)]^\gamma}{P^\gamma(x,y,z,T)}\right\}\right)+f_C(x,y,z)\delta(t)+$$

$$+\Omega\frac{\partial}{\partial x}\left\{\frac{D_S}{kT}\nabla_S\mu_1(x,y,z,t)\int_0^{L_z}[\alpha_{2C}+C(x,y,W,t)]\,dW\right\}+$$

$$+\Omega\frac{\partial}{\partial y}\left\{\frac{D_S}{kT}\nabla_S\mu_1(x,y,z,t)\int_0^{L_z}[\alpha_{2C}+C(x,y,W,t)]\,dW\right\}\tag{1d}$$

$$\frac{\partial I_2(x,y,z,t)}{\partial t}=\frac{\partial}{\partial x}\left[D_I(x,y,z,T)\frac{\partial I_1(x,y,z,t)}{\partial x}\right]+\frac{\partial}{\partial y}\left[D_I(x,y,z,T)\frac{\partial I_1(x,y,z,t)}{\partial y}\right]+$$

$$+\frac{\partial}{\partial z}\left[D_I(x,y,z,T)\frac{\partial I_1(x,y,z,t)}{\partial z}\right]-k_{I,I}(x,y,z,T)[\alpha_{1I}+I_1(x,y,z,t)]^2-k_{I,V}(x,y,z,T)\times$$

$$\times[\alpha_{1I}+I_1(x,y,z,t)][\alpha_{1V}+V_1(x,y,z,t)]+\Omega\frac{\partial}{\partial x}\left\{\nabla_S\mu(x,y,z,t)\int_0^{L_z}[\alpha_{2I}+I_1(x,y,W,t)]\,dW\times\right.$$

$$\left.\times\frac{D_{IS}}{kT}\right\}+\Omega\frac{\partial}{\partial y}\left\{\frac{D_{IS}}{kT}\nabla_S\mu(x,y,z,t)\int_0^{L_z}[\alpha_{2I}+I_1(x,y,W,t)]\,dW\right\}\tag{3d}$$

$$\frac{\partial V_2(x,y,z,t)}{\partial t}=\frac{\partial}{\partial x}\left[D_V(x,y,z,T)\frac{\partial V_1(x,y,z,t)}{\partial x}\right]+\frac{\partial}{\partial y}\left[D_V(x,y,z,T)\frac{\partial V_1(x,y,z,t)}{\partial y}\right]+$$

$$+\frac{\partial}{\partial z}\left[D_V(x,y,z,T)\frac{\partial V_1(x,y,z,t)}{\partial z}\right]-k_{V,V}(x,y,z,T)[\alpha_{1V}+V_1(x,y,z,t)]^2-k_{I,V}(x,y,z,T)\times$$

$$\times[\alpha_{1I}+I_1(x,y,z,t)][\alpha_{1V}+V_1(x,y,z,t)]+\Omega\frac{\partial}{\partial x}\left\{\nabla_S\mu(x,y,z,t)\int_0^{L_z}[\alpha_{2V}+V_1(x,y,W,t)]\,dW\times\right.$$

$$\left.\times\frac{D_{VS}}{kT}\right\}+\Omega\frac{\partial}{\partial y}\left\{\frac{D_{VS}}{kT}\nabla_S\mu(x,y,z,t)\int_0^{L_z}[\alpha_{2V}+V_1(x,y,W,t)]\,dW\right\}$$

$$\frac{\partial \Phi_{2I}(x,y,z,t)}{\partial t}=\frac{\partial}{\partial x}\left[D_{\Phi_I}(x,y,z,T)\frac{\partial \Phi_{1I}(x,y,z,t)}{\partial x}\right]+\frac{\partial}{\partial y}\left[D_{\Phi_I}(x,y,z,T)\frac{\partial \Phi_{1I}(x,y,z,t)}{\partial y}\right]+$$

$$+\Omega\frac{\partial}{\partial x}\left\{\frac{D_{\Phi_I S}}{kT}\nabla_S \mu\,(x,y,z,t)\int\limits_0^{L_z}\left[\alpha_{2\Phi_I}+\Phi_{1I}(x,y,W,t)\right]dW\right\}+k_{I,I}(x,y,z,T)I^2(x,y,z,t)+$$

$$+\Omega\frac{\partial}{\partial y}\left\{\frac{D_{\Phi_I S}}{kT}\nabla_S \mu\,(x,y,z,t)\int\limits_0^{L_z}\left[\alpha_{2\Phi_I}+\Phi_{1I}(x,y,W,t)\right]dW\right\}+k_I(x,y,z,T)I(x,y,z,t)+$$

$$+\frac{\partial}{\partial z}\left[D_{\Phi_I}(x,y,z,T)\frac{\partial \Phi_{1I}(x,y,z,t)}{\partial z}\right]+f_{\Phi_I}(x,y,z)\delta(t) \qquad (5d)$$

$$\frac{\partial \Phi_{2V}(x,y,z,t)}{\partial t}=\frac{\partial}{\partial x}\left[D_{\Phi_V}(x,y,z,T)\frac{\partial \Phi_{1V}(x,y,z,t)}{\partial x}\right]+\frac{\partial}{\partial y}\left[D_{\Phi_V}(x,y,z,T)\frac{\partial \Phi_{1V}(x,y,z,t)}{\partial y}\right]+$$

$$+\Omega\frac{\partial}{\partial x}\left\{\frac{D_{\Phi_V S}}{kT}\nabla_S \mu\,(x,y,z,t)\int\limits_0^{L_z}\left[\alpha_{2\Phi_V}+\Phi_{1V}(x,y,W,t)\right]dW\right\}+k_{V,V}(x,y,z,T)V^2(x,y,z,t)+$$

$$+\Omega\frac{\partial}{\partial y}\left\{\frac{D_{\Phi_V S}}{kT}\nabla_S \mu\,(x,y,z,t)\int\limits_0^{L_z}\left[\alpha_{2\Phi_V}+\Phi_{1V}(x,y,W,t)\right]dW\right\}+k_V(x,y,z,T)V(x,y,z,t)+$$

$$+\frac{\partial}{\partial z}\left[D_{\Phi_V}(x,y,z,T)\frac{\partial \Phi_{1V}(x,y,z,t)}{\partial z}\right]+f_{\Phi_V}(x,y,z)\delta(t).$$

Integration of the both sides of Eqs. (1*d*), (3*d*) and (5*d*) leads to obtaining following relations for the considered concentrations

$$C_2(x,y,z,t)=\frac{\partial}{\partial x}\int\limits_0^t\left\{1+\xi\frac{\left[\alpha_{2C}+C_1(x,y,z,\tau)\right]^\gamma}{P^\gamma(x,y,z,T)}\right\}\left[1+\varsigma_1\frac{V(x,y,z,\tau)}{V^*}+\varsigma_2\frac{V^2(x,y,z,\tau)}{(V^*)^2}\right]\times$$

$$\times D_L(x,y,z,T)\frac{\partial C_1(x,y,z,\tau)}{\partial x}d\tau+\frac{\partial}{\partial y}\int\limits_0^t D_L(x,y,z,T)\left[1+\varsigma_1\frac{V(x,y,z,\tau)}{V^*}+\varsigma_2\frac{V^2(x,y,z,\tau)}{(V^*)^2}\right]\times$$

$$\times\frac{\partial C_1(x,y,z,\tau)}{\partial y}\left\{1+\xi\frac{\left[\alpha_{2C}+C_1(x,y,z,t)\right]^\gamma}{P^\gamma(x,y,z,T)}\right\}+\frac{\partial}{\partial z}\int\limits_0^t\left[1+\varsigma_1\frac{V(x,y,z,\tau)}{V^*}+\varsigma_2\frac{V^2(x,y,z,\tau)}{(V^*)^2}\right]\times$$

$$\times D_L(x,y,z,T)\frac{\partial C_1(x,y,z,\tau)}{\partial z}\left\{1+\xi\frac{\left[\alpha_{2C}+C_1(x,y,z,\tau)\right]^\gamma}{P^\gamma(x,y,z,T)}\right\}d\tau+f_C(x,y,z)+$$

$$+\Omega\frac{\partial}{\partial x}\int_0^t\frac{D_S}{kT}\nabla_S\mu\left(x,y,z,\tau\right)\int_0^{L_z}\left[\alpha_{2C}+C_1\left(x,y,W,\tau\right)\right]dW\,d\tau+\frac{\partial}{\partial y}\int_0^t\nabla_S\mu\left(x,y,z,\tau\right)\times$$

$$\times\Omega\frac{D_S}{kT}\int_0^{L_z}\left[\alpha_{2C}+C_1\left(x,y,W,\tau\right)\right]dW\,d\tau \qquad (1e)$$

$$I_2(x,y,z,t)=\frac{\partial}{\partial x}\int_0^t D_I(x,y,z,T)\frac{\partial I_1(x,y,z,\tau)}{\partial x}d\tau+\frac{\partial}{\partial y}\int_0^t D_I(x,y,z,T)\frac{\partial I_1(x,y,z,\tau)}{\partial y}d\tau+$$

$$+\frac{\partial}{\partial z}\int_0^t D_I(x,y,z,T)\frac{\partial I_1(x,y,z,\tau)}{\partial z}d\tau-\int_0^t k_{I,I}(x,y,z,T)\left[\alpha_{2I}+I_1(x,y,z,\tau)\right]^2 d\tau-$$

$$-\int_0^t k_{I,V}(x,y,z,T)\left[\alpha_{2I}+I_1(x,y,z,\tau)\right]\left[\alpha_{2V}+V_1(x,y,z,\tau)\right]d\tau+\frac{\partial}{\partial x}\int_0^t\nabla_S\mu\left(x,y,z,\tau\right)\times$$

$$\times\Omega\frac{D_{IS}}{kT}\int_0^{L_z}\left[\alpha_{2I}+I_1(x,y,W,\tau)\right]dW\,d\tau+\frac{\partial}{\partial y}\int_0^t\nabla_S\mu\left(x,y,z,\tau\right)\int_0^{L_z}\left[\alpha_{2I}+I_1(x,y,W,\tau)\right]\times$$

$$\times\Omega\frac{D_{IS}}{kT}d\,W\,d\tau \qquad (3e)$$

$$V_2(x,y,z,t)=\frac{\partial}{\partial x}\int_0^t D_V(x,y,z,T)\frac{\partial V_1(x,y,z,\tau)}{\partial x}d\tau+\frac{\partial}{\partial y}\int_0^t D_V(x,y,z,T)\frac{\partial V_1(x,y,z,\tau)}{\partial y}d\tau+$$

$$+\frac{\partial}{\partial z}\int_0^t D_V(x,y,z,T)\frac{\partial V_1(x,y,z,\tau)}{\partial z}d\tau-\int_0^t k_{V,V}(x,y,z,T)\left[\alpha_{2V}+V_1(x,y,z,\tau)\right]^2 d\tau-$$

$$-\int_0^t k_{I,V}(x,y,z,T)\left[\alpha_{2I}+I_1(x,y,z,\tau)\right]\left[\alpha_{2V}+V_1(x,y,z,\tau)\right]d\tau+\frac{\partial}{\partial x}\int_0^t\nabla_S\mu\left(x,y,z,\tau\right)\times$$

$$\times\Omega\frac{D_{VS}}{kT}\int_0^{L_z}\left[\alpha_{2V}+V_1(x,y,W,\tau)\right]dW\,d\tau+\frac{\partial}{\partial y}\int_0^t\nabla_S\mu\left(x,y,z,\tau\right)\int_0^{L_z}\left[\alpha_{2V}+V_1(x,y,W,\tau)\right]\times$$

$$\times\Omega\frac{D_{VS}}{kT}d\,W\,d\tau+f_V\left(x,y,z\right)$$

$$\Phi_{2I}(x,y,z,t)=\frac{\partial}{\partial x}\int_0^t D_{\Phi_I}(x,y,z,T)\frac{\partial\Phi_{1I}(x,y,z,\tau)}{\partial x}d\tau+\frac{\partial}{\partial y}\int_0^t\frac{\partial\Phi_{1I}(x,y,z,\tau)}{\partial y}\times$$

$$\times D_{\Phi_I}(x,y,z,T)d\tau+\frac{\partial}{\partial z}\int_0^t D_{\Phi_I}(x,y,z,T)\frac{\partial\Phi_{1I}(x,y,z,\tau)}{\partial z}d\tau+\Omega\frac{\partial}{\partial x}\int_0^t\nabla_S\mu\left(x,y,z,\tau\right)\times$$

$$\times \frac{D_{\Phi_I S}}{kT}\int_0^{L_z}\left[\alpha_{2\Phi_I}+\Phi_{1I}(x,y,W,\tau)\right]dW\,d\tau+\Omega\frac{\partial}{\partial y}\int_0^t\frac{D_{\Phi_I S}}{kT}\int_0^{L_z}\left[\alpha_{2\Phi_I}+\Phi_{1I}(x,y,W,\tau)\right]dW\times$$

$$\times\nabla_S\mu(x,y,z,\tau)d\tau+\int_0^t k_{I,I}(x,y,z,T)I^2(x,y,z,\tau)\,d\tau+\int_0^t k_I(x,y,z,T)I(x,y,z,\tau)\,d\tau+$$

$$+f_{\Phi_I}(x,y,z) \qquad (5e)$$

$$\Phi_{2V}(x,y,z,t)=\frac{\partial}{\partial x}\int_0^t D_{\Phi_V}(x,y,z,T)\frac{\partial\Phi_{1V}(x,y,z,\tau)}{\partial x}d\tau+\frac{\partial}{\partial y}\int_0^t\frac{\partial\Phi_{1V}(x,y,z,\tau)}{\partial y}\times$$

$$\times D_{\Phi_V}(x,y,z,T)d\tau+\frac{\partial}{\partial z}\int_0^t D_{\Phi_V}(x,y,z,T)\frac{\partial\Phi_{1V}(x,y,z,\tau)}{\partial z}d\tau+\Omega\frac{\partial}{\partial x}\int_0^t\nabla_S\mu(x,y,z,\tau)\times$$

$$\times\frac{D_{\Phi_V S}}{kT}\int_0^{L_z}\left[\alpha_{2\Phi_V}+\Phi_{1V}(x,y,W,\tau)\right]dW\,d\tau+\Omega\frac{\partial}{\partial y}\int_0^t\frac{D_{\Phi_V S}}{kT}\int_0^{L_z}\left[\alpha_{2\Phi_V}+\Phi_{1V}(x,y,W,\tau)\right]dW\times$$

$$\times\nabla_S\mu(x,y,z,\tau)\,d\tau+\int_0^t k_{V,V}(x,y,z,T)V^2(x,y,z,\tau)\,d\tau+\int_0^t k_V(x,y,z,T)V(x,y,z,\tau)\,d\tau+$$

$$+f_{\Phi_V}(x,y,z).$$

Averaged values of the considered approximations could be obtain framework following relation [26]

$$\alpha_{2\rho}=\frac{1}{\Theta L_xL_yL_z}\int_0^\Theta\int_0^{L_x}\int_0^{L_y}\int_0^{L_z}\left[\rho_2(x,y,z,t)-\rho_1(x,y,z,t)\right]dz\,dy\,dx\,dt. \qquad (10)$$

Now we will use relations (1*e*), (3*e*), (5*e*) into the above relation (10). After that we obtain the following relations for the considered average values a_{2r}

$$a_{2C}=0,\ a_{2FI}=0,\ a_{2FV}=0,\ \alpha_{2V}=\sqrt{\frac{(b_3+E)^2}{4b_4^2}-4\left(F+\frac{\Theta a_3F+\Theta^2L_xL_yL_zb_1}{b_4}\right)}-\frac{b_3+E}{4b_4},$$

$$\alpha_{2I}=\frac{C_V-\alpha_{2V}^2S_{VV00}-\alpha_{2V}\left(2S_{VV01}+S_{IV10}+\Theta L_xL_yL_z\right)-S_{VV02}-S_{IV11}}{S_{IV01}+\alpha_{2V}S_{IV00}}.$$

Here $b_4=\frac{1}{\Theta L_xL_yL_z}S_{IV00}^2S_{VV00}-\frac{1}{\Theta L_xL_yL_z}S_{VV00}^2S_{II00}$, $b_3=-\frac{S_{II00}S_{VV00}}{\Theta L_xL_yL_z}\left(2S_{VV01}+S_{IV10}+\right.$

$$\left.+\Theta L_xL_yL_z\right)+\frac{S_{IV00}S_{VV00}}{\Theta L_xL_yL_z}\left(S_{IV01}+2S_{II10}+S_{IV01}+\Theta L_xL_yL_z\right)+\frac{S_{IV00}^2}{\Theta L_xL_yL_z}\left(2S_{VV01}+S_{IV10}+\right.$$

$$+\Theta L_x L_y L_z\big)-\frac{S_{IV00}^2 S_{IV10}}{\Theta^3 L_x^3 L_y^3 L_z^3},\; b_2=\frac{S_{II00}S_{VV00}}{\Theta L_x L_y L_z}\left(S_{VV02}+S_{IV11}+C_V\right)-\left(S_{IV10}-2S_{VV01}+\Theta L_x L_y\times\right.$$

$$\left.\times L_z\right)^2+\frac{S_{IV01}S_{VV00}}{\Theta L_x L_y L_z}\left(\Theta L_x L_y L_z+2S_{II10}+S_{IV01}\right)+\frac{S_{IV00}}{\Theta L_x L_y L_z}\left(S_{IV01}+2S_{II10}+2S_{IV01}+\Theta L_x L_y\times\right.$$

$$\left.\times L_z\right)\left(2S_{VV01}+\Theta L_x L_y L_z+S_{IV10}\right)-\frac{S_{IV00}^2}{\Theta L_x L_y L_z}\left(C_V-S_{VV02}-S_{IV11}\right)+\frac{C_I S_{IV00}^2}{\Theta^2 L_x^2 L_y^2 L_z^2}-\frac{2\,S_{IV10}}{\Theta L_x L_y L_z}\times$$

$$\times S_{IV00}S_{IV01},\; b_1=S_{II00}\frac{S_{IV11}+S_{VV02}+C_V}{\Theta L_x L_y L_z}\left(2S_{VV01}+S_{IV10}+\Theta L_x L_y L_z\right)+\frac{S_{IV01}}{\Theta L_x L_y L_z}\left(\Theta L_x L_y\times\right.$$

$$\left.\times L_z+2S_{II10}+S_{IV01}\right)\left(2S_{VV01}+S_{IV10}+\Theta L L_y L_z\right)-\frac{S_{IV10}S_{IV01}^2}{\Theta L_x L_y L_z}-\frac{S_{IV00}}{\Theta L_x L_y L_z}\left(3S_{IV01}+2S_{II10}+\right.$$

$$\left.+\Theta L_x L_y L_z\right)\left(C_V-S_{VV02}-S_{IV11}\right)+2\,C_I S_{IV00}S_{IV01},\; b_0=\frac{S_{II00}}{\Theta L_x L_y L_z}\left(S_{IV00}+S_{VV02}\right)^2-\frac{S_{IV01}}{L_x L_y L_z}\times$$

$$\times\frac{1}{\Theta}\left(\Theta L_x L_y L_z+2\,S_{II10}+S_{IV01}\right)\left(C_V-S_{VV02}-S_{IV11}\right)+2C_I S_{IV01}^2-S_{IV01}\frac{C_V-S_{VV02}-S_{IV11}}{\Theta L_x L_y L_z}\times$$

$$\times\left(\Theta L_x L_y L_z+2\,S_{II10}+S_{IV01}\right),\; C_I=\frac{\alpha_{1I}\alpha_{1V}}{\Theta L_x L_y L_z}S_{IV00}+\frac{\alpha_{1I}^2 S_{II00}}{\Theta L_x L_y L_z}-\frac{S_{II20}S_{II20}}{\Theta L_x L_y L_z}-\frac{S_{IV11}}{\Theta L_x L_y L_z},$$

$$C_V=\alpha_{1I}\alpha_{1V}S_{IV00}+\alpha_{1V}^2 S_{VV00}-S_{VV02}-S_{IV11},\; E=\sqrt{8y+\Theta^2\frac{a_3^2}{a_4^2}-4\Theta\frac{a_2}{a_4}},\; F=\frac{\Theta\,a_2}{6\,a_4}+$$

$$+\sqrt[3]{\sqrt{r^2+s^3}-r}-\sqrt[3]{\sqrt{r^2+s^3}+r},\; r=\frac{\Theta^3 b_2}{24b_4^2}\left(4b_0-\Theta\,L_x L_y L_z\frac{b_1 b_3}{b_4}\right)-\frac{\Theta^3 b_2^3}{54b_4^3}-b_0\frac{\Theta^2}{8b_4^2}\times$$

$$\times\left(4\,\Theta\,b_2-\Theta^2\frac{b_3^2}{b_4}\right)-L_x^2 L_y^2 L_z^2\frac{\Theta^4 b_1^2}{8b_4^2},\; s=\Theta^2\frac{4b_0 b_4-\Theta\,L_x L_y L_z b_1 b_3}{12\,b_4^2}-\frac{\Theta\,b_2}{18\,b_4}.$$

Now we will calculate components of displacement vector by solution of equations of system (8). We will calculate approximations with the first-order of the considered components by using standard method of averaging of function corrections. We calculate approximations with the first-order of the above components by using method of averaging of function corrections. To calculate the approximations one shall replace the required functions on their not yet known average values a_i in right sides of appropriate equations. After the replacement we obtain Eqs. (8) in the following form

$$\rho(z)\frac{\partial^2 u_{1x}(x,y,z,t)}{\partial t^2}=-K(z)\beta(z)\frac{\partial T(x,y,z,t)}{\partial x},\ \rho(z)\frac{\partial^2 u_{1y}(x,y,z,t)}{\partial t^2}=$$

$$=-K(z)\beta(z)\frac{\partial T(x,y,z,t)}{\partial y},\ \rho(z)\frac{\partial^2 u_{1z}(x,y,z,t)}{\partial t^2}=-K(z)\beta(z)\frac{\partial T(x,y,z,t)}{\partial z}.$$

Integration of these equations on time gives a possibility to obtain required approximations in the following form

$$u_{1x}(x,y,z,t)=u_{0x}+K(z)\frac{\beta(z)}{\rho(z)}\frac{\partial}{\partial x}\int_0^t\int_0^{\vartheta}T(x,y,z,\tau)\,d\tau\,d\vartheta-$$

$$-K(z)\frac{\beta(z)}{\rho(z)}\frac{\partial}{\partial x}\int_0^{\infty}\int_0^{\vartheta}T(x,y,z,\tau)\,d\tau\,d\vartheta,$$

$$u_{1y}(x,y,z,t)=u_{0y}+K(z)\frac{\beta(z)}{\rho(z)}\frac{\partial}{\partial y}\int_0^t\int_0^{\vartheta}T(x,y,z,\tau)\,d\tau\,d\vartheta-$$

$$-K(z)\frac{\beta(z)}{\rho(z)}\frac{\partial}{\partial y}\int_0^{\infty}\int_0^{\vartheta}T(x,y,z,\tau)\,d\tau\,d\vartheta,$$

$$u_{1z}(x,y,z,t)=u_{0z}+K(z)\frac{\beta(z)}{\rho(z)}\frac{\partial}{\partial z}\int_0^t\int_0^{\vartheta}T(x,y,z,\tau)\,d\tau\,d\vartheta-$$

$$-K(z)\frac{\beta(z)}{\rho(z)}\frac{\partial}{\partial z}\int_0^{\infty}\int_0^{\vartheta}T(x,y,z,\tau)\,d\tau\,d\vartheta.$$

The next-order approximations could be obtained by analogous replacement on the following sums $a_i+u_{i-1}(x,y,z,t)$ [26]. Eqs. (8) for the second-order approximations takes the form

$$\rho(z)\frac{\partial^2 u_{2x}(x,y,z,t)}{\partial t^2}=\left\{K(z)+\frac{5E(z)}{6[1+\sigma(z)]}\right\}\frac{\partial^2 u_{1x}(x,y,z,t)}{\partial x^2}+\left\{K(z)-\frac{E(z)}{3[1+\sigma(z)]}\right\}\times$$

$$\times\frac{\partial^2 u_{1y}(x,y,z,t)}{\partial x\,\partial y}+\frac{E(z)}{2[1+\sigma(z)]}\left[\frac{\partial^2 u_{1y}(x,y,z,t)}{\partial y^2}+\frac{\partial^2 u_{1z}(x,y,z,t)}{\partial z^2}\right]-\frac{\partial T(x,y,z,t)}{\partial x}\times$$

$$\times K(z)\beta(z)+\left\{K(z)+\frac{E(z)}{3[1+\sigma(z)]}\right\}\frac{\partial^2 u_{1z}(x,y,z,t)}{\partial x\partial z}$$

$$\rho(z)\frac{\partial^2 u_{2y}(x,y,z,t)}{\partial t^2}=\frac{E(z)}{2[1+\sigma(z)]}\left[\frac{\partial^2 u_{1y}(x,y,z,t)}{\partial x^2}+\frac{\partial^2 u_{1x}(x,y,z,t)}{\partial x\partial y}\right]-\frac{\partial T(x,y,z,t)}{\partial y}\times$$

$$\times K(z)\beta(z)+\frac{\partial}{\partial z}\left\{\frac{E(z)}{2[1+\sigma(z)]}\left[\frac{\partial u_{1y}(x,y,z,t)}{\partial z}+\frac{\partial u_{1z}(x,y,z,t)}{\partial y}\right]\right\}+\frac{\partial^2 u_{1y}(x,y,z,t)}{\partial y^2}\times$$

$$\times\left\{\frac{5E(z)}{12[1+\sigma(z)]}+K(z)\right\}+\left\{K(z)-\frac{E(z)}{6[1+\sigma(z)]}\right\}\frac{\partial^2 u_{1y}(x,y,z,t)}{\partial y\partial z}+K(z)\frac{\partial^2 u_{1y}(x,y,z,t)}{\partial x\partial y}$$

$$\rho(z)\frac{\partial^2 u_{2z}(x,y,z,t)}{\partial t^2}=\frac{E(z)}{2\,[1+\sigma(z)]}\left[\frac{\partial^2 u_{1z}(x,y,z,t)}{\partial x^2}+\frac{\partial^2 u_{1z}(x,y,z,t)}{\partial y^2}+\frac{\partial^2 u_{1x}(x,y,z,t)}{\partial x\partial z}+\right.$$

$$\left.+\frac{\partial^2 u_{1y}(x,y,z,t)}{\partial y\partial z}\right]+\frac{\partial}{\partial z}\left\{K(z)\left[\frac{\partial u_{1x}(x,y,z,t)}{\partial x}+\frac{\partial u_{1y}(x,y,z,t)}{\partial y}+\frac{\partial u_{1x}(x,y,z,t)}{\partial z}\right]\right\}+$$

$$+\frac{E(z)}{6[1+\sigma(z)]}\frac{\partial}{\partial z}\left[6\frac{\partial u_{1z}(x,y,z,t)}{\partial z}-\frac{\partial u_{1x}(x,y,z,t)}{\partial x}-\frac{\partial u_{1y}(x,y,z,t)}{\partial y}-\frac{\partial u_{1z}(x,y,z,t)}{\partial z}\right]-$$

$$\left.\left.-\frac{\partial u_{1x}(x,y,z,t)}{\partial x}-\frac{\partial u_{1y}(x,y,z,t)}{\partial y}-\frac{\partial u_{1z}(x,y,z,t)}{\partial z}\right]\right\}\frac{E(z)}{1+\sigma(z)}-K(z)\beta(z)\frac{\partial T(x,y,z,t)}{\partial z}.$$

Integration of these equations on time gives a possibility to obtain the required second-order approximations of considered components of displacement vector

$$u_{2x}(x,y,z,t)=\frac{1}{\rho(z)}\left\{K(z)+\frac{5E(z)}{6[1+\sigma(z)]}\right\}\frac{\partial^2}{\partial x^2}\int_0^t\int_0^{\vartheta}u_{1x}(x,y,z,\tau)\,d\tau\,d\vartheta+\frac{1}{\rho(z)}\left\{K(z)-\right.$$

$$\left.-\frac{E(z)}{3\,[1+\sigma(z)]}\right\}\frac{\partial^2}{\partial x\partial y}\int_0^t\int_0^{\vartheta}u_{1y}(x,y,z,\tau)\,d\tau\,d\vartheta+\frac{E(z)}{2\,\rho(z)}\left[\frac{\partial^2}{\partial y^2}\int_0^t\int_0^{\vartheta}u_{1y}(x,y,z,\tau)\,d\tau\,d\vartheta+\right.$$

$$\left.+\frac{\partial^2}{\partial z^2}\int_0^t\int_0^{\vartheta}u_{1z}(x,y,z,\tau)\,d\tau\,d\vartheta\right]\frac{1}{1+\sigma(z)}+\frac{1}{\rho(z)}\frac{\partial^2}{\partial x\partial z}\int_0^t\int_0^{\vartheta}u_{1z}(x,y,z,\tau)d\tau\,d\vartheta\,\left\{K(z)+\right.$$

$$\left.+\frac{E(z)}{3\,[1+\sigma(z)]}\right\}-K(z)\frac{\beta(z)}{\rho(z)}\frac{\partial}{\partial x}\int_0^t\int_0^{\vartheta}T(x,y,z,\tau)\,d\tau\,d\vartheta-\frac{\partial^2}{\partial x^2}\int_0^{\infty}\int_0^{\vartheta}u_{1x}(x,y,z,\tau)d\tau\,d\vartheta\times$$

$$\times\frac{1}{\rho(z)}\left\{K(z)+\frac{5E(z)}{6\,[1+\sigma(z)]}\right\}-\left\{K(z)-\frac{E(z)}{3[1+\sigma(z)]}\right\}\frac{\partial^2}{\partial x\partial y}\int\limits_0^{\infty}\int\limits_0^{\vartheta}u_{1y}(x,y,z,\tau)\,d\,\tau\,d\,\vartheta\times$$

$$\times\frac{1}{\rho(z)}-\frac{E(z)}{2\,\rho(z)[1+\sigma(z)]}\left[\frac{\partial^2}{\partial y^2}\int\limits_0^{\infty}\int\limits_0^{\vartheta}u_{1y}(x,y,z,\tau)d\,\tau\,d\,\vartheta+\frac{\partial^2}{\partial z^2}\int\limits_0^{\infty}\int\limits_0^{\vartheta}u_{1z}(x,y,z,\tau)d\,\tau\,d\,\vartheta\right]-$$

$$-\frac{1}{\rho(z)}\left\{K(z)+\frac{E(z)}{3\,[1+\sigma(z)]}\right\}\frac{\partial^2}{\partial x\partial z}\int\limits_0^{\infty}\int\limits_0^{\vartheta}u_{1z}(x,y,z,\tau)d\,\tau\,d\,\vartheta+u_{0x}+K(z)\frac{\beta(z)}{\rho(z)}\times$$

$$\times\frac{\partial}{\partial x}\int\limits_0^{\infty}\int\limits_0^{\vartheta}T(x,y,z,\tau)\,d\,\tau\,d\,\vartheta$$

$$u_{2y}(x,y,z,t)=\frac{E(z)}{2\,\rho(z)}\left[\frac{\partial^2}{\partial x^2}\int\limits_0^{t}\int\limits_0^{\vartheta}u_{1x}(x,y,z,\tau)d\,\tau\,d\,\vartheta+\frac{\partial^2}{\partial x\partial y}\int\limits_0^{t}\int\limits_0^{\vartheta}u_{1x}(x,y,z,\tau)d\,\tau\,d\,\vartheta\right]\times$$

$$\times\frac{1}{1+\sigma(z)}+\frac{K(z)}{\rho(z)}\frac{\partial^2}{\partial x\partial y}\int\limits_0^{t}\int\limits_0^{\vartheta}u_{1y}(x,y,z,\tau)d\,\tau\,d\,\vartheta+\frac{1}{\rho(z)}\left\{\frac{5E(z)}{12[1+\sigma(z)]}+K(z)\right\}\times$$

$$\times\frac{\partial^2}{\partial y^2}\int\limits_0^{t}\int\limits_0^{\vartheta}u_{1x}(x,y,z,\tau)\,d\,\tau\,d\,\vartheta+\frac{1}{2\rho(z)}\frac{\partial}{\partial z}\left\{\frac{E(z)}{1+\sigma(z)}\left[\frac{\partial}{\partial z}\int\limits_0^{t}\int\limits_0^{\vartheta}u_{1y}(x,y,z,\tau)\,d\,\tau\,d\,\vartheta+\right.\right.$$

$$\left.\left.+\frac{\partial}{\partial y}\int\limits_0^{t}\int\limits_0^{\vartheta}u_{1z}(x,y,z,\tau)d\,\tau\,d\,\vartheta\right]\right\}-K(z)\frac{\beta(z)}{\rho(z)}\int\limits_0^{t}\int\limits_0^{\vartheta}T\,(x,y,z,\tau)d\,\tau\,d\,\vartheta-\left\{\frac{E(z)}{6[1+\sigma(z)]}-\right.$$

$$\left.-K(z)\right\}\frac{1}{\rho(z)}\frac{\partial^2}{\partial y\partial z}\int\limits_0^{t}\int\limits_0^{\vartheta}u_{1y}(x,y,z,\tau)d\,\tau\,d\,\vartheta-\frac{E(z)}{2\,\rho(z)}\left[\frac{\partial^2}{\partial x^2}\int\limits_0^{\infty}\int\limits_0^{\vartheta}u_{1x}(x,y,z,\tau)d\,\tau\,d\,\vartheta+\right.$$

$$\left.+\frac{\partial^2}{\partial x\partial y}\int\limits_0^{\infty}\int\limits_0^{\vartheta}u_{1x}(x,y,z,\tau)d\,\tau\,d\,\vartheta\right]\frac{1}{1+\sigma(z)}-K(z)\frac{\beta(z)}{\rho(z)}\int\limits_0^{\infty}\int\limits_0^{\vartheta}T\,(x,y,z,\tau)\,d\,\tau\,d\,\vartheta-\frac{K(z)}{\rho(z)}\times$$

$$\times\frac{\partial^2}{\partial x\partial y}\int\limits_0^{\infty}\int\limits_0^{\vartheta}u_{1y}(x,y,z,\tau)d\,\tau\,d\,\vartheta-\frac{1}{\rho(z)}\frac{\partial^2}{\partial y^2}\int\limits_0^{\infty}\int\limits_0^{\vartheta}u_{1x}(x,y,z,\tau)\,d\,\tau\,d\,\vartheta\left\{\frac{5E(z)}{12\,[1+\sigma(z)]}+\right.$$

$$\left.+K(z)\right\}-\frac{\partial}{\partial z}\left\{\frac{E(z)}{1+\sigma(z)}\left[\frac{\partial}{\partial z}\int\limits_0^{\infty}\int\limits_0^{\vartheta}u_{1y}(x,y,z,\tau)\,d\,\tau\,d\,\vartheta+\frac{\partial}{\partial y}\int\limits_0^{\infty}\int\limits_0^{\vartheta}u_{1z}(x,y,z,\tau)\,d\,\tau\,d\,\vartheta\right]\right\}\times$$

$$\times\frac{1}{2\rho(z)}-\frac{1}{\rho(z)}\left\{K(z)-\frac{E(z)}{6[1+\sigma(z)]}\right\}\frac{\partial^2}{\partial y\partial z}\int_0^\infty\int_0^\vartheta u_{1y}(x,y,z,\tau)\,d\tau\,d\vartheta+u_{0y}$$

$$u_z(x,y,z,t)=\frac{E(z)}{2[1+\sigma(z)]}\left[\frac{\partial^2}{\partial x^2}\int_0^\infty\int_0^\vartheta u_{1z}(x,y,z,\tau)\,d\tau\,d\vartheta+\frac{\partial^2}{\partial y^2}\int_0^\infty\int_0^\vartheta u_{1z}(x,y,z,\tau)\,d\tau\,d\vartheta+\right.$$

$$\left.+\frac{\partial^2}{\partial x\partial z}\int_0^\infty\int_0^\vartheta u_{1x}(x,y,z,\tau)\,d\tau\,d\vartheta+\frac{\partial^2}{\partial y\partial z}\int_0^\infty\int_0^\vartheta u_{1y}(x,y,z,\tau)\,d\tau\,d\vartheta\right]\frac{1}{\rho(z)}+\frac{1}{\rho(z)}\times$$

$$\times\frac{\partial}{\partial z}\left\{K(z)\left[\frac{\partial}{\partial x}\int_0^\infty\int_0^\vartheta u_{1x}(x,y,z,\tau)\,d\tau\,d\vartheta+\frac{\partial}{\partial y}\int_0^\infty\int_0^\vartheta u_{1x}(x,y,z,\tau)\,d\tau\,d\vartheta+\right.\right.$$

$$\left.\left.+\frac{\partial}{\partial z}\int_0^\infty\int_0^\vartheta u_{1x}(x,y,z,\tau)\,d\tau\,d\vartheta\right]\right\}+\frac{1}{6\rho(z)}\frac{\partial}{\partial z}\left\{\frac{E(z)}{1+\sigma(z)}\left[6\frac{\partial}{\partial z}\int_0^\infty\int_0^\vartheta u_{1z}(x,y,z,\tau)\,d\tau\,d\vartheta-\right.\right.$$

$$\left.\left.-\frac{\partial}{\partial x}\int_0^\infty\int_0^\vartheta u_{1x}(x,y,z,\tau)\,d\tau\,d\vartheta-\frac{\partial}{\partial y}\int_0^\infty\int_0^\vartheta u_{1y}(x,y,z,\tau)\,d\tau\,d\vartheta-\frac{\partial}{\partial z}\int_0^\infty\int_0^\vartheta u_{1z}(x,y,z,\tau)\,d\tau\,d\vartheta\right]\right\}-$$

$$-K(z)\frac{\beta(z)}{\rho(z)}\frac{\partial}{\partial z}\int_0^\infty\int_0^\vartheta T(x,y,z,\tau)\,d\tau\,d\vartheta+u_{0z}.$$

In this paper we calculate distributions of concentrations of radiation defects and dopant, components of displacement vector. We used method of averaging of function corrections and obtain the second-order approximations. These approximations are usually sufficient approximations for qualitative analysis and obtaining some quantitative results. We check our analytical result by comparison with numerical simulations.

DISCUSSION

Now we will consider redistributions of radiation defects and dopant during annealing and with account stress between layers of heterostructure. We present some typical distributions of dopant concentrations on Figs. 2 and 3 for both types of doping. These figures corresponds to larger value of dopant diffusion coefficient in the epitaxial layer in comparison with dopant diffusion coefficient in the substrate. Figs. 2 and 3 show, that inhomogeneity of heterostructure leads to increasing gradient of concentration of dopant near interface between layers. At the same time homogeneity of distribution of dopant increased. Increasing of the considered gradient leads to decreasing of switching time of the considered transistors. Another effect leads to decreasing of local overheat of materials of heterostructures during functioning of transistors. It should be noted, that optimization of annealing of dopant and/or radiation defects should be done during manufacture of considered field-effect transistors. To describe the optimization we consider two limiting cases. One of them corresponds to small annealing time. In this situation dopant has not enough time to achieve nearest interface between layers of heterostructure. In this case inhomogeneity of heterostructure and modification of distribution of concentration of dopant can not be used. The second limiting case is large annealing time. In this situation concentration of dopant is too homogenous. So, we consider optimization of annealing time by choosing of it's compromise value framework approach, which was recently introduces in Refs. [15, 25-32]. To use the criterion we used approximation of dopant concentration distribution by step-wise function. The approximation is presented on Figs. 4 and 5 for diffusion and ion types of doping, respectively. After introduction of the approximation one shall minimize the mean-squared error

$$U = \frac{1}{L_x L_y L_z} \int_0^{L_x} \int_0^{L_y} \int_0^{L_z} \left[C(x, y, z, \Theta) - \psi(x, y, z) \right] d\,z\,d\,y\,d\,x \, . \tag{15}$$

Here function $\psi(x,y,z)$ describes considered approximation function.

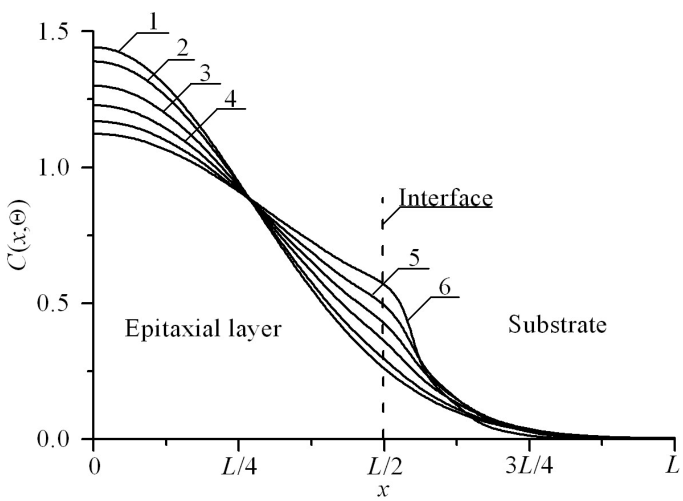

Figure 2: Infused dopant distributions in perpendicular direction to interface between layers of heterostructure. Difference between values of dopant diffusion coefficient increases with increasing of number of curves

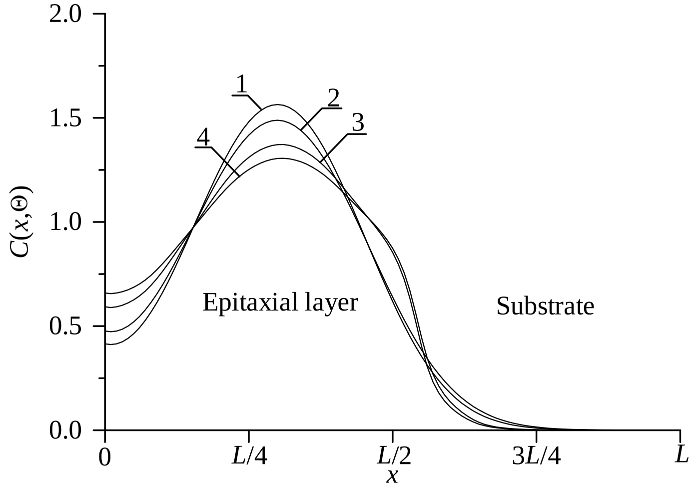

Figure 3: Implanted dopant distributions in perpendicular direction to interface between layers of heterostructure. Difference between values of dopant diffusion coefficient increases with increasing of number of curves. Curves 1 and 3 were calculated for annealing time Q = 0.0048(L_x^2+L_y^2+L_z^2)/D_0. Curves 2 and 4 were calculated for annealing time Q = 0.0057(L_x^2+L_y^2+L_z^2)/D_0. Curves 1 and 2 were calculated for homogenous sample. Curves 3 and 4 were calculated for considered on Fig. 1 heterostructure

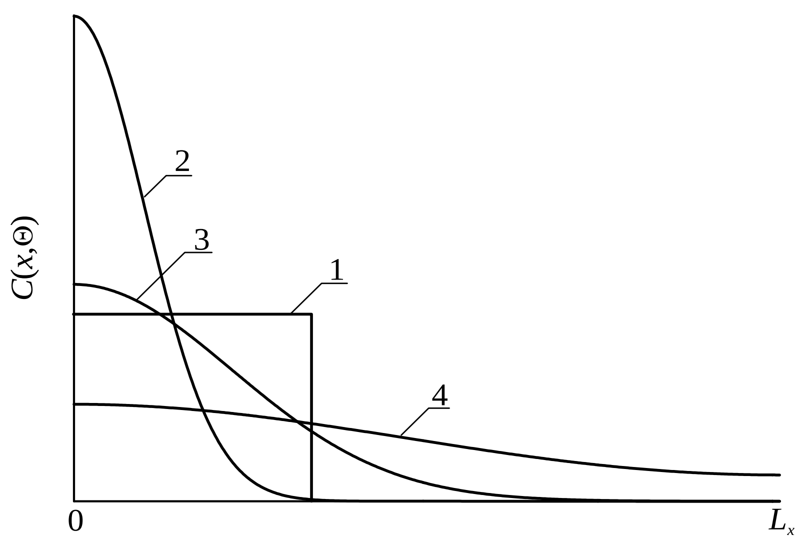

Figure 4: Distributions of concentration of infused dopant in space. Curve 1 describes step-wise approximation function. Curves 2-4 describe distributions of concentration of infused dopant in space. Increasing of number of curve corresponds to increasing of annealing time

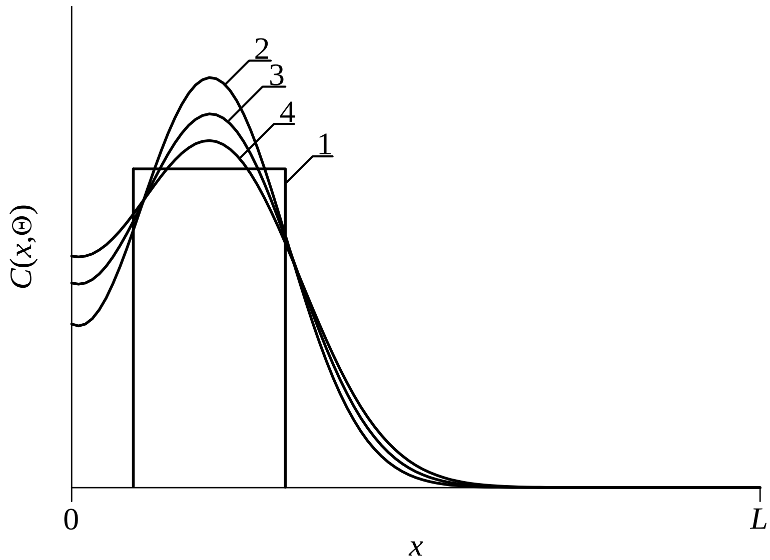

Figure 5: Distributions of concentration of implanted dopant in space. Curve 1 describes step-wise approximation function. Curves 2-4 describe distributions of concentration of infused dopant in space. Increasing of number of curve corresponds to increasing of annealing time

We present some dependences of compromise annealing time on several parameters. These dependences were presented on Figs. 6 for diffusion type of doping and on Figs. 7 for ion type of doping. It is known, that radiation defects, generated during ion implantation, should be annealed. The annealing leads to spreading of distribution of concentration of dopant. If dopant achieves required interfaces between materials of heterostructure during radiation defects annealing, than we have ideal case. If dopant has no enough time for this achievements during the annealing, than additional annealing of dopant required. Accounting of such regime of annealing leads to decreasing of value of optimal annealing time of dopant for ion type of doping in comparison with diffusion one.

Now we will analyze changing of concentration of dopant distribution under influence of mechanical stress relaxation in the considered heterostructure (see Fig. 1). If $\varepsilon_0 < 0$, than compression dopant concentration distribution could be find. Contrary (at $\varepsilon_0 > 0$) one can find vise versa effect. The described changing of dopant concentration distribution could be partially compensated by using laser or microwave annealing [29]. Both types of annealing leads to acceleration of dopant diffusion and other processes, which are existing during technological process. Analysis of changing of concentration of dopant distribution shows, that mechanical stress in

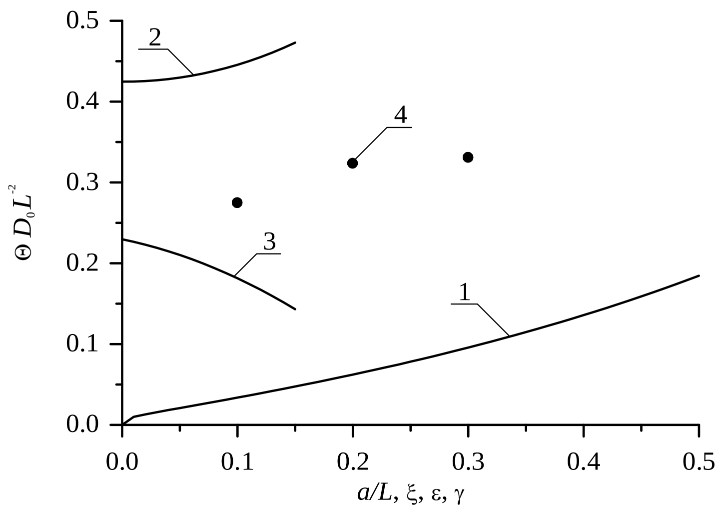

Figure 6: Dependences of optimal annealing time after dimensioning for diffusion type of doping. Curve 1 describes optimal annealing time after dimensioning as a function of ratio a/L at $\xi = \gamma = 0$ for averaged dopant diffusion coefficient. Curve 2 describes optimal annealing time after dimensioning as a function of parameter e at $\xi = \gamma = 0$ for a/L=1/2. Curve 3 describes optimal annealing time after dimensioning as a function of parameter ξ at $\varepsilon = \gamma = 0$ for a/L=1/2. Curve 4 describes optimal annealing time after dimensioning as a function of parameter γ at $\varepsilon = \xi = 0$ for a/L=1/2.

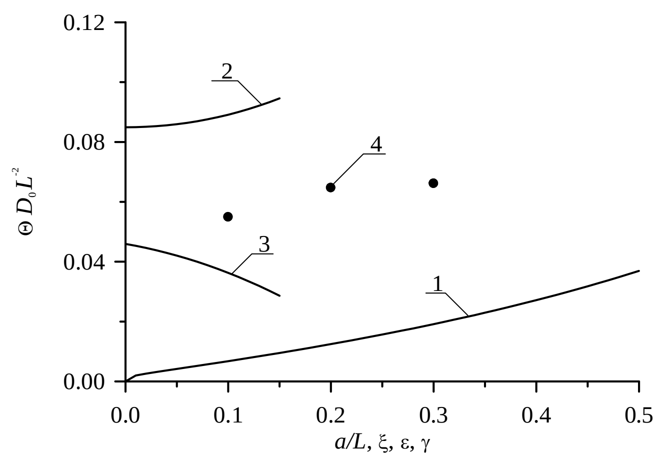

Figure 7: Dependences of optimal annealing time after dimensioning for ion type of doping. Curve 1 describes optimal annealing time after dimensioning as a function of ratio a/L at $\xi = \gamma = 0$ for averaged dopant diffusion coefficient. Curve 2 describes optimal annealing time after dimensioning as a function of parameter ε at $\xi = \gamma = 0$ for a/L=1/2. Curve 3 describes optimal annealing time after dimensioning as a function of parameter ξ at $\varepsilon = \gamma = 0$ for a/L=1/2. Curve 4 describes optimal annealing time after dimensioning as a function of parameter γ at $\varepsilon = \xi = 0$ for a/L=1/2

heterostructure is a reason of changing of recently considered optimal values of annealing time. It should be noted, that using mechanical stress gives a possibility to increase elements density framework integrated circuits. At the same time the stress is a reason of generation discrepancy dislocations. Fig. 8 illustrates dependence of perpendicular to interface between epitaxial layer and substrate displacement vector component u_z on the analogous coordinate. The figure shows, that porosity of materials of heterostructure leads to decreasing of miss-match induced stress.

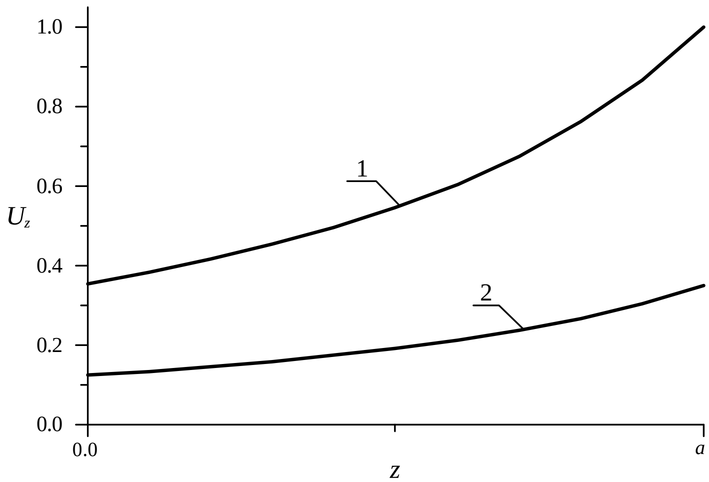

Figure 8: Coordinate dependences of component u_z after dimensioning for the case, when buffer layer is nonporous and porous, respectively (curve 1 and 2, respectively)

CONCLUSION

We introduce an analytical approach to model manufacturing of an enhanced swing differential Colpitts oscillator to optimize this process for increasing of density of field-effect heterotransistors framework the considered circuit with decreasing of their dimensions. The approach gives a possibility to take into account spatial and at the same time temporal variations of parameters of technological process. At the same time the approach gives a possibility to take into account nonlinearity of physical processes. The approach gives a possibility to formulate recommendations to optimize technological processes of manufacturing of the considered integrated circuit.

References

[1] V.I. Lachin, N.S. Savelov. *Electronics*. Rostov-on-Don: Phoenix, 2001.

[2] A. Polishscuk. Modern Electronics. Issue 12. P. 8-11 (2004).

[3] G. Volovich. Modern Electronics. Issue 2. P. 10-17 (2006).

[4] A. Kerentsev, V. Lanin, Power Electronics. Issue 1. P. 34 (2008).

[5] A.O. Ageev, A.E. Belyaev, N.S. Boltovets, V.N. Ivanov, R.V. Konakova, Ya.Ya. Kudrik, P.M. Litvin, V.V. Milenin, A.V. Sachenko. Semiconductors. **Vol. 43** (7). P. 897-903 (2009).

[6] Jung-Hui Tsai, Shao-Yen Chiu, Wen-Shiung Lour, Der-Feng Guo. Semiconductors. **Vol. 43** (7). P. 971-974 (2009).

[7] O.V. Alexandrov, A.O. Zakhar'in, N.A. Sobolev, E.I. Shek, M.M. Makoviychuk, E.O. Parshin. Semiconductors. **Vol. 32** (9). P. 1029-1032 (1998).

[8] I.B. Ermolovich, V.V. Milenin, R.A. Red'ko, S.M. Red'ko. Semiconductors. **Vol. 43** (8). P. 1016-1020 (2009).

[9] P. Sinsermsuksakul, K. Hartman, S.B. Kim, J. Heo, L. Sun, H.H. Park, R. Chakraborty, T. Buonassisi, R.G. Gordon. Appl. Phys. Lett. **Vol. 102** (5). P. 053901-053905 (2013).

[10] J.G. Reynolds, C.L. Reynolds, Jr.A. Mohanta, J.F. Muth, J.E. Rowe, H.O. Everitt, D.E. Aspnes. Appl. Phys. Lett. **Vol. 102** (15). P. 152114-152118 (2013).

[11] N.I. Volokobinskaya, I.N. Komarov, T.V. Matyukhina, V.I. Reshetnikov, A.A. Rush, I.V. Falina, A.S. Yastrebov. Semiconductors. **Vol. 35** (8). P. 1013-1017 (2001).

[12] E.L. Pankratov, E.A. Bulaeva. Reviews in Theoretical Science. **Vol. 1** (1). P. 58-82 (2013).

[13] S.A. Kukushkin, A.V. Osipov, A.I. Romanychev. Physics of the Solid State. **Vol. 58** (7). P. 1448-1452 (2016).

[14] E.M. Trukhanov, A.V. Kolesnikov, I. D. Loshkarev. Russian Microelectronics. **Vol. 44** (8). P. 552-558 (2015).

[15] E.L. Pankratov, E.A. Bulaeva. Reviews in Theoretical Science. **Vol. 3** (4). P. 365-398 (2015).

[16] K.K. Ong, K.L. Pey, P.S. Lee, A.T.S. Wee, X.C. Wang, Y.F. Chong. Appl. Phys. Lett. **Vol. 89** (17). P. 172111-172114 (2006).

[17] H.T. Wang, L.S. Tan, E. F. Chor. J. Appl. Phys. **Vol. 98** (9). P. 094901-094905 (2006).

[18] Yu.V. Bykov, A.G. Yeremeev, N.A. Zharova, I.V. Plotnikov, K.I. Rybakov, M.N. Drozdov, Yu.N. Drozdov, V.D. Skupov. Radiophysics and Quantum Electronics. **Vol. 43** (3). P. 836-843 (2003).

[19] Y.W. Zhang, A.F. Bower. Journal of the Mechanics and Physics of Solids. **Vol. 47** (11). P. 2273-2297 (1999).

[20] L.D. Landau, E.M. Lefshits. *Theoretical physics*. 7 (*Theory of elasticity*) (Physmatlit, Moscow, 2001).

[21] Z.Yu. Gotra, *Technology of microelectronic devices* (Radio and communication, Moscow, 1991).

[22] P.M. Fahey, P.B. Griffin, J.D. Plummer. Rev. Mod. Phys. **Vol. 61** (2). P. 289-388 (1989).

[23] V.L. Vinetskiy, G.A. Kholodar', *Radiative physics of semiconductors*. (Naukova Dumka, Kiev, 1979).

[24] A.G. Roy, K. Mayaram, T.S. Fiez. Analog. Integr. Circ. Sig. Process. **Vol. 82**. P. 691-703 (2015).

[25] E.L. Pankratov, E.A. Bulaeva. Int. J. Micro-Nano Scale Transp. **Vol. 4** (1). P. 17-31 (2014).

[26] Yu.D. Sokolov. Applied Mechanics. **Vol.1** (1). P. 23-35 (1955).

[27] E.L. Pankratov. Russian Microelectronics. **Vol. 36** (1). P. 33-39 (2007).

[28] E.L. Pankratov, E.A. Bulaeva. Int. J. Nanoscience. **Vol. 16** (4). P. 1650039-1650054 (2017).

[29] E.L. Pankratov, E.A. Bulaeva. Int. J. Micro-Nano Scale Transp. **Vol. 3** (3). P. 119-130 (2012).

[30] E.L. Pankratov. Nano. **Vol. 6** (1). P. 31-40 (2011).

[31] E.L. Pankratov, E.A. Bulaeva. Multidiscipline modeling in materials and structures. **Vol. 12** (4). P. 672-677 (2016).

[32] E.L. Pankratov, E.A. Bulaeva. Journal of computational and theoretical nanoscience. **Vol. 14** (7). P. 3548-3555 (2017).