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CROSS-BORDER INVESTMENT UNDER EXCHANGE RATE UNCERTAINTY

Udo Broll[1]

Soumyatanu Mukherjee[2]

[1]*Department of Business and Economics, School of International Studies (ZIS), Technische Universität Dresden, 01062 Dresden, Germany*
Email: udo.broll@tu-dresden.de

Department of Banking & Finance, Southampton Business School, University of Southampton, SO17 1BJ, UK. E-mail: S.Mukherjee@soton.ac.uk

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ABSTRACT

This note analyses the decision problem of a risk-averse foreign firmregarding the optimum cross-borderinvestment decisionunder uncertainties stemmed from the exchange rate movementsin a mean-variance decision-theoretic model. This framework is based upon the utility from the expected value and the standard deviation of the firm's random final profit. This modelling approach allows us to explore how mucha risk-averse firm optimally invests abroad when facing uncertainties regarding the exchange rate movements. All comparative static effects are described in terms of the relativetrade-offs between risks and returns.

INTRODUCTION

Cross-border investments are one of the key components of the globalisation process. The higher fixed costs (often sunk) of locating abroad generally comprise uncertainties associated with exchange rate movements (Broll and Wong, 2006; Schmidt and Broll, 2009; Russ, 2007; 2012; and so on). Not only to the OECD countries like USA, but also the investment flows from the more industrialized nations into the developing country like India (which has typically faced, a' la Rodrik, 2016; the "premature deindustrialization" pattern of evolution) over the last two decades have caughtsubstantial attentions. This short note, in such context, modelsthe optimum decision-making process of a risk-averse firm, who considers investing abroadin a low-income developing country, under uncertainties stemmed from fluctuations in nominal foreign exchange rates.

The unique contribution of this paper is to explore the decision problem of a risk-averse foreign firmregarding how much to invest optimally, under uncertainties stemmed from the exchange rate movements, in a two-moment decision model. Over the time, the two-moment decision model, or the mean - variance decision-theoretic model has been acknowledged as a meaningful alternative model of risk preferences to the expected utility approach, given that the following conditions (Ormiston and Schlee, 2001; Epstein, 1985) are satisfied in this context:

(1) all feasible distributions of any random variable differ only with respect to the mean and the standard deviation,

and

(2) the stochastic variable (spot exchange rate)is linearly related tothe mean and standard deviation of final profit (see Meyer, 1987; for the validity of this assumption).

The major advantage is its intuitive simplicity: everything can be comprehended in terms of the trade-offs between return and risk.

The comparative static effects are describedin terms of trade-offs between return and risk. The paper is organised as follows. Section 2 discusses the modelling framework in details. Section 3 evaluates the optimuminvestment decision owing to the changes in exchange rate risk distribution.Section3illustrates the significance of the results of the comparative static exercises in sections 3 in terms of a parametric example, and finally section 6 concludes.

2. THE MODEL

Our investigation rests on the following set of assumptions: We consider a risk-averse firm (operating in developed high-income country) that investsK (in domestic currency) in a developing country (foreign/destination/host country). The expression for the firm's revenue in foreign currency is given by $R(K)$, with $R'(K) > 0$, $R''(K) < 0$, $R(0) = 0$. $K > 0$ is positive under the well-known Inada-condition (Inada, 1963; Uzawa, 1963; Takayama, 1985). Therefore, the revenue in the sourcecountry's currency becomes $\tilde{e}R(K)$, where the nominal or spot exchange rate ($\tilde{e}$), defined in terms of the currency in the source country, per unit of the currency in the host country, is uncertain. In this context, we consider there is a single time horizon with two dates, i.e., $t = 0$ and $t = 1$. To begin with, we assume $t = 0$, the firm decides to invest, while the realizations of the random spot foreign exchange rate takes place at $t = 1$. Theuser cost of financing capitalis rK is defined in source country's currency.It results in the uncertain total profit of the firm, which can be written as

$$\tilde{\pi} = \tilde{e}R(K) - rK. \tag{1}$$

Expected profit is given by:

$$\mu = E(\tilde{\pi}) = \mu_e R(K) - rK. \tag{2}$$

Profit risk is defined as follows:

$$v = v_e R(K)^2 \tag{3}$$

For any random variable $\tilde{W}$, the mean and variance are denoted by respectively, μ_W and v_W.

The preference function of the firm is $U = U(v, \mu)$, with $U_\mu(v, \mu) > 0$, $U_v(v, \mu) < 0$, and $U(v, \mu)$ is strictly quasi-concave in (v, μ)-space. In other words, the firm exhibits non-satiation and risk aversion. It is reasonable to assume risk-averse behaviour. Even cases like the firm is publicly listed, corporate taxes, costs of financial distress, capital market imperfections and so on (Froot et al., 1993) - we would have a concave objective function of the decision-maker. We use risk aversion as an approximation for these imperfections (see, also, Broll and Wong, 2006).

The marginal rate of substitution (MRS) between risk and return is defined by

$$S = -\frac{U_v(v,\mu)}{U_\mu(v,\mu)}.$$

$S > 0$ is the two-parameter equivalent to Arrow-Pratt (Arrow, 1965; Pratt, 1964)measure of absolute risk aversion. The firm solves the following problem

$$\max_{(K \geq 0)} U(v, \mu) \tag{4}$$

subject to (2) and (3).

Since, we are concerned about only $K > 0$ (owing to the Inada condition), we are going to restrict our attention only to the interior solution of the maximisation programme. Hence, the first ordercondition (F.O.C.) entails the following:

$$U_\mu(v^*, \mu^*)(\mu_e R'(K^*) - r) + U_v(v^*, \mu^*)(\partial_v(K^*) / \partial K) = 0 \tag{5}$$

Where,

$$(\partial v(K^*) \;/\; \partial K) = 2v_e\, R(K^*)\, R'\,(K^*)$$

From Eq. 5 we obtain

$$\{\mu_e\, R'\,(K^*) - r\}/((\partial v(K^*)/K)\,) = S(v(K^*), \mu(K^*)) \tag{6}$$

$\{\mu_e R'\,(K^*) - r\}$ denotes the risk-premium of the firm, which is positive, given that the firm is risk-averse. Therefore, we always have $(\partial v(K^*) \;/\; \partial K) > 0$, or the scenario of a "risky" venture offoreign investment. However, Eq. 6 implies that the efficiency frontier, with slope $\{\mu_e\, R'\,(K^*) - r\} \;/\; ((\partial v(K^*)/K)\,)$, must be a tangent to the indifference curve, with slope $S(v^*, \mu^*)$.

Before proceeding to the comparative static exercises, let us introduce fewer concepts that will be used in the analyses.

Definition 1. The elasticity of the marginal rate of substitution between risk and return with respect to the variability of the randomfinal profit is

$$\epsilon_v(v,\mu) = \frac{\partial S(v,\mu)}{\partial v}\frac{\mu}{S(v,\mu)}.$$

The elasticity $\epsilon_\mu\,(v, \mu)$ indicates the percentage change in the "willingness-to-pay" (WTP hereafter) for a reduction in risk, over the percentage change in the variability of final profit, keeping the mean(μ) constant.

Definition 2. The elasticity of the marginal rate of substitution between risk and return with respect to the mean of final profit is defined as

$$\epsilon_v(v,\mu) = \frac{\partial S(v,\mu)}{\partial \mu}\frac{\mu}{S(v,\mu)}.$$

The elasticity $\epsilon_\mu\,(v, \mu)$ indicates the percentage change in the relative WTP for a reduction in risk, over the percentage change in expected final profit, keeping the variance (v) constant.

With these definitions in hand, let us begin with our first set of comparative static exercises.

3. PERTURBATIONS IN THE DISTRIBUTION OF $\tilde{e}$

Now let us first trace out the change in optimum investment abroad owing to the increase in the exchange rate risk.

Proposition 1. Higher exchange rate volatility leads to a decrease in optimum investment if and only if $\epsilon_v\,(v^*, \mu^*) > -1$.

Proof. Implicit differentiation of the F.O.C. in (6) gives

$$\text{sgn}(\partial K^* / \partial v_e) = -\text{sgn}\left[S_v \frac{\partial v}{\partial v_e} + \frac{\partial^2 v}{\partial K \partial v_e} \frac{\{\mu_e R'(K^*) - r\}}{(\partial v(K^*) / \partial K)^2} \right]$$

As $\tilde{Z} = 0$, substituting $(\partial v / (\partial v_e)) = R(K^*)^2$, $((\partial^2 v) / (\partial K v_e)) = 2R(K^*)$ R^' (K^*)and using the F.O.C., after some simple manipulations, we obtain

$$\text{sgn}((\partial K^*) / (\partial v_e)) = -\text{sgn}[\varepsilon_v + 1]$$

Therefore, $((\partial K^*) / (\partial v_e)) < 0$, whenever $\varepsilon_v > -1$.

(Q.E.D.)

An increase in v_e leads to lesser exposure to the exchange rate fluctuations, provided the relative WTP for a risk reduction, must not be "too elastic" to the increase in v_e.

Now we are going to examine the relationship between the firm's optimum investment allocation decision with respect to a change in the expected foreign exchange rate, i.e., μ_e.

Proposition 2. An increase in the expected exchange rate will lead to an increase in optimum investmentif and only if $\epsilon_\mu (v^*, \mu^*) < 1$.

Proof. Implicitly differentiating the F.O.C. in (6) we get

$$\text{sgn}\left(\frac{\partial K^*}{\partial \mu_e}\right) = \text{sgn}\left(1 - \alpha^* \epsilon_\mu (v^*, \mu^*)\right)$$

$$\alpha^* = \sigma^* S(v^*, \mu^*) / \mu^* > 0$$

This is because, $(\mu_e R'(K^*) - r) > 0$ (the decision-maker is risk averse). Also, since $R(.)$ is strictly concave, $R(K^*) / K^* > R'(K^*)$. Hence,

$$0 < \mu_e R'(K^*) - r < \mu_e \left(\frac{R(K^*)}{K^*}\right) - r,$$

In other words, $\alpha^* \in (0, 1)$.

Therefore, $(\partial K^* / (\partial \mu_e) > 0$, if and only if $\varepsilon_\mu < 1$.

(Q.E.D.)

An increase in the expected exchange rate wouldcausean increase in the firm's exposure to the exchange rate fluctuations, if and only ifthe relative WTP for a reduction in exchange rate risk does not increase too elastically in μ.

What are the significance of propositions 1 and 2? Any change in the distribution of the spot exchange rate in the world market leads to an unambiguously negative substitution effect (lower investment due to higher risk) and an income effect (or wealth effect) that could be either positive or negative. Thus, the total effect on K^* depends on the relative magnitudes of the income and substitution effects.

4. A PARAMETRIC EXAMPLE

Let us exemplify our propositions and their significance by a specific preference function, as pioneered inSaha (1997).

$$U = \mu^a - v^b \tag{8}$$

Likewise demonstrated in Saha (1997), (8) allows us to have the most flexibility, since it does not require to presume any specific assumption on the pattern of riskpreference structure.

Hence, the expression for the MRS between risk and return for the investing firm would become

$$S(\mu, v) = (b \,/\, a)\, \mu^{1-a}\, v^{b-1} \tag{9}$$

and the F.O.C. for the optimum investment to be made by the firm turns out to be

$$a\{\mu_e\, R'(K^*) - r\}\mu^{*(a-1)} - b\left(\frac{\partial v(K^*)}{\partial K}\right)v^{*(b-1)} = 0 \tag{10}$$

Given 10, we can come up with the following results equivalent to Propositions (1) - (5).

According to the definitions 1 and 2 the relative changes in the degree of risk a version with respect to the standard deviation and mean of the random final profit are respectively,

$$\varepsilon_v = b - 1,\ \varepsilon_\mu = 1 - a.$$

Hence, from the F.O.C. we can derive the following results as corollaries to the propositions 1-5.

Corollary 2: Under the preferences given by (8), we have

(a) A higher v_e will induce the firm to behave in more risk averse fashion if and only if $b > 0$.

Proof.

$$\frac{\partial K^*}{\partial v_e} = -2bv^{*(b-1)}R(K^*)R'(K^*)[(b-1)+1] < 0,$$ if and only if $b > 0$, or $(b-1) > -1$. In other words, $\frac{\partial K^*}{\partial v_e} < 0$, if and only if $(b-1) = \varepsilon_\sigma > -1$. This result is also consistent with Proposition 1.

(b) An increase in μ_e induces the firm into more risk-taking behaviour if and only if, $a > 0$.

Proof.

$$\frac{\partial K^*}{\partial v_e} = a[\mu_e R'(K^*) - rK^*]^{a-1} R'(K^*) > 0,$$ if and only if, $a > 0$, or $(1-a) < 1$. In other words, $\frac{\partial K^*}{\partial v_e} > 0$, if and only if $(1-a) = \varepsilon_\mu < 1$. This is consistent with Proposition 2.

Thus, all our comparative static results in section 3 are represented in terms of the preference parameters a and b in our example.

5. CONCLUDING REMARKS

The aim of this paper has been to explore the decision of a risk-averse firm on how much to be optimally invested abroadunder nominal exchange rate fluctuations in the world market. The risk-averse firm's optimum investment decision is contingent upon the relative sensitivity of the risk aversion, i.e. the willingness to invest abroad, owing to the changesin the distribution of the spot exchange rate.

An attractive feature of the conditions we derive for the decision problem of the risk-averse firm using the two-moment approach is their simplicity in interpretation: with minimal assumption on preference structure like monotonicity and quasi-concavity, the sufficiency conditions based on the firm's relative sensitivity towards risks are more intuitive and appealing as empirically testable predictions; in contrast to the alternative (such as expected utility) approaches, which would depend on higher-order derivatives of utility functions and their composites.

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