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Detecting ARCH Effects: Power versus Frequency of Observation. Some Monte Carlo results

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Abstract: This short paper using simulation techniques studies the effects of increasing the frequency of observation and the data span on the testing the ARCH effects on a time series. According to our simulation results, the power of the Lagrange Multiplier test for detecting ARCH effects depends strongly on the level of temporal aggregation and the number of the available observations.

Keywords: Monte Carlo; Time Aggregation Span; Power; Arch(p) specifications.

JEL classification codes: C15; C22

1. Introduction

Practitioners often have to decide whether to use weekly, monthly, quarterly or annual data when testing for the ARCH effects on a time series. They face the question of temporal aggregation problem.

A number of authors have addressed this question for the effects of temporal aggregation, from different perspectives. Only to mention some of them, useful references include the papers by Sims (1971), Wei (1982), Christiano & Eichenbaum (1987), Marcellino (1999), Breitung and Swanson (2002), Gulasekaran and Abeysinghe (2002) and Tserkezos and others (1992,1998,2006,2007).

This short paper using simulation techniques studies the effects of increasing the frequency of observation and the data span on the testing the ARCH effects on a time series.

A study related with our research is the paper of Drost and Nijman (1993). They approach the time aggregation effect problem more from a theoretical point of view and they do not refer directly to the time aggregation effects on testing ARCH effects.

The results of this paper show the importance of the time aggregation level in applied time series analysis and especially in financial time series data. Using Monte Carlo techniques, we found that temporal aggregation and especially the temporal aggregation level could lead to wrong conclusions of the ARCH effects on a time series.

The paper is organized as follows. Section 2 contains the simulation results and Section 3 offers some concluding remarks.

2. Design of the Monte Carlo experiment and results.

In order to test the possible temporal aggregation effects on the power of a Lagrange multiplier test, we generate *n*=4000 observations at the highest level of temporal disaggregation for a variable y according to the DGP[1]:

$$y_t = 2 + random(\sqrt{a_0 + a_1 y^2_{t-1}}\ ) \tag{1}$$

$$Var(y_t) = random(a_0 / (1 - a_1)) \tag{2}$$

This DGP draws the first value from the unconditional distribution (which is mean 0, variance $a_0 / (1 – a_1)$ and the remaining ones are generated recursively using the previous value.

Simple Lagrange Multiplier tests were performed to test for the existence of ARCH effects in a simulated time series. The test for ARCH(p) effects is based on an iterative procedure for different values of p (p = 1, 2 ,…) testing the statistical significance of the parameters $\alpha_0, \alpha_1, \alpha_2, ... \alpha_p$ of the:

$$y_t^2 = \alpha_0 + \alpha_1 y_{t-1}^2 + \alpha_2 y_{t-2}^2 + ... + \alpha_p y_{t-p}^2 + u_t \tag{4}$$

with

$$u_t \sim NID(0, \sigma^2_u) \tag{5}$$

The hypotheses of the test are the following:

H_0: $\alpha_1 = \alpha_2 = ... = \alpha_P = 0$, is accepted if (N-P)$R^2 \langle X^2_\omega(p)$ (6)

H_1: ARCH(p), reject (N-P)$R^2 \rangle x^2_\omega(p)$ (7)

ω is the significance level

p is the number of degrees of freedom

and *N* is the number of available observations

Temporal aggregates of the simulated variable are obtained following two procedures:

In the first procedure temporal aggregated data are formed by averaging basic observations over nonoverlapping intervals. The temporally aggregated data were obtained as follows:

$$y_T{}^A = C y_t \tag{8}$$

where $y_T{}^A$ is the temporal aggregated data, *m* is the time aggregation level and y_t the simulated data, *C* is a time aggregation[2] matrix of the form:

$$C = (1/m)\begin{bmatrix} 11...11000............00000000000000 \\ 00000011....11000......0000000000 \\ 00000000000011....1100....000000 \\ .... \\ 0000................0000000000111...11 \end{bmatrix} \quad m = 1, \ldots, 20 \qquad (9)$$

Using the Aggregation matrix C the aggregated data are the mean values of the period. In the second procedure temporally aggregated data ware obtained at each iteration, using a random starting period in the whole sample data and averaging the selected observations using the C matrix at the 20 different time aggregation levels.

Our simulation experiments are based on 10000 iterations. Analytical results are summarized arithmetically and graphically in Tables 1 to 2 and Figures 1

Table 1: Percentages of accepting the true hypothesis, i.e. the existence of ARCH effects under different time aggregation levels (0.025 Significance Level)

	Number of observations						
Aggregation level	*400*	*600*	*700*	*1000*	*1200*	*1400*	*1600*
1	99,87	100,00	100,00	100,00	100,00	100,00	100,00
2	40,00	63,47	78,10	87,43	92,37	94,77	97,20
3	17,10	28,23	36,97	44,70	50,70	56,77	62,03
4	10,27	15,87	19,87	23,80	26,73	31,30	34,07
5	6,53	10,77	12,97	16,20	18,10	20,40	21,50
6	5,13	8,80	10,17	11,97	13,67	14,23	15,43
7	4,47	7,57	8,03	10,40	11,77	11,83	12,63
8	4,20	6,73	8,07	9,63	10,53	11,00	11,03
9	4,10	6,07	7,53	9,00	9,50	9,80	9,40
10	3,97	5,57	6,53	7,07	8,07	8,40	9,00
11	3,23	5,60	6,20	6,93	7,57	7,87	8,77
12	3,00	4,97	6,30	6,80	7,80	7,93	8,10
13	2,73	4,63	6,53	6,17	6,67	6,90	7,47
14	2,40	4,17	5,20	5,97	7,07	7,03	8,20
15	2,13	4,50	5,87	6,00	6,53	6,63	7,53
16	2,00	3,97	5,30	5,80	6,57	6,50	6,83
17	1,90	3,93	4,23	5,53	6,33	6,80	7,30
18	1,87	3,93	5,33	5,50	6,30	6,67	7,07
19	1,83	3,70	4,13	5,03	5,53	6,07	6,80
20	1,50	4,63	4,80	5,03	5,40	5,47	6,73

Source: Our estimates. Data entries are 'probabilities' of detecting ARCH effects. The size of the test is 0.025. Data entries are given by n/10000, where n is the number of the 10000 times we detect ARCH effects.

to 6 in the text and in the Appendix, for 20 different time aggregation levels $m = 1,2,...,20$, three statistical significant levels (0.025,0.05 and 0.1) and different number of available data ($t = 400,500,.....1600$) with an increasing step of 200 observations.

Table 2 reports the percentages of accepting the true hypothesis, i.e, the existence of ARCH effects under different time aggregation levels, using a random starting period selection data and the number of the available data at 0.025significance level. Analogous graphical results for the different significance level (0.05 and 0.10) are given in the Appendix.

In general, the weakening of the power of the test is obvious, independently of the time aggregation level, the number of the available observations and the significant level. The test is very powerful only in the case we use the simulated data in the highest level of temporal disaggregation, i.e. the period the data were simulated.

Table 2: Percentages of accepting the true hypothesis, i.e. the existence of ARCH effects under different time aggregation levels, using a random starting period selection data. (0.025 Significance Level)

	Number of observations						
Aggregationlevel	*400*	*600*	*700*	*1000*	*1200*	*1400*	*1600*
1	99,83	100,00	100,00	100,00	100,00	100,00	100,00
2	38,50	60,87	78,53	87,10	92,20	93,13	95,17
3	17,40	27,80	36,90	44,63	52,33	57,20	61,30
4	10,20	15,23	22,47	25,43	31,33	34,83	39,37
5	7,40	11,73	14,47	17,17	20,80	24,67	28,00
6	6,03	10,17	11,20	13,70	17,57	19,90	22,47
7	4,13	7,33	9,07	12,47	14,47	17,17	20,30
8	4,53	6,37	8,87	11,13	12,90	16,40	17,37
9	3,63	6,30	7,17	9,60	12,17	15,17	18,20
10	3,43	5,63	7,00	10,03	11,37	13,77	15,97
11	2,63	4,83	6,33	8,90	9,73	13,47	15,83
12	3,07	4,33	5,70	7,67	9,87	13,53	15,43
13	3,07	4,23	5,20	7,97	9,37	13,30	15,37
14	2,40	5,03	5,87	7,57	9,50	12,90	14,17
15	2,70	4,67	5,17	6,97	9,00	11,77	14,27
16	2,23	4,90	5,03	7,23	9,10	11,87	13,30
17	2,37	4,20	4,43	6,90	8,43	10,37	13,33
18	1,87	3,50	5,77	5,87	8,30	10,67	13,20
19	2,47	3,40	4,03	6,20	8,43	11,43	12,47
20	1,67	3,87	5,13	6,90	8,57	10,70	12,10

Source: Our estimates. Data entries are 'probabilities' of detecting ARCH effects. The size of the test is 0.025. Data entries are given by n/10000, where n is the number of the 10000 times we detect ARCH effects.

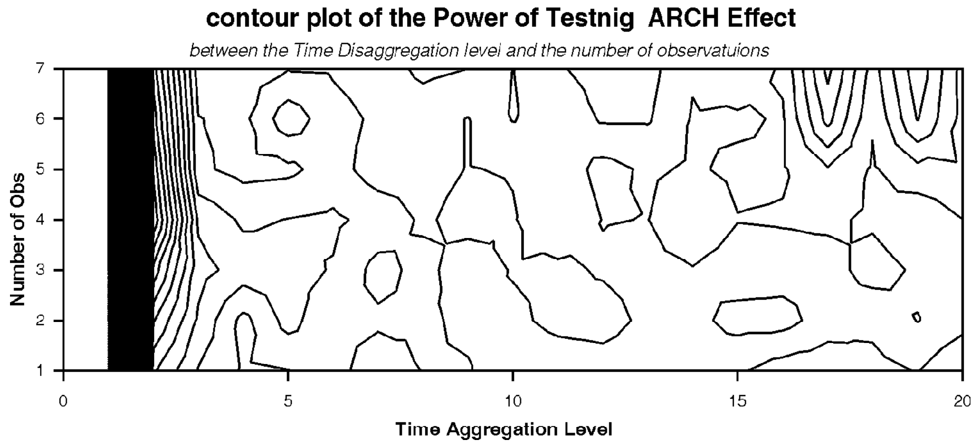

Figure 1: The Counter plot of the power of the ARCH effects tests in relation to different number of available data and temporal disaggregation level, at 0.025 significance level.

Source: Our estimates

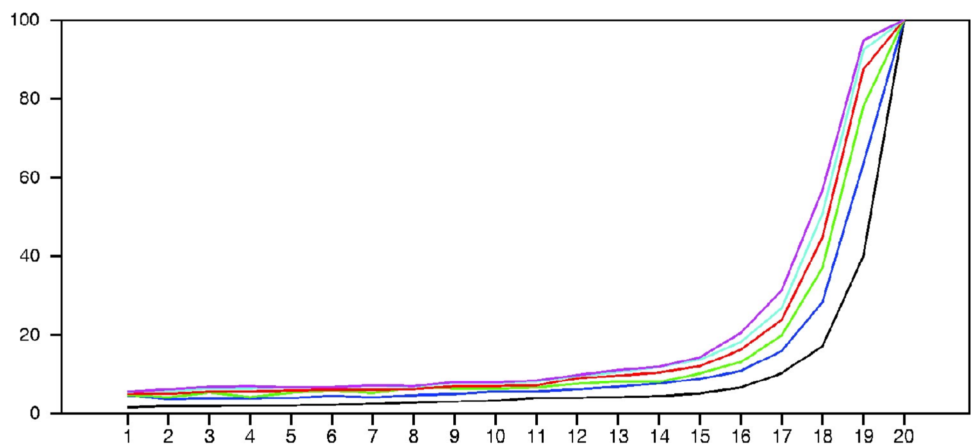

Figure 2: The power of the ARCH effects tests in relation to different number of available data and temporal disaggregation level at 0.025 significance level.

Source: Our estimates

3. Conclusions

The results of this paper show the importance of the time aggregation level in applied time series analysis and especially in financial time series data. Using Monte Carlo techniques, we found that temporal aggregation and especially the temporal aggregation level could lead to wrong conclusions of the ARCH effects in a time series.

On the basis of our Monte Carlo results, we may conclude that as the span of time aggregation widens, the power of the Lagrange Multipliers test for testing ARCH effects, is getting wakening. The test is very powerful in the case we use the simulated data in the highest level of temporal disaggregation, i.e. the period the data were simulated.

More specifically, the issue of temporal aggregation in financial time series is of great importance, especially when in our analysis we use time aggregated data, i.e. yearly, monthly or even weakly data. Using the time aggregated data there is a high probability to reject the ARCH effects on the properties of a time series.

Lastly, the conclusions of this paper are in line with the more general and special findings of similar studies on the negative effects of time aggregation on the power of some 'every day' used tests in applied economic research. Of course, our experiments could be extended to different estimates of the parameters in equation (3).

Notes

1. For the RATS (*www.estima.com*) users the implementation of equation (1)to(3) is the following : set (first=%ran(sqrt(a0/(1-a1)))) y =2+%ran(sqrt(a0+a1*y{1}**2))
2. For more about these Time Aggregation relations using matrix approach, see: Gilbert (1977) and Tserkezos (1998).

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Appendix

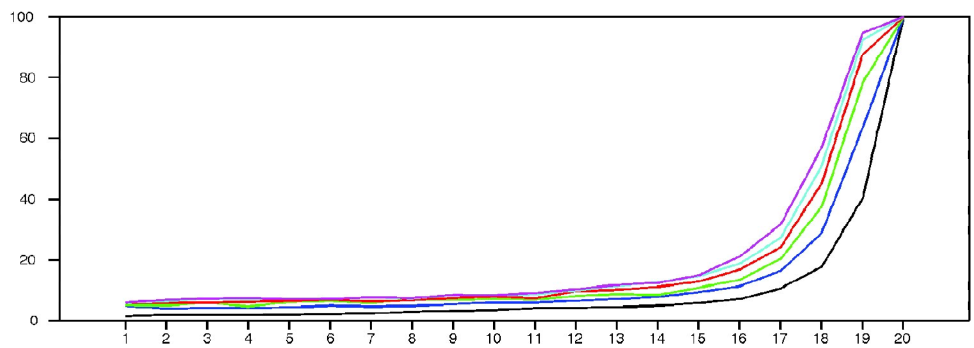

Figure 3: The power of the ARCH effects tests in relation to different number of available data and temporal disaggregation level at 0.05 significance level.

Source: Our estimates.

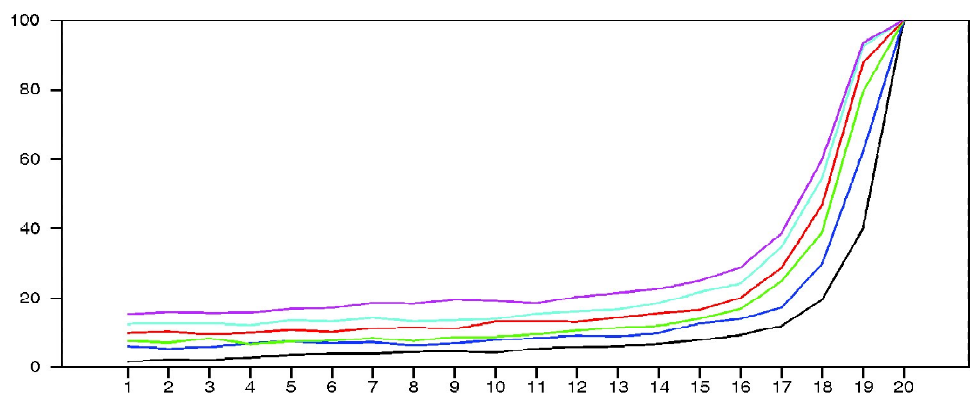

Figure 4: The power of the ARCH effects tests in relation to different number of available data and temporal disaggregation level at 0.05 significance level using randomly the starting period.

Source: Our estimates

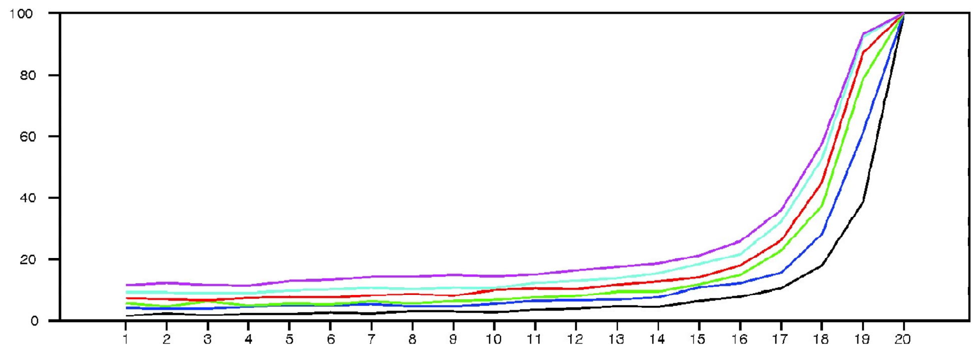

Figure 5: The power of the ARCH effects tests in relation to different number of available data and temporal disaggregation level at 0.10 significance level using randomly the starting period.

Source: Our estimates

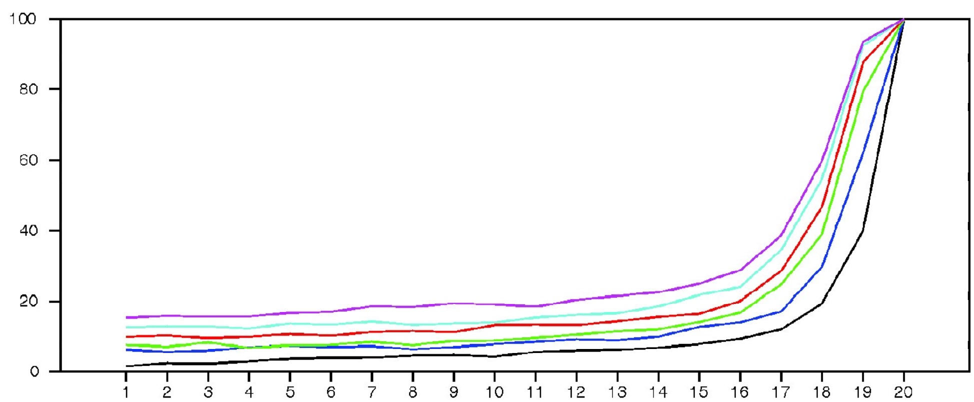

Figure 6: The power of the ARCH effects tests in relation to different number of available data and temporal disaggregation level at 0.10 significance Level using randomly the starting period.

Source: Our estimates