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# The Volatility Spillovers between Zimbabwe, The United States of America, South Africa, Botswana and China: Copula GARCH Model

**[1]Ndava Constantine Mupondo, [1]Brian C Jones & [2]Caston Sigauke**

[1]*Department of Statistics and Operations Research, National University of Science and Technology, P O Box AC 939, Ascot, Zimbabwe*
[2]*Department of Statistics, University of Venda, Private Bag X5050, Thohoyandou, 0950, South Africa*
*Corresponding author E-mail: constantine.mupondo@nust.ac.zw*

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***Abstract:*** *This study explores both the static and dynamic conditional dependency between stock market return series for Zimbabwe and its four supposedly major trading partners. A broad set of the most popular linear and nonlinear copula-type models that capture dependency structure under different circum-stances is applied to investigate this relevancy. To measure the constant dependence structure, we used the Normal and Student-t copulas, and the Generalized Autoregressive Score model with the Student-tcopula was used to capture the dynamic dependence. Our results show that volatility of stock return series among these economies can be best described by Student-tcopula models. Zimbabwe's stock return is positively influenced by the South African counterpart implying that such a core movement greatly affects Zimbabwe. Several implications for stock market risks, policy implication and hedging strategies can be singled out and implemented from the results obtained. The value at risk VaR and expected shortfall was performed and results revealed that the VaR for all the pairs starts at the 3% mark and generally decreases to very low levels of 6% at the start of 2015 going along with a further decline to impassioned levels of approximately 7.5% at the middle of the year 2016.*

***Key words:*** *dependence structure; copula, co-volatility, stock return, equity market*

## 1. INTRODUCTION

The adoption of trade investment liberalization in line with globalization by all nations, international capital market integration, and the expeditious flow of capital across markets especially between developing and developed worlds through bilateral trade agreements have intensified in recent years. The increased connections between international stock and foreign exchange markets has inadvertently strengthened the linkages between the stock market and exchange rate fluctuations. Of late this phenomenon has become a hot topic of research by academics trying to figure out the true dependence structure between international markets and this puzzle has remained a challenging and interesting task for both academic researchers and market participants. In this regard, by examining the structure of dependence between international markets specifically between developed world and a proxy of sub-Saharan economies, policymakers and investors can improve their forecasts, their portfolio hedging and the asset pricing of derivatives in the case where these are applicable. Modelling dependence between related securities and trading partners in the recent literature has widely increased. In most but not all cases these studies have focused amongst others, on modelling the market co-movement of capital markets ((Liu, Lister and Pang, 2013); (Righi, and Ceretta, 2015)), between equity markets and energy markets ((Apergis, Nicholas, and Stephen Miller, 2009); (Kilian, and Park, 2009); (Arouri, and Nguyen, 2010)), and/or even between different types of energy series ((Gŕegoire, and Fisher, 2008)). Generally, there is no concordance reached in the findings of these researches on the existence of such a relationship. There are a number of factors attributed to this which include empirical methodology employed, the type of securities investigated (derivatives, options, stocks, energy, and commodities), and the developmental level of markets examined (underde-veloped, developed, and/or developing markets). On the empirical side, although there are numerous studies that investigated the dependence structure between international stock markets, little or no attention has been focused on the dependence structure between stock market return of the supposedly developed world and the sub-Saharan region proxy (see (Choudry, 1996); (Vivian, Wohar, 2012)).

Given the aforementioned scenario, this research focuses on investigating the stock markets indices' dependence structure between the developed western and Asian economies (China and United States of America (USA)) and sub-Saharan economies (South Africa, Botswana and Zimbabwe). Zimbabwe, the country that was once the breadbasket of Southern Africa is in serious humanitarian and economic meltdown. She has experienced a plethora of economic challenges which have been attributed to like some schools of thought allotted, either economic mismanagement or its failure to oil job creation machinery so as to boost production. Others blame the country's misfortunes squarely on its poor negotiation power to attract favorable bilateral trading terms that have seen its trading partners gaining a competitive advantage over her. It is in the interest of this research to establish the sources of Zimbabwe's economic adversity by looking at it from an stock security perspective since equities act as a blood life and an economic barometer of any economy. In actual fact this research will try to explain sources of stock market volatilities in the Zimbabwean market specifically on those variables that can be attributed to the impact of established bilateral trade agreements between two economies. Precisely the current study will use two robust methods to examine the relationships of price co-volatility applying the static copula and dynamic copula models to find dependence parameter estimate based on a rolling window sample approach. Since all economies universally are prone to both political and financial upheavals that may negatively impact the findings, this study adopted structural breaks tests developed by ((Bai, and Perron, 1998); (Bai, and Perron, 2003)). The existence of dynamic behaviour in the stock return series of the estimated dependence parameter will be confirmed by such tests. Findings of this research will have strong policy implications that can help investors and policymakers in pricing and hedging international capital investments as well as in improving their forecast of stock index prices.

Hamao, Masulis and Ng (1990) considered that Wallerstein characterises the evolving world system as punitive mechanisms, which redistribute additional value from what is termed the periphery to the core. The esteemed cores are classified as developed or industrialised economies with vast technological advancement whereas the periphery refer to the 'underdeveloped', typically raw materials-exporting, poor or least developed countries generally categorised with cheap labour. The periphery in this research encompass countries in the sub-Saharan region specifically Botswana, South Africa and Zimbabwe while the core include USA and China whose economies are basically into processing/manufacturing. It is well documented that the historical economic relationship has been that of the core exploiting economic resources of the periphery. In this regard, with reference to the bilateral economic and political relations, it is the core or a country that is in a better economic standing that dictates the direction of this bilateral relationship. This economic dominance of one country over the other(s) creates conditions for the exploitation of the country's resources, particularly minerals with relatively little cost and easy political persuasion. For example in line with the world systems theory, core countries which are basically raw material importing ones have the aptitude to prescribe the prices for the generally low priced exportables from comparatively poor countries. This is in agreement with Martinez-Vela (2001) who stated that among the most important structures of the current world system is a power hierarchy between core and periphery, in which powerful and wealthy industrial societies dominate and exploit weak and poor peripheral countries. This nevertheless imply that there is considerable dependency syndrome that goes in line with the world system theory as the peripheral countries which are structurally constrained and vulnerable to be exploited by developed economies heavily rely on the core for economic and political developmental necessities. Stock market indices act as an economic barometer of the economy. It reflects the general performance of the country's trade and investment relationship with its trading counterparts. The volatility of any country's stock index is attributable to the complexity structure of both idiosyncratic and systematic factors. Given the aforementioned scenario, the dependence structure and co-movements of stock indexes between trading partners should be thoroughly investigated so as to accurately and properly identify the attributes to fluctuations in one stock index to the other. Specifically, both core and peripheral economies' volatility are not immune from other stock market indices largely influenced by unbalanced trade. Such fluctuations cannot take place in isolation. Hence, there is dire need for a rigorous exploration into the long and short term co-movements between capital stock markets that will enable governments and investors universally to develop appropriate policies and capital investment portfolios that minimises detrimental effects on the national economy.

Our study of stock market return series co-volatilities contributes to the related academic litera-ture along three principal dimensions. First and foremost, it uses copula models to examine the stock market return dependence structure. This methodological approach is more consistent relative to para-metric bivariate distribution functions because it takes the different distributions and merges them together to form the joint probability distribution which is mathematically compatible. Additionally, copulas allow the examination of joint extreme co-volatilities an aspect principally important in hatch-ing portfolio allocation decisions that will enable an investor to design appropriate risk-management strategies, and at the same time they proffer adequate information on tail dependence. Furthermore, rolling windows are used to examine time-varying conditional correlations between capital markets. Finally, the study will also focus on volatility spillovers and asymmetric effects between stock markets primarily related to crisis. To the best of our knowledge, copulas have not been used to investigate de-pendence structure, volatility spillover and asymmetry issues by applying rolling window-time varying approach to examine stock market return series.

This paper is divided into five sections: Section 1 is the introduction. The second is the summary of the literature reviewed. The third one explains Data and Methodology. Section 4 is Results and Discussion. The fifth is the conclusion and recommendations.

## 2. LITERATURE REVIEW

The genesis of literature that champion internationally diversified investment portfolios premised on the low correlation among markets were fundamentally the aftermath of preliminary researches in the late 1960s and the early 1970s (see e.g. (Grubel, and Fadnar, 1971); (Grubel, and Fadner, 1971); (Agmon, 1972); (Solnik, 1974)). In the 1990s, special attention by financial researchers has been directed to the interdependence connecting various capital stock markets cognisance of average return spillovers to and volatility spillovers (see eg. Theodossiou, and Lee (1995); (Liu, Lister and Pang, 2013)). In the meantime, most developing stock markets have liberalized their operations primarily as a considerable reform effort and this will inevitably result in increased international investors' investment opportunity that will generate foreign currency in the countries. Notwithstand-ing favourable investment returns in the face of diversification opportunities in upcoming capital markets, this research is at the epicenter of finding stock market volatility inter-dependencies linking countries in three different development categories that are high-income (developed) countries, newly emerging economies (emerging) and low-income countries (developing). There are predominantly three conditions that impact investing internationally, viz. market volatilities, future fluctuations in foreign currency risks and cross-country correlations ((Abdul *et al.*, 2009)). Nevertheless, the interconnection between developed and developing markets has been explored before but this strand in literature falls short of filling the existing aperture and there is a need to consider the whole spectrum of countries in different developmental levels. Furthermore, literature based on developing countries' capital markets interconnections is very inadequate. This research will explicitly model conditional first moment (mean) and conditional first moment spillovers (volatility) of the capital stock markets. Markets that are closely integrated have substantial evidence of mean and volatility spillovers. The countries to be examined are Zimbabwe, South Africa, Botswana, China, and USA.

The literature of asset market linkages has been surveyed in several papers, such as Andersen *et al.* (2003) and Ehrmann, Fratzscher and Rigobon (2005). Measuring volatility is a common phenomenon in the financial market, where because of the realisation that due to globalisation, movements in price index in these markets are not immune from each other that the interrelationships among different stock markets are examined by primarily academic researchers. For instance, Eun, and Shim (1989) in his research alluded that the bullish stock market in the USA is one of the primary source of international volatility transmission that can, in turn, affect the majority of other foreign equity markets. Nevertheless, the study further states that there is inadequate evidence of other foreign stock markets to explain variations within the US stock market, which could suggest a possible foreign stock market variations cannot explain variations within the US stock market, which may fall short of suggesting a possible unidirectional effect. To other stock markets such as the UK, Germany, Canada, and Japan, Theodossiou, and Lee (1993) in their research paper have proven that the USA stock market has positive transmission effects. Another research, Kearney (2000) furthermore illustrated that the fluctuations in the majority of stock markets worldwide is steered from stock market variations in the USA and Japan, which are then transferred to Europe and beyond. As pinpointed by Andersen et al. (2003); the USA, UK, and Japan are identi-fied as leaders universally accounting for 75% of the total global long-term or short-term capital traded.

There have been several papers that analyse international spillovers on individual asset prices in isolation and stock market returns. Various papers investigate the correlations across international stock markets, such as Longin, and Solnik (1995), Daly (2003), McAleer *et al.* (2008), and Kearney, and Poti (2004). Most of these authors have investigated the case of developed countries with the exception of Daly (2003). These researchers used both unconditional and conditional correlations and their findings suggested that correlations between stock markets increase over time, except for the correlations of the Hang Seng and Nikkei markets, which are constant (see (McAleer *et al.*, 2008)).

Several authors have examined the mean and volatility spillovers across international stock markets, in developed markets, emerging markets, or both, using various univariate and multivariate GARCH models (see, for example, (Hamao, Masulis and Ng, 1990), (Koutmos, 1996), (Choudry, 1996), (Koutmos, 1996), (Ng, 2000), (In *et al.*, 1990), (In, Kim and Yoon, 2003), (Miyakoshi, 2003), (Bala, and Premaratne, 2004), (Worthington, and Higgs, 2004), and (da Veiga, and McAleer, 2005)). These authors have suggested that spillovers move in the direction of developed to emerging markets. Moreover, emerging markets have been shown to be more integrated, so that volatility spillovers across emerging markets in the same region have tended to strengthen.

The aforementioned studies used either simple regression frameworks or multivariate A/GARCH models to examine cross volatilities in the stock markets. None of the above-mentioned empirical studies has made momentous attempts to use a rolling dynamic/time-varying approach and allows for the choice of appropriate copulas among the most commonly used copula families. Moreover, none of the cited studies have consequentially investigated structural breaks in the dependence behaviour. However, a relatively divorced study by Aloui *et al.* (2013) is most related to ours; it uses time-varying copulas to investigate whether dependence exists between commodity and stock markets principally focusing on only one commodity, and their basis was on stock markets in Central and Eastern Europe (CEE) transition economies. There are studies that investigated the relationship between economic variables and socially responsible investing (SRI) using a relatively robust methodology, the multivariate extreme value technique. For example, Bai, and Perron (1998) investigated volatility transmission and time-varying correlations between sustainability stock indices (SSIs) and conven-tional stocks using daily data from January 2004 through September 2015 for North America, Europe and Asia-Pacific countries. They found that SRI provided diversification benefits for conventional portfolios globally. On the methodological part, there relatively few studies that have applied either copula or multivariate extreme value technique to investigate dependence structure between related data sets. Sarwar *et al.* (2019) investigated the dependence structure and spillover effect between major stock market returns for China, Japan and India and the WTI crude oil market by employing the GARCH family. Their findings revealed that these Asian countries were minimally affected by oil shocks; India was the most affected. Among other researchers, the dynamic conditional correlation (DCC)-Multivariate Generalized Autoregressive Conditional Heteroscedasticity (MGARCH) model appears to be the most popular since it has specifications to investigate the time-varying conditional correlation Caporin, and McAleer (2008). Many studies, such as (Ashfaq, Tang and Maqbool (2019), (Ghosh, Sanyal and Jana (2020), and (Khalfaoui, Sarwar, and Tiwari (2019)), have as well employed the DCC-MGARCH model. However, our study is different from that of Aloui *et al.* (2013) in that it investigates the dependence structure between developed and emerging economies stock markets and it has paid particular attention to tail dependence structure as well, whereas the study of Aloui *et al.* (2013) considers only the linkage between the oil and stock markets.

To the best of our knowledge, there are no studies that consider the co-volatility of stock return series between Zimbabwe and its major trading partners. In addition, previous studies analysing the relationship between stock return series using copula methodology and likewise extreme value theory are limited. Therefore, our study is the first to analyze the dynamic interrelationship and volatility spillover between Zimbabwe and its partners.

## 3. METHODOLOGY

To achieve our goal, three multivariate models will be estimated, namely the autoregressive moving average-asymmetric generalised autoregressive conditional heteroskedasitc (ARMA-AGARCH) model of Mcaleer, Hoti, Chan (2009), the vector autoregressive moving average-generalised autoregres-sive conditional heteroskedasitc (VARMA-GARCH) model of Ling and McAleer (2003) and the copula-based GARCH models of Fischer *et al.* (2009). In a bid to detect whether conditional variances of the stock market returns follow the GARCH process, two basic univariate models will be estimated, viz. ARMA(p,q)-GARCH(1,1), and ARMA(p,q)-GJR(1,1). Supposing that the univari-ate model properties are satisfied, extending them to their multivariate counterparts would be sensible.

### 3.1. Sklar's Theorem

The importance of copulas in statistics is described in Sklar's theorem Sklar (1959). In this sense, this theorem is considered as the central theorem of copula theory whereby the $n$-dimensional distribution function can be decomposed into two primary parts, namely, the marginal distribution and the copula.

**Theorem 3.1:** (Sklar's Theorem-1). *Let H be an n-dimensional distribution function with marginals* $F_1, F_2,\ldots,F_n$. *Then there exists an n-copula C such that for all* $x_1, x_2,\ldots,x_n \in \Re$,

$$H(x_1, x_2,\ldots,x_n) = C(F_1(x_1), F_2(x_2),\ldots, F_n(x_n)).$$

Conversely, if $C$ is an $n$-copula and $F_1, F_2,\ldots,F_n$ are distribution functions, then the function H defined by Theorem 3.1 is an $n$-dimensional distribution with marginals $F_1, F_2,\ldots,F_n$. Furthermore, if the marginals are all continuous, then $C$ is unique. Otherwise $C$ is uniquely determined on $RanF_1 \times RanF_2 \times \ldots \times RanF_n$, where $RanF_i$ is the range of the function $F_i$.

For $n = 2$, we have the corresponding theorem in two dimensions.

**Theorem 3.2** (Sklar's Theorem-2). *Let H be a joint distribution function with the marginals F and G. There exists a copula C such that for all x and y in* $\Re$,

$$H(x,y) = C(F\,(x),G(y)).$$

If $F$ and $G$ are continuous, then the copula $C$ is unique; otherwise it is uniquely determined on $RanF \times RanG$. Conversely, if $C$ is a copula, and $F$, $G$ are distribution functions, then the function $H$ defined by Theorem 3.2 is a distribution function with marginals $F$ and $G$.

With this important theorem we see that the copula function is one of the most useful tools for dealing with multivariate distribution functions with given or known univariate marginals.

Following Sklar's theorem, there exists a copula $C : [0, 1]^n \to [0, 1]$ with uniform marginals that can map the univariate marginal distribution $F_i$ to the multivariate distribution function $F$.

The density of the multivariate distribution function $F$ is

$$f(x_1,\ldots,x_n) = c(F_1(x_1),\ldots, F_n(x_n)) \times f_1(x_1) \times \ldots \times f_n(x_n). \tag{1}$$

Firstly we obtained the standardized residuals by applying an autoregressive moving-average GARCH (ARMA-GARCH) model for the daily returns of stock price return series. Then these were transformed into a uniform distribution. Finally, we input the transformed uniform variates into the copula function to model the dependence structure.

The effect of fluctuation cannot be distinguished individually very clearly in the traditional mul-tivariate GARCH model. The VARMA-GARCH model is expressed as:

$$Y_t = E(Y_t F_{t-1}) + \varepsilon_t \tag{2}$$

$$\varepsilon_t = D_{t\eta t} \tag{3}$$

$$H_t = \omega + \sum_{j=1}^{r} \alpha_{jj} \varepsilon_{j,t-1} + \sum_{j-1}^{p} \beta_{jj} H_{j,t-1}. \tag{4}$$

And VARMA-AGARCH model is in the following form:

$$H_t = \omega + \sum_{j=1}^{r} \alpha_{jj} \varepsilon_{j,t-1} + \sum_{j=1}^{r} CjjI_{jj} \varepsilon_{j,t-1} + \sum_{j-1}^{p} \beta_{jj} H_{j,t-1}, \tag{5}$$

where $H_t = (h_{1t}, h_{2t}, \ldots, h_{mt}),\ \eta_t = (\eta_{1t}, \eta_{2t}, \ldots, \eta_{mt}),\ D_t = diag\left(h_{1t}^{\frac{1}{2}}, h_{1t}^{\frac{1}{2}}, \ldots, h_{1t}^{\frac{1}{2}}\right).$

For this study, the full model is in the following form:

$$At = \Upsilon_{A0} + \Upsilon_{A1} A_{t-1} + \Upsilon_{A2} B_{t-1} + \Upsilon_{A3} C_{t-1} + \Upsilon_{A4} D_{t-1} + \Upsilon_{A5} E_{t-1} + \epsilon_{At}. \tag{6}$$

$$\begin{pmatrix} \epsilon_{At} \\ \epsilon_{Et} \end{pmatrix} | \Omega_{t-1} \sim N(0, H_t), \tag{7}$$

where $A$, $B$, $C$, $D$ and $E$ are the stock indices for Zimbabwe, USA, Botswana, South Africa and China respectively and $\epsilon$ is error term. Since the nature of financial return series data is known to be leptokur-tic (stylised facts of financial data), we use the heavy tailed distributions such as Student-$t$, skewed Student-$t$ etc and MLE (Maximum Likelihood Estimation) procedure to estimate the parameters of this model.

$$\hat{\theta} = \arg\min \frac{1}{2} \sum_{t=1}^{\eta} (\log | Q_t | + \epsilon_t' Q_t^{-1} \epsilon_t), \tag{8}$$

where θ is the vector of parameters to be estimated on the conditional log-likelihood function, and $|Q_t|$ is the determinant of $Q_t$, the conditional covariance matrix.

### 3.2. GARCH model

The GARCH model which put conditional variance of lags into ARCH model and make it general was proposed by Bollerslev (1986). It is given by:

$$R_t = \mu_{t-1} + \varepsilon_t \tag{9}$$

$$\varepsilon_t / \Omega_{t-1} \sim N(0, h_t) \tag{10}$$

$$h_t = \alpha_0 + \alpha_1 \varepsilon^2{}_{t-1} + \beta_1 h_{t-1} \tag{11}$$

When $\alpha_0 > 0$, $\alpha_1 \geq 0$ and $\beta_1 \geq 0$, $\beta_1 + \beta_1 < 1$, then the GARCH model is stable.

GARCH ($p$, $q$) model can be described as follows:

$$\begin{cases} R_t = \mu_{t-1} + \varepsilon_t & \\ \varepsilon_t / \Omega_{t-1} & \sim N(0, h_t) \\ h_t = \alpha_0 + \Sigma_{j=1}^{q} \alpha_j \varepsilon_{t-j}^2 + \Sigma_{j=1}^{p} \beta_j h_{t-j} & \end{cases} \tag{12}$$

In the model above, $\mu_t$ is the conditional mean of $R_t$ at time $t$, $h_t$ is the conditional variance of the residual (error term) and is the all useful information set at time $t-1$, $\alpha_0$ is a constant, $\alpha_j$ is the coefficient of the previous $i^{th}$ period's squared residual, $\beta_j$ is the coefficient of the $h_{t-1}$ i.e. first period lag of $ht$, $\varepsilon^2{}_{t-1}$ is the previous first period's squared residual (ARCH term) and $h_{t-j}$ is the previous first day's residual variance of volatility (GARCH term). The GARCH (1,1) model can be described as follows:

$$\begin{cases} r_{i,t} = c_0 + c_1 r_{i,t-1} + \varepsilon_{i,t}' \varepsilon_{i,t} \Psi_{t-1} = h_{i,t}' z_{i,t}' z_{i,t} & \sim skewed - t(z_t \eta_t, \lambda_t) \\ \varepsilon_{i,t} & \sim N(0, h_{i,t}^2) \\ h_{i,t}^2 = \omega_{i,t} + \alpha \varepsilon_{i,t}^2 + \beta h_{i,t-1}, & \end{cases} \tag{13}$$

where $\omega_{i,t} > 0$, $\alpha$, $\beta \geq 0$ and $\alpha + \beta < 1$.

It is assumed that error terms $\varepsilon_{i,t}$ follows a Skewed-$t$ distribution that can be used to describe the asymmetric and heavy tail characteristics of each variable.

### 3.2.1. Marginal Distribution

We are going to consider both parametric and non-parametric models in modeling the marginal distributions and this takes the route of Patton and Andrew (2013).

For parametric estimation, the skewed-*t*distribution following Hansen and Bruce (1994) is used. The density function is

$$skewed-t(z,\eta\lambda)=\begin{cases} HI\left(1+\frac{1}{\eta-2}\left(\frac{Hz+G}{1-\lambda}\right)^2\right)^{-\frac{\eta+1}{2}} & z<-\frac{G}{H} \\ HI\left(1+\frac{1}{\eta-2}\left(\frac{Hz+G}{1+\lambda}\right)^2\right)^{-\frac{\eta+1}{2}} & z\geq-\frac{G}{H}, \end{cases}$$

where $2 < \eta < \infty$ and $-1< \lambda < 1$. The constants $G$, $H$ and $I$ are given by: $G \equiv 4\lambda H\frac{\eta-2}{\eta-1}, H\equiv 1+2\lambda^2$ and $I\equiv\frac{\Gamma\left(\frac{\eta+1}{2}\right)}{\sqrt{\pi(\eta-2)}\,\Gamma\left(\frac{\eta}{2}\right)}$,

where $\lambda$ and $\eta$ are the asymmetry and kurtosis parameters, separately. Those are restricted to be $-1 < \lambda < 1$ and $2 < \eta < \infty$. When $\lambda = 0$, it will turn to the Student $-t$ distribution. If $\lambda = 0$, $\eta \to \infty$, it will be the normal distribution.

For the non-parametric estimation, we use the empirical distribution function (EDF):

$$\hat{F}r_i=\frac{1}{T+1}\sum_{t=1}^{T}1\{\hat{z}_{it}\leq z\} \tag{14}$$

where $T$ is the length of the data and $\hat{z}_{it}$ it indicates the standardized residuals from the GARCH model.

## 3.3. Copula Model

There are three families of copula models which comprise elliptical copulas (Normal and Student-*t*), Archimedean copulas (Gumbel, Clayton, and Frank), and quadratic copulas (Plackett). The copulas are in fact the dependence structure of the model, implying that all the information about the depen-dence is contained in the copula function. In light of the preceding statement, the choice of the copula that will fit the data is very important. Generally, one takes a parametric family of copulas among many existing others for both parametric and non-parametric and fit it to the data by estimating the parameters of the family. However, there exist a few systematic rigorous methods for the choice of copulas that will ensure that the selected family of copula will converge to the real structure dependence underlying the data. These include the method of the maximum likelihood, inference functions for margins, estimation based on the dependence measures and canonical maximum likelihood (CML) method amongst others. For each model, we estimate the parameters by the CML method and we could define the 'optimal' copula which gives the maximum value ($\alpha_{CML}$). The CML estimation gives results in Table 1.

**Table 1**
**Canonical Maximum Likelihood (CML) estimation results**

| | *Copula* | *Canonical Maximum Likelihood function* | $\hat{\alpha}_{CML}$ |
|---|---|---|---|
| *C*1 | Cook-Johnson | $\alpha^{-1}(u^{-\alpha}-1)$ | 0.132708430 |
| *C*2 | Gaussian | $\Phi_2\{\Phi^{-1}(u_1),\ \Phi^{-1}(u_1);\ \theta\}$ | 3.922417536 |
| C3 | Frank | $-\ln\left(\dfrac{\exp(-\alpha u)-1}{\exp(-\alpha)-1}\right)$ | 0.916382524 |
| C4 | Joe | $-\log\{1-(1-t)^{\theta}\}$ | 1.542897251 |
| C5 | Rotated-Gumbel | $(-\ln u)^{\alpha}$ | 2.701462803 |
| C6 | Student–*t* | $\Sigma_{i=1}^{n}\log c_{v,P}(\widehat{U}_i)$ | 4.360578972 |
| *C*7 | Clayton | $\theta^{1}(t^{-\theta}-1)$ | 0.932678197 |

*Note:* Notes: For Student $-t$, $v$ and $P$ are the parameters and $c_{v,P}$ denotes the density of the $t$ copula

In this study, we chose to adopt the Student-*t*which is then our 'optimal' copula as reflected in the results displayed in Table 1 the Gaussian, Rotated Gumbel, and and Joe copulas to explore the dependence structure between Zimbabwe, South Africa, Botswana, China, and USA stock return index.

### *3.3.1. Constant Copula Model*

Based on an elliptical distribution are the Gaussian and Student $-t$ distributions. The symmetric Gaussian copula has no tail dependence. The values of $\theta > 0$ and $\theta < 0$ lead to positive and negative dependence, respectively. To capture tail dependence in extreme events, the Student's $-t$ which is symmetrical is adopted. In a situation where $\theta_1 > 0$ and $\theta_2 \to \infty$, the dependency between respective variables becomes independent. The Plackett copula cannot capture either the lower or upper tail dependence just like the Gaussian in view of the fact that it is also symmetric. A structure in which $0 < \theta < 1$ implies negative dependence while $\theta > 1$ reflect positive dependence. To capture extreme lower dependence, the Gumbel copula is used. If $\theta = 1$ and $\theta \to \infty$ it indicates independence and perfectly negative dependence, respectively.

Asummary of the above mentioned bivariate copula model properties are reflected in Table 2:

### *3.3.2. Dynamic Copula Model*

In this research, we applied the Student's„*t* copula time-varying model in conjunction with the Gen-eralized Autoregressive Score (GAS) model as proposed by Creal *et al.* (2013) because it provides a general framework for modeling time variation. We contemplate $\delta_t$ to be the parameter vector of the time-varying copula models component, that can be expressed as:

$$\delta_t = h(f_t) \tag{15}$$

where $f_t$ is the time-varying parameter vector in the GAS model. Following Creal *et al.* (2013), without loss of generality we can assume that $f_t$ is given by the familiar autoregressive updating equation:

$$f_{t+1} = \omega + \alpha S_{t-1}.\nabla_t \beta f_t, \tag{16}$$

$$\nabla_t = \frac{\partial \ln c(u_t; f_t)}{\partial f_{t-1}}, \tag{17}$$

$$f_t = h^{-1}(\nabla_t), \tag{18}$$

where $\omega$ is a constant, $S_{t-1}$ is a time-dependent scaling matrix, $c()$ is the density function of the copula model, and $u_t$ is the vector of the probability integral transforms using the univariate marginals. According to Creal *et al.* (2013), we can set the scaling matrix $S_{t-1}$ to be equal to the pseudo-inverse information matrix:

$$S_{t+1} = I_{t-1}^{-1} = \mathrm{I}\,\nabla_t \nabla_t'] = -E_{t-1}\left[\frac{\partial \ln c(u_t; f_t)}{\partial f_{t-1}\, \partial f_{t-1}'}\right]. \tag{19}$$

To ensure that the parameter lies within (–1, 1), the copula function $\nabla_t = [1-\exp(-f_t)] / [1+\exp(-f_t)]$ for the time-varying Student's–*t* is used.

The copula function is used in discussing problems between many variables and is also called the dependence function (Deheuvels (1978),

**Table 2: Summary Statistics**

| *Type* | *Parameter(s)* | *Function* | *Lower Tail Dependence* | *Upper Tail Dependence* |
|---|---|---|---|---|
| Cook-Johnson | $\theta \in (-1, 1)$ | $\max\left(\left[u_1^{-\alpha} + u_2^{-\alpha} - 1\right]^{\frac{-1}{\alpha}}, 0\right)$ | 0 | $2 - 2^{\frac{1}{\beta+\alpha}}$ |
| Normal | $\theta \in (-1, 1)$ | $\int_{-\infty}^{\phi^{-1}(u_1)} \int_{-\infty}^{\phi^{-1}(v_2)} \frac{1}{2\pi\sqrt{1-\theta^2}} \exp\left(-\frac{s^2 - 2\theta st + t^2}{2(1-\theta^2)}\right) dsdt$ | $\left(u^{-\theta} + v^{-\theta} - 1\right)^{-\frac{1}{\theta}}$ | 0 |
| Frank | $\theta \in (0, 1)$ | $-\alpha \ln(1 + (e^{-\alpha} - 1)^{-1}.(e^{-\alpha u} - 1)(e^{-\alpha v} - 1))$ | 0 | 0 |
| Joe | $\theta \in (1, \infty)$ | $1 - \left\{(\tilde{u}_1)^{\theta} + (\tilde{u}_2)^{\theta} - (\tilde{u}_1\tilde{u}_2)^{\theta}\right\}^{\frac{1}{\theta}}$ | $2^{-\frac{1}{\Upsilon^k}}$ | $2 - 2^{\frac{1}{\Upsilon}}$ |
| Rotated-Gumbel | $\theta \in (1, \infty)$ | $\exp\left\{-\left[(-\ln u_1')^{\theta} + (-\ln u_2')^{\theta}\right]^{\frac{1}{\theta}}\right\}$ | $2 - 2^{\frac{1}{\theta}}$ | 0 |
| Student – $t$ | $\rho(\theta_1) \in (-1, q)$ | $\int_{-\infty}^{t^{-1}(u_1)} \int_{-t}^{\phi^{-1}(u_2)} \frac{1}{2\pi\sqrt{1-\rho^2}} \exp\left(-\frac{s^2 - 2\rho st + t^2}{2(1-u_2^2)}\right)^{\frac{u_2+2}{2}} dsdt$ | $2 \times \left(-\sqrt{(u_2+1)\frac{\rho-1}{\rho+1}}\right)$ | $2 \times \left(-\sqrt{(u_2+1)\frac{\rho-1}{\rho+1}}\right)$ |
| Clayton | $\theta \in (-1, \infty)$ | $\left(u_1^{-\theta} + u_2^{-\theta}\right)^{-\frac{1}{\theta}}$ | $2^{-\frac{1}{\theta}}$ | 0 |

Note: This table presents the basic properties of the Cook-Johnson, Normal, Frank,Joe, Rotated-Gumbel, Student $-t$ and Clayton copula models. $u_1$ and $u_2$ are uniform variates. In the Rotated Gumbel copula, $u_1' = 1 - u_1$, $u_2' = 1 - u_2$. $\phi^{-1}(.)$ denotes the inverse cumulative distribution function of the univariate standard normal distribution, and $t^{-1}(.)$ indicates the inverse cumulative distribution function of the univariate Student-$t$distribution. $F_t(.)$ denotes the probability density function of the standard Student-$t$distribution. For Joe, $\tilde{u}_j = 1 - u_j$. This table is based on Aloui *et al.* (2013), Patton and Andrew (2013) and Nelson (2006).

Sklar (1959)) advances the copula theory, pointing out that one unit distribution can be analyzed to $n$ marginal distributions and one copula function. Given that the number of parameters can be large, two-step methods are generally employed. Thus, in this paper, the marginal parameters were first estimated by optimizing the marginal log-likelihoods independently of each other. Second, the copula parameters were estimated by optimizing the corresponding copula log-likelihood at the second step.

The marginal log likelihoods function:

$$m\mathcal{L}(\theta,x)=\sum_{i=1}^{p}\sum_{j=1}^{r}\log(F_t(x_{1,t};\phi_t)) \tag{20}$$

The copula log-likelihood function:

$$c\mathcal{L}(\theta, \mu, \phi) = \log(c(\mathcal{F}_1(x_1, t,\ldots, \mathcal{F}_p(x_{p'}, t; \theta_t)) \tag{21}$$

Therefore, the log likelihoods of two elliptical copula, the Gaussian and Student $-t$ copula are given by:

$$\mathcal{L}_c(R;u_t)=-\frac{1}{2}\sum_{t=1}^{\tau}(\log R+\epsilon_t'(R^{-1}-I)\epsilon_t) \tag{22}$$

$$\mathcal{L}_{st}(R,d,u_t)=-T\log\frac{\Gamma\left(\frac{d+p}{g}\right)}{\Gamma\left(\frac{d}{g}\right)}-pT\log\frac{\Gamma\left(\frac{d+1}{g}\right)}{\Gamma\left(\frac{d}{g}\right)}-\frac{d+p}{2}\sum_{t=1}^{T}\log\left(1+\frac{\epsilon_t' R^{-1}\epsilon_t}{d}\right)$$
$$-\sum_{t=1}^{T}\log R+\frac{d+1}{2}\sum_{t=1}^{p}\sum_{t=1}^{T}\log\left(1+\frac{\epsilon_{1,t}'}{d}\right) \tag{23}$$

where $\epsilon_t$ is the vector of the transformed standardized residuals which depends on the copula specification. For the Gaussian copula, the vector $\epsilon_t$ is defined as: $\epsilon_t = (^{-1}(u_{1}, _t),\ldots,^{-1}(u_{p,t})$, where $^{-1}(u_{i,t})$ is the inverse univariate standard normal distribution. For the Student„t copula, it is defined analogously as: $\epsilon_t = t_d^{-1}(u_{1,t}),\ldots,t_d^{-1}(u_{p,t})$, where $t_d^{-1}$ is the inverse student-$t$distribution with $d$ degrees of freedom. In both of likelihoods $R$ denotes the correlation matrix of $\epsilon_t$.

The DCC (1.1) model of Engle (1982) defined that the degree of freedom parameter is static for the Student $-t$ copula and the correlation $R^t$ evolves through time.

$$Q_t = (1-\alpha-\beta)\cdot\overline{Q} + \alpha\epsilon_{t-1}\cdot\epsilon'_{t-1} + \beta_{t-1} \tag{24}$$

$$R_t = \tilde{Q}_t^{-1} Q_t \tilde{Q}_t^{-1}, \tag{25}$$

where $\overline{Q}$ is sample covariance of $\epsilon_t$, $\tilde{Q}_t$ is a square p × $p$ matrix with zeros as off-diagonal elements and diagonal element the square root of those of $Q_t$. The parameter constraints for the DCC are the same as for the univariate GARCH (1,1) models.

$$\alpha+\beta<1,\ \alpha,\beta\in(0,1) \tag{26}$$

## 4. EMPIRICAL ANALYSIS

### 4.1. Data and Summary Statistics

We analyse the stock markets of 5 countries believed to be associated with Zimbabwe using daily time series from https://www.investing.com database containing entirely all the selected proxy stock indices representing respective countries. The countries are Zimbabwe, Botswana, China, South Africa and USA. After 2008, Zimbabwe reformed its currency regime more than twice. The first time was in 2009, after a galloping inflation when the authorities adopted a multiple currency, and the second time was in 2014 when she introduced the bond note as a legal tender and the third time was in 2019 when real-time gross settlement (RTGS) systems were instituted into the economy. The aftermath of any change in currency regime are contemporary high fluctuations associated with the stock market returns. We thus chose stock indices for our analysis. The sample size is sufficiently large stretching from 1 February 2009 to 31 December 2020, to match the data avail-ability for the data for all stock markets under study. We downloaded all data from the same database.

The following function was used to obtained the stationary return series for all stock indices:

$$r_{i,t} = 100\times\left(\frac{p_{i,t}}{p_{i,t-1}}\right) \tag{27}$$

where $r_{i,t}$ is the return of stock index $i$ at time $t$, $p_{i,t}$ is the price index of stock $i$ at time $t$ and $p_{i,t-1}$ is the price index of stock $i$ at time $t-1$.

We applied Bai, and Perron (1998), Bai, and Perron (2003) methodology to test for structural breaks when we suspect that there is more than one, in order to avert unexpected changes in our dataset, since many events occurred during the sample period. Zimbabwe has gone through various currency regimes and in the same vein in 2015 China's central bank devalued it's currency, USA's monetary reforms of 2011 in light of public debt etc. which may positively or negatively impact on the volatility co-movement in the stock returns.

**Table 3**
**Structural break tests**

| *Country* | *Zimbabwe* | *Botswana* | *USA* | *South Africa* | *China* |
|---|---|---|---|---|---|
| *p*–value | 2.2e-16 | 3.4e-42 | 1.5e-29 | 4.5e-97 | 8.7e-76 |

*Note:* This table presents the p-values of structural break tests using the methodology developed by Bai, and Perron (1998), Bai, and Perron (2003).

Results of structural break tests performed on the stock return series (Table 3) yielded a confir-mation of the null hypothesis which asserts that there are no breaks. This preliminary investigation using structural breaks suggests that we can use the sampled period's data for further analysis. A deeper investigation using tools such as unit root test, constant copula and time varying copula test, however, to explore dependency structure between stock returns over the entire time period will be carried out in later sub/ sections.

**Table 4**
**Descriptive Statistics**

| *Index* | *Mean* | *Std Dev* | *Sk.* | *Kur.* | *SW-test* | *J.B* | *Q(10)* | *Q2(10)* | *ARCH(10)* |
|---|---|---|---|---|---|---|---|---|---|
| Zim | 0.00063 | 0.01945 | -1.53992 | 283.05582 | 0.36344*** | 0.000 | 0.030 | 0.000 | 0.000 |
| Bot | 0.000054 | 0.00369 | 0.10923 | 70.705068 | 0.53298*** | 0.000 | 0.130 | 0.000 | 0.000 |
| USA | 0.000472 | 0.00950 | -0.29555 | 4.62727 | 0.93824*** | 0.000 | 0.011 | 0.000 | 0.000 |
| China | 0.000217 | 0.01360 | -0.43894 | 6.455204 | 0.90071*** | 0.000 | 0.000 | 0.000 | 0.000 |
| SA | 0.000348 | 0.01005 | -0.07323 | 1.667821 | 0.98237*** | 0.000 | 0.001 | 0.000 | 0.000 |

*Note:* This table presents the descriptive statistics for the daily stock returns of Zimbabwe, Botswana, USA, China and South Africa respectively. The sample period is from 2 February 2009 to 30 June 2020 (2876 observations). For the SW-test which refers to the Shapiro-Wilk normality test, we report the *p*-value of this statistic. *** indicates a rejection of the null hypothesis, which states that the data is normally distributed at the 1% level of significance. $Q(10)$ and $Q^2(10)$ are Ljung-Box tests for 10th-order serial correlation in the returns and squared returns. ARCH (10) is the Engle (1982) test for the $10^{th}$-order ARCH. These three tests are distributed as $\chi^2(10) = 18.3$.

The data in Table 4 are a summary of the descriptive statistics of the variables. The standard deviation of Zimbabwe and China stock index returns are relatively higher than those of the other three stock index returns. Apart from Botswana, all the remaining stocks manifest negative skewness whilst Zimbabwe exhibits heavy negativity and South Africa's skewness near zero. By and large, these disclose that they are outstandingly skewed to the left. Zimbabwe and Botswana have comparatively high excess kurtosis statistics although all of the variables incorporated in this study are positive. This, nevertheless, implies that the distribution of stock returns are leptokurtic, that is it has larger, thicker tails than the normal distribution implying that there is a higher probability of extreme outlier values. This typically show a relatively higher value at risk (VaR) in the left tail due to the larger amount of value under the curve in the worst-case scenario. In the case of South Africa, its return series is platykurtic thereby implying that it has thinner tails than a normal distribution, leading to less extreme positive or negative returns. This kind of market is more tolerable for investment by more risk-averse investors because they are less likely to yield severe results. Consequently, the presupposition of skewed-*t* is pertinent in this study. The two statistics (skewness and kurtosis) are as well captured and summarized in the Shapiro-Wilk statistic, which likewise confirms the rejection of the null hypothesis of normality for all-time series. Summarily, though violated in the case of South African stock return series, it is a fact that the dual combination of negative values of skewness statistics and its associated leptokurtic nature exhibited are stylised facts of financial data.

The Ljung-Box (LB) statistic reported in Table 4, to test the absence of serial autocorrelation in the stock return series and corresponding squared return series. It can be categorically stated that this test can be used for testing market efficiency, whether to apply the GARCH or not, diagnostic test for the OLS residuals and for checking the independence of observations in the stock return series. The results reveal the existence of significant

**Table 5**
**Rank correlation and linear correlation**

| *Pair* | *Kendall's $\tau$* | *Spearman's $\rho$* | *Pearson* |
|---|---|---|---|
| Zimbabwe-South Africa | -0.03096036** | -0.0470248** | 0.002358378 |
| USA-South Africa | 0.02362634* | 0.03484893* | 0.03861345** |
| China-South Africa | -0.02004004* | -0.03069187* | -0.01693841* |

*Note:* This table represents the Kendall's τ and the Spearman's ρ correlation co-efficient, which measures strength and direction of association that exists between two variables measured on at least an ordinal scale; and the Pearson correlation coefficient, which measures linear correlation between variables. ** and * designate relevance at 0.05 and 0.10 level respectively.

contemporaneous autocorrelation for the entire return series. In consideration with squared returns series; there is significant presence of autocorrelation, which exhibit powerful volatility clustering and strong dependence structure in the long horizon. These aspects are confirmed by the ARCH test results.

In this section, we discuss the results of both the rank correlation and linear correlation in Table 5 for comparison purposes. Fundamentally before the beginning of any serious copula analysis is done the correlation juxtaposition needs to be upheld. Kendall and Spearman's rank correlation are used because they are appropriate when one or both variables are skewed and is robust when extreme values are present. Pearson's product moment correlation coefficient, on the other hand is affected by extreme values, which may exaggerate or dampen the strength of the relationship, and is primarily appropriate when either or both variables are not normally distributed. All significant coefficients are displayed. Since all stock return series are either leptorkutic or platykurtic, comments for the correlations are principally based on the Kendall's and Spearman's rank correlation coefficient. There exist a negative correlation between Zimbabwe-South Africa and China-South Africa stock return series as exhibited by both the Kendall and Spearman's correlation coefficient whilst a positive USA-South Africa correlation exists. This matched set has the highest coefficient for both the rank and linear correlations, very likely because South Africa is a strategic trading partner of the United States and they enjoy a solid bilateral relationship with strong collaboration in the areas of health, education, environment, and digital economy than other countries in our sample.

### *4.1.1. Stationarity of stock returns*

To test for the traditional unit roots, the Augmented Dickey Fuller (ADF), Phillips-Perron (PP) and Kwiatkowski-Phillips-Schmidt-Shin (KPSS) are used, where the null hypothesis states that a unit root is present in all data series. The ADF is specified as:

$$\Delta Y_t = \alpha + \beta_t + \theta Y_{t-1} + \sum_{t=1}^{p} \phi \Delta Y_{t-1} + \epsilon_t \tag{28}$$

The null hypothesis is θ = 0 which, if not rejected when the $p$-value > 0.05, means that the data series is not stationary. After running the data, results acquired shows that all series data are stationary reflected by the estimated value of θ of all the stock returns that are significantly less than zero at the 5% level. These results imply that the log of the first difference is adequate to induce stationarity in price series. Details are in Table 6. We will only employ stationary time series (stock return series) in the remaining parts of this research.

**Table 6**
**ADF, PP and KPS test for stationarity**

| *Variable* | *ADF* | *PP* | *KPSS* |
|---|---|---|---|
| Zimbabwe | -12.302*** | -12.31*** | 0.054 |
| Botswana | -11.378*** | -11.38*** | 0.249 |
| USA | -14.559*** | -14.54*** | 0.072 |
| China | -13.708*** | -13.68*** | 0.087 |
| South Africa | -13.899*** | -13.91*** | 0.097 |

*Note:* The ADF, PP and KPSS refer to the augmented Dickey and Fuller (1981), Phillips and Perron (1988) and Kwiatkowski *et al.* (1992) unit root tests, corresponding to the process with an intercept but without a trend. *, **, ***, stand for 10%, 5% and 1% critical values, respectively.

For the daily stock return series, Figure 1 exhibits a plot of its time-path, whilst Table 2 displays corresponding descriptives. It is of interest to note that the mean of Botswana is comparatively close to zero, and similarly its standard deviation is the least of all stock return series. This reflects a stable stock return series over the period of study under government regulations. Contrary to this, Zimbabwe which share borders with Botswana, has the most fluctuation among the five stock market return series predominantly at the end of 2015 when growth fell from 1.4 percent in 2015 to 0.7 percent in 2016 continuing to plummet in per capita income growth coupled with fiscal imbalances which when conjoined with a large volume of domestic borrowing which is nurtured and threaten to weaken the financial sector that are at the core of Zimbabwe's ongoing financial crisis. Besides Botswana, all stock return series are negatively skewed with very high kurtosis apart from South Africa. This reflects non-normality and asymmetric characteristics financial return time series distributions. The Shapiro-Wilk test performed on the stock return series furthermore verifies this analysis since the threshold of $p$-are less than the pre-defined values.

## 4.2. Static Copula Results

In this section we will fit both the parametric and the non-parametric copulas to historical data on variables under consideration. Copulas are very powerful mathematical tools for capturing linear and non-linear correlation structure in data. It allows a multidimensional framework, giving up the restrictive gaussian assumption which we know is incorrect in most financial

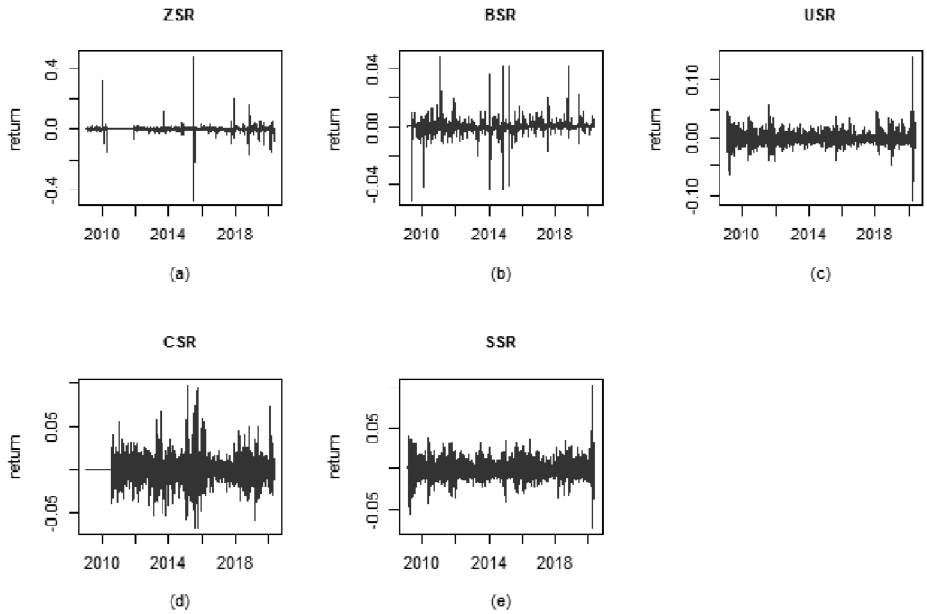


**Figure 1: Stock indices' Daily returns of all series. (a-e) refer to return series of Zimbabwe, Botswana, USA, China and South Africa, respectively**

data and the study of extremal events. When these correlation structures are captured, they can be built in risk analysis models depending on one's application. In our research we have five sets of 2838 data points. With interest in analysing this data whether there are some dependency structures or not we can build a model that can be used to project some of these parameters into the future. To come up with the most appropriate fit to the data we consider using statistical measures of fit called information criteria. These are Schwarz information criterion (SIC), Alkaike information criterion (AIC), Hannan-Quinn information criterion (HQIC) and the Likelihood ratio (LR).

- Schwarz information criterion (SIC), also known as Bayesian information criterion (BIC)

$$SIC = ln[n]k - 2\ln[L_{max}] \tag{29}$$

- Alkaike information criterion (AIC)

$$AIC_c = \left(\frac{2n}{n-k-1}\right)k - 2\ln[L_{\max}] \tag{30}$$

- Hannan-Quinn information criterion (HQIC)

$$HQIC = 2ln[ln[n]]k - 2ln[L_{max}] \tag{31}$$

where $n$ = number of observations, $k$= number of parameters to be estimated and $L_{max}$ = the maximised value of the log-likelihood for the estimated model (i.e. fit the parameters by MLE and record the natural log of the Likelihood).

The aim here is to find the model with the least value of the selected information criterion. The component $-2\ln[L_{max}]$ that appears in each formulae act as an estimate of the deviance of the fitted model. The coefficient of $k$ shows the degree to which the number of model parameters is being penalised. The values are displayed in Table 7.

**Table 7**
**SIC, HQIC, AIC and LR information criterion**

| *Type* | *SIC* | *AIC* | *HQIC* | *LR* |
|---|---|---|---|---|
| Normal | -82.45 | -23.03 | -44.42 | 0.0174 |
| Student's t | -82.30 | -16.91 | -40.46 | 1.0000 |
| Frank | -8.13 | -2.18 | -4.33 | 0.0693 |
| Clayton | -7.11 | -1.17 | -3.31 | 0.1153 |
| Rotated Gumbel | -8.08 | -2.09 | -4.24 | 0.0726 |

Basing on the values of SIC, HQIC and AIC in Table 7, the Gaussian and the Student-*t* depen-dence structure exhibits better explanatory power than other copula distributions. This reveals the dominance of Elliptical Copula under which both the Gaussian and the Student-*t*falls.

There is broadly a scattered symmetric dependence which correlates with higher trade dependence in general between variables using parametric copula best fitted models (normal and student-*t*copula) judging largely on the information criterion as reflected in Table 7. This, for comparison purposes justify the adoption of non-parametric empirical copulas which takes a shape exclusively from historical data.

The two static copula estimations (Normal and Student's *t*) for four stock return pairs that span Zimbabwe and Botswana, USA, China and South Africa, respectively, are shown in Table 8. The copula model parameter estimates are all significant at the 1% level that are based on the maximum likelihood method in which *inter alia* the Student-*t* copula has the highest value while the Gaussian copula has the lowest. In terms of the log-likelihood function, the Student's *t* copula appears to be the most appropriate model for describing the dependence structure between Zimbabwe stock index returns and each major trading partner's stock returns series, followed by the Gaussian copula. Therefore, the revelation driven from the two copula models shows a significant inter-temporal

**Table 8**
**Static copula parameter estimation**

| *Type* | *Parametric* | | | *Non-Parametric* | | |
|---|---|---|---|---|---|---|
| | $\theta$ | | $Log\,\xi$ | $\theta$ | | $Log\,\xi$ |
| | Zimbabwe-Botswana | | | | | |
| Normal | 0.049*** | | 53.87 | 0.0487*** | | 52.45 |
| Student's $t$ ($Q$, $v^{-1}$) | -0.016*** | 0.0309** | 57.92 | -0.0159** | 0.0301 * | 57.87 |
| | Zimbabwe-USA | | | | | |
| Normal | -0.009*** | | 9.89 | -0.0089*** | | 9.02 |
| Student's $t$ ($Q$, $v^{-1}$) | -0.022*** | 0.0910 *** | 19.76 | -0.0219*** | 0.0940 *** | 19.99 |
| | Zimbabwe-China | | | | | |
| Normal | -0.043*** | | 47.27 | -0.0427*** | | 46.82 |
| Student's $t$ ($Q$, $v^{-1}$) | -0.005** | 0.0505 ** | 54.14 | -0.00499** | 0.0405 ** | 53.98 |
| | Zimbabwe-South Africa | | | | | |
| Normal | 0.001*** | | 61.28 | 0.00099*** | | 62.62 |
| Student's $t$ ($Q$, $v^{-1}$) | 0.014** | 0.0416 ** | 63.11 | 0.01397** | 0.0421 ** | 63.23 |

*Note:* Constant copula estimations results are presented for the Zimbabwe-Botswana, Zimbabwe-USA, Zimbabwe-China, and Zimbabwe-South Africa pairs in this table. $\hat{\theta}$ represent the estimated coefficient. $Log\,\xi$ represents the log-likelihood of each constant copula model. The values in paren-theses are the standard error of the parameter. The values in bold indicate the highest loglikelihood values. **, and *** indicate significance at 5%, and 1% levels, respectively.

dependence structure. Apart from Zimbabwe-South Africa pair, as reflected by the results, there is a negative dependence among all other pairs under consideration for the two cases i.e. parametric and semi-parametric.

The two Southern African Development Community (SADC) neighbouring countries and members of the SADC Free Trade Area (SADC FTA); Zimbabwe and South Africa have bilateral trade agreement dating back from 1964, which give provision for reciprocal trade access to certain agricultural products, manufactured goods, machinery, mineral fuels, vehicles, plastic products, electrical equipment among others. Zimbabwe is currently experiencing one of the worst economic collapse in the history of mankind primarily exports, low value natural resource based products and imports largely value added manufactured goods implying that the manufacturing backbone for her is too weak to process raw material into finished usable goods. This has apparently created a positive coefficient as revealed from the copula output as shown in Table 8. Thus a positive shock in Zimbabwe stock return, positively affects South African stock returns and vice versa. This is in line with economic theory which suggests that whenever there are strong trade ties between two economies, then the dependence structure of the two is positive. This is also in line with the relatively strong positive correlation that exists between these trading partners as revealed in Table 3. Notwithstanding the cordial trade relation there are weak ties between these economies. Looking through the lenses of bilateral relations between the countries concerned, this could be as a result of Zimbabwe's dual combination of failure to have a clear export promotion strategy and its anti-import position as demonstrated by interventions that it has of late introduced since 2012 to regulate imports as a device to boost local industry. According to Tshuma (2016), to this extend, statutory instruments were introduced whereby surtaxes on selected products were effected, although these detrimental to the interests of the economy and people that has ultimately resulted in sort of balanced trade.

The Zimbabwe-China relations have developed over the years when Zimbabwe evolves into the Look East Policy, prioritising economic and political sectors. The association is generally referred to as a microcosm of China-Africa relation because of its economic and resources aspects Chun (2014). There is a negative relationship that exist between the two stock markets. This implies that a positive shock in the Zimbabwe stock market return, negatively affects Chinese stock returns and contrariwise. Such kind of relationship may result predominantly from the biased nature of the bilateral trade where China continues to manipulate and exploit to its advantage the relationship knowing that Zimbabwe has no other friend and partners from the developed first world economies for development and cooperation. Hence the skewed nature of trade agreements that yields unsatisfactory results to the Zimbabwean economy indebted to the political and economic weaknesses of policy makers and leadership.

Zimbabwe-Botswana bilateral trade has substantially increased to become the leading trading pair in the whole of SADC. However, this increase still fall short of people's expectations as there is potential for further growth of this relationship which is marred by persistent disputes and conflicts to the extent of threatening to disrupt the entire trade by closing borders henceforth a negative copula coefficient for both the parametric and non-parametric models. On the Zimbabwe-USA front, Zimbabwe has been enjoying friendly trade relations with its USA counterpart for quite sometime up to 2000 when the USA administration took a leading role in condemning the Zimbabwean Government on account of increased assault on human rights and the rule of law, a position it has maintained to date. This hostile relation could have culminated to the negative dependence structure between the two trading partners. The aforementioned suggest that a negative shock on the Zimbabwe stock return has negatively impacted on the USA stock return and the other way round.

**Table 9**
**Student-*t* copula model, tail dependence coefficients**

| *Pair* | *Parametric* | | *Non-Parametric* | |
|---|---|---|---|---|
| | $\hat{\lambda}^L$ | $\hat{\lambda}^U$ | $\hat{\lambda}^L$ | $\hat{\lambda}^U$ |
| Zimbabwe-Botswana | 0.4198 | 0.4219 | 0.4216 | 0.4223 |
| Zimbabwe-USA | 0.2129 | 0.2120 | 0.2356 | 0.2349 |
| Zimbabwe-China | 0.1232 | 0.1231 | 0.1372 | 0.1366 |
| Zimbabwe-South Africa | 0.3254 | 0.3275 | 0.4103 | 0.4135 |

*Note:* Student-*t*copula model tail dependence coefficients estimated from the static.

The Student-*t*copula model reigns supreme in modeling stock return dependence structure of stock markets considered in this research as demonstrated in Tables 8 and 9. Parameter $v$ represents the estimated degrees of freedom. The value of the degrees of freedom is significantly high for the entire stock returns pairs. This is an indication of extreme co-movements tendency between the concerned pairs. This, nevertheless, supports joint extreme co-movements irrespective of their individual marginal behaviour which is a strength resolutely captured by copula models. The results presented in Table 8 are in concurrence with the preceding analysis which show the predominance of fat tailed Student-*t*dependence structure both in the upper and the lower tail. The lack of proper concordance between the lower and upper tail parameter for both the parametric and non-parametric entices one to conclude that there is a presence of asymmetric dependence between stock return pairs under discussion which impacts much on investment finance and risk diversification. Basically asymmetric dependence reflects the characteristic of the joint return distribution whereby dependence between paired stock returns during the bearish downturn phase differs from that observed during bullish upturn. On the other hand lower-tail asymmetric dependence refers the situation in which dependence in the lower tail is higher that in the upper tail whereas upper-tail asymmetric dependence refers to the opposite situation. From Table 8 we can conclude that these dependencies Zimbabwe-Botswana and Zimbabwe-South Africa exhibit upper-tail asymmetric dependence (UTAD) whilst Zimbabwe-USA and Zimbabwe-China exhibit lower-tail asymmetric dependence. The prevalence and price of upper-tail asymmetric dependence indicates that in general investors in these stock markets place substantial value on UTAD, implying that the returns of these markets are exhibiting this distinctive characteristic. Consequently, this may cascade down to the firms with large cap equities in the market environment that can to a larger dimension not only explained by changes in systematic risks, but additionally by changes in the value of the stock displaying UTAD trending with the market.

These results seem to possess important practical implications to both investors and other stock market participants in line with their future endeavours to manage tail risks. Tail-risk protection buyers and sellers need to be vigilant and consider the probable magnitude of systematic risk changes inherent to changes in asymmetric dependence and craft a strategy that hedges these changes in linear dependence risks that should be completely different from the strategy that can be embraced to manage changes in tail dependence. When properly carried out, the relationship that might exist between the magnitude of sensitivity of returns to asymmetric dependence, comparable to the responsiveness of returns in line with systematic risk, stock participants may be enabled to assess the sufficiency of systematic risk hedge to alleviate the prospective losses characterized by tail dependence. The analysis of the volatility dependency structure of Zimbabwe Stock return series in line with the volatility of the other stock return was undertaken using the Gaussian and Student's $t$ copula models. Parametric and non-parametric on both static and dynamic copula results were estimated. The results of the Gaussian and Student-$t$ copula are presented in Table 8, and in the same vein the number of volatility spillovers and asymmetric effects of stock index return were estimated and are summarised in Table 9. This table also suggest the presence of volatility spillovers in eight of the ten pairs. Of principal significance is the evidence emanating from South Africa to Zimbabwe, from USA to South Africa and from China to South Africa. This evidence is in agreement with the knowledge that Zimbabwe heavily depends on South Africa while on the other hand South Africa inherently depends on USA and China that set such a dependency structure. The significant interdependences in the conditional volatilities among stock returns series are 3 of 5 cases for both the Gaussian copula model and Student-$t$. In addition, asymmetric effects are evident in 1 of the 5 cases. Accordingly, the dual evidence of volatility spillovers and asymmetric effects of negative and positive shocks on conditional variance however suggest that Student's $t$ copula is superior to Gaussian copula in examining the volatility of stock price return series.

### 4.3. Dynamic Copula Results

**Table 12**
**Dynamic copula parameter estimation.**

| *Parametric* | | | | | *Non-Parametric* | | | | |
|---|---|---|---|---|---|---|---|---|---|
| $\widehat{\omega}^{U}$ | $\widehat{\alpha}$ | $\widehat{\beta}$ | $\widehat{v}$ | $Log\,\xi$ | $\widehat{\omega}$ | $\widehat{\alpha}$ | $\widehat{\beta}$ | $\widehat{v}$ | $Log\,\xi$ |
| | | | | Zimbabwe-Botswana | | | | | |
| -0.0304 | 0.0162 * | 0.867*** | 0.0794* | 89.7362 | -0.0944 | 0.0536 *** | 0.1237 *** | 0.0798 * | 97.5681 |
| (0.0321) | (0.0254) | (0.0025) | (0.0638) | - | (0.0010) | (0.0022) | (0.0000) | (0.0752) | - |
| | | | | Zimbabwe-USA | | | | | |
| -0.2158 ** | 0.0876 *** | 0.1312 *** | 0.1968*** | 229.8312 | -0.0371*** | 0.0690 *** | 0.7178*** | 0.0907*** | 230.1090 |
| (0.0012) | (0.0279) | (0.0397) | (0.0405) | - | (0.0000) | (0.0364) | (0.0107) | (0.0978) | - |
| | | | | Zimbabwe-China | | | | | |
| -0.2004*** | 0.0201 | 0.4061 | 0.0760*** | 33.1408 | -0.2031*** | 0.0489 | 0.2465 | 0.0405** | 33.1562 |
| (0.0116) | (0.0215) | (0.1777) | (0.0922) | - | (0.0000) | (0.0719) | (0.6083) | (0.0137) | - |
| | | | | Zimbabwe-South Africa | | | | | |
| -0.0069 | 0.0194 | 0.1109*** | 0.0655*** | 27.39076 | -0.0316*** | 0.0142 | 0.8123*** | 0.0914*** | 27.3269 |
| (0.0702) | (0.0156) | (0.2173) | (0.01968) | - | (0.0001) | (0.0957) | (0.2970) | (0.0617) | - |

*Note:* Results of dynamic Gaussian and Student-$t$ copulas are presented in this table. $\widehat{\omega}, \widehat{\alpha}, \widehat{\beta}$, and $\widehat{v}$ represent the coefficients estimated. The log-likelihood of each Dynamic copula model are represented by $Log\xi$. The standard error of the parameter are given in the corresponding parentheses (). ***, **, and * indicate levels of significance at the 11%, 5%, and 10% respectively.

[H]

The results obtained in Table 11 show the dynamic Student-*t* copula model using the Generalised Additive Model (GAM model). In some cases, using the model we discover that the $\widehat{\beta}$ estimate is highly significant and has a considerably large value, implying that we can derive the conclusion that dynamic copula model parameters in the current period have a significant impact on the copula parameters of the next period.

To further increase the statistical significance of our results, we plot the time-varying estimated parameters from the dynamic Student-*t*copula in the non-parametric case. As can be seen from Figure 2, these analyses further stress the fact of negative dependence structures; a result that amounts to concordance with static copula model results. The same conclusion may also be driven by the rolling windows technique analysis. The performance of dynamic copula models compared to traditional static copula model is stronger especially in high volatility periods when the need for reducing variance is particularly important. [a][b][c][d]

Finally, Figure 3 plot the dynamic paths of the conditional tail dependence with time-varying in the non-parametric case. All the conditional tail dependence display significant variability, which suggests that the assumption of constant conditional tail dependence is not valid. It is interesting to note that the tail dependence is positive for all pairs of

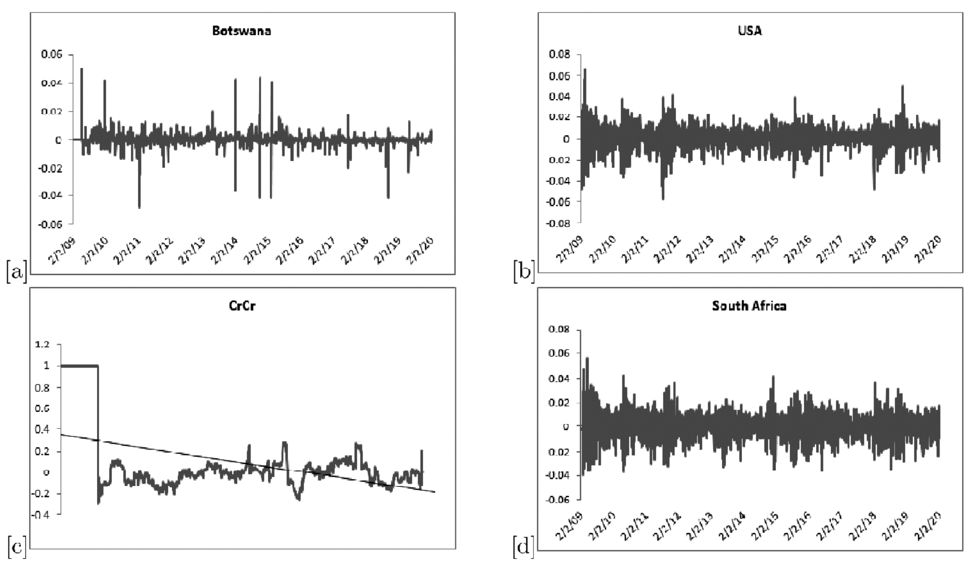


**Figure 2: Dynamic parameters from the time-varying Student-*t*copula models. (a–d) refer to Zimbabwe-Botswana, Zimbabwe-USA, Zimbabwe-China and Zimbabwe-South Africa pairs, respec-tively**

Zimbabwe-Botswana, Zimbabwe-USA, Zimbabwe-China and Zimbabwe-South Africa.

### 4.4. Value at Risk and Excepted Shortfall

The riskiness of a portfolio can be best quantified by the dual statistical measures of value-at-risk (VaR) and expected shortfall (ES). In this regard VaR is a measure of maximum expected loss for investments with a certain degree of confidence, i.e. it estimates the extend to which a set of investments might lose, given normal market conditions per given time frame. Expected shortfall evaluate the market risk of a portfolio in the worst of cases suggesting that it computes the expected loss when the portfolio return is greater than the value of the VaR calculated with that confidence level. In order to study the market correlation, we are going to focus on forecasting value-at-risk of the Zimbabwe market, being conditional on the other four markets (Botswana, USA, China and South Africa) having leading effect on volatilities of the Zimbabwe market. To attain this goal we consider the pairs as equally weighted portfolios and embrace Monte Carlo simulation to compute the dynamic VaR and ES by employing the procedure the following four sequential order:

1. Time-varying Student„*t* copula model parameter estimates are adopted to come up with 3000 uniformly distributed random digits for each time *t* (*t*is from 1 to T, where T is the length of the stock return series).
2. the uniform randomly generated numbers are transformed into a string of standardized residuals. A new stock return series is generated by embracing the parameters estimated from the GARCH models.
3. apply the portfolio investment weights (1% in this study) alongside the current stock returns to compute a series of returns on the portfolio and employing all these compute the VaR and ES.
4. replicate stages 1–3 up to T times (where T is the length of the stock return series).

Results of the conditional value-at-risk (CVaR) are plotted as shown in Figure 3. The left panel exhibit a 1% CVaR whilst the right panel shows expected shortfall (ES) for an equally weighted portfolio centered on the parameters obtained from the dynamic Student„*t*copula model. The worst loss at the p=99% confidence level reflect that an investor on the stork market is 99% unquestionable that at the end of a pre-defined risk horizon that there will be no greater loss than the recognised VaR. The output reveals that the VaR for all the pairs starts at the 3% mark. For the Zimbabwe-Botswana pair the VaR decreases to very low levels of 6% at the start of 2015 going along with a further dwindle to impassioned levels of approximately 7.5% at the middle of the year 2016. Responding to the further deterioration of the Zimbabwe economic performance the Zimbabwe stork market has been exhibiting this characteristic throughout all the pairs and furthermore as the world economic situation keeps on changing. In like

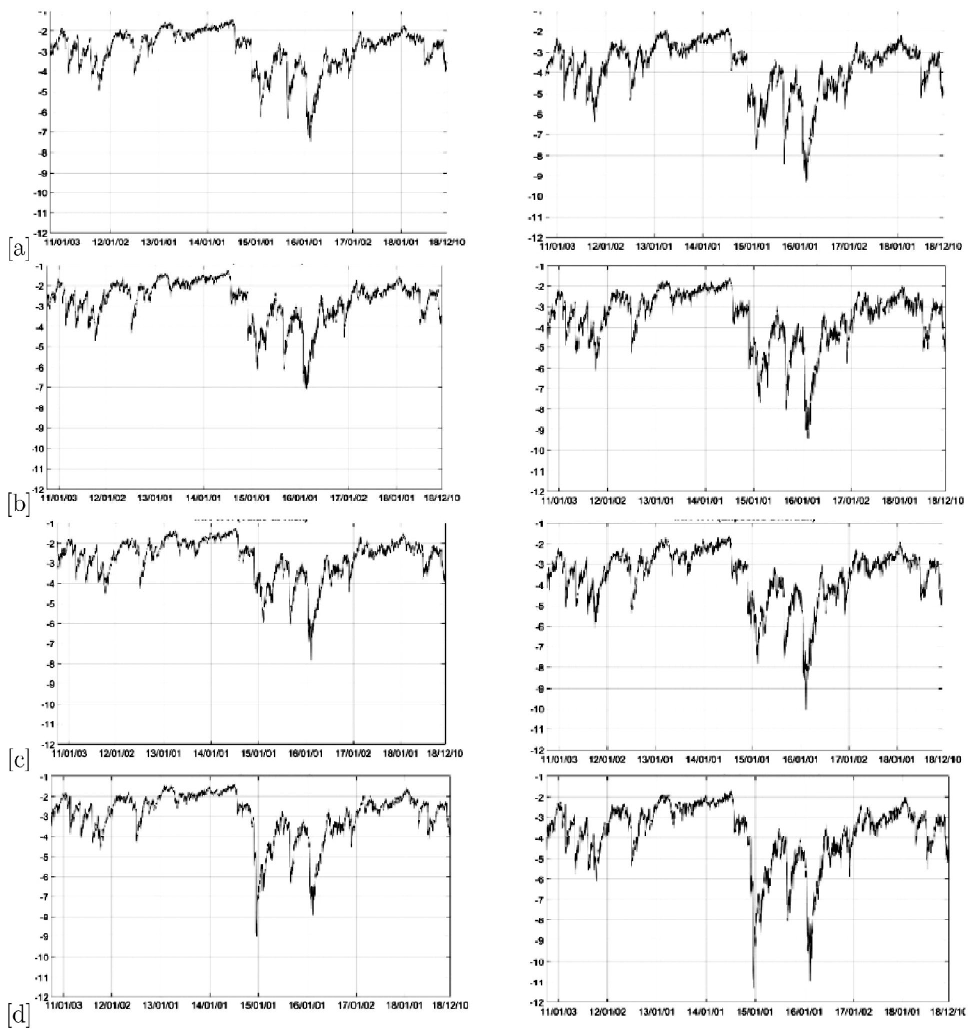


**Figure 3: The left panel display the conditional 1% Value at Risk whilst the right panel reflect the Expected Shortfall for an equal-weighted portfolio based on the Student –*t* copula model that have been developed. (a-d) refer to Zimbabwe-Botswana, Zimbabwe-USA, Zimbabwe-China and Zimbabwe-South Africa pairs, respectively**

manner the behaviour of the expected shortfall (ES) reacted correspondingly to the changes occurring in these stock market fluctuations/volatilities.

### 4.5. Concluding Remarks and Recommendations

The paper investigated the volatility spillovers that exist between stock return series of Zimbabwe and its major trading partners that include among others South Africa, Botswana, USA and China for the period 1 February 2009 to 30 April 2020. The empirical results have revealed that a significant depen-dence existed in almost all stock market pairs, and most of these dependences are positive and just a few are negative, which imply that there is a possibility of a hedge against rising volatilities in markets exhibiting different signs in their dependency structures. As an example, when one market's volatility increases, investors with investment portfolios in that market will find the market highly risk that may negatively impact on investor confidence which may ultimately reduce demand for stock, which will lead to a decline in the stock price. The second aspect is that in both the static and dynamic copula models, the Zimbabwe-South African and USA-South African pairs have the strongest dependence, possibly because South Africa is a pivotal economy in its trade with Zimbabwe making itself dependent on it for its exports and also has established USA as one of its major trading partner in embracing its exports. Therefore, Zimbabwe should seriously consider the volatilities in the South African stock market returns well as USA when building models for predicting and forecasting its future stock returns. USA stock return volatilities have very strong significant effects on the South African market. This research has examined both the static and dynamic dependencies between these economies. It has employed both the Gaussian and Student's *t* copula-based models to capture the nonlinear, asymmetric, and tail dependence structure between the paired stock index returns. Both the Gaussian and Student's *t* estimates were statistically significant for all returns and most of the estimates of the asymmetric effects were significant. The copula based model has revealed that the Student's *t* dependence has a better explanatory power than the Gaussian dependence structure. Based on the asymptotic standard errors, the Gaussian and Student's *t* copula-based models showed evidence of volatility spillovers and asymmetric effects of negative and positive shocks on the conditional variances.

Nevertheless, it is prudent mentioning that in this paper, the researcher only considered bivariate copula models. For the future extension of this research should consider using high-dimensional copula models, which have the capacity of providing a much more flexible analysis. This would provide a more complete picture of the complexities associated with the analyses and a check of the robustness of the empirical results presented in this paper. Investors, government agencies, policy makers, as well as other key stakeholders in the stock market environment.

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