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An Application of SPS Allocation

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Abstract: (Marjit & Sarkar, 2017) have proved the existence of a unique inequality-preserving redistribution allocation in the contract curve. The allocation is proven for those measures of inequalities which satisfy certain criterion. We investigate whether their Strong Pareto Superior (SPS) allocation hold in a special scenario, which moves away from their stated axioms. We find the evidence to the contrary; however, the theoretical foundation for such an analysis remains an open question.

Keywords: Strong Pareto Superior allocation, Fairness, Inequality.

JEL Classifications: C00, D63

Introduction

(Marjit & Sarkar, 2017) have proved the existence of a unique inequality-preserving redistribution allocation in the contract curve. They call the allocation as Strong Pareto Superior allocation (SPS) and argue that the result is true irrespective of whether one considers an absolute measure of inequality or the leftist measure, or a relative measure of inequality or the rightist measure, even though they show that the results are different for both. The measures are called ASPS allocation and RSPS allocation respectively. In this paper, we apply their idea and investigate the existence of an SPS-like allocation in a special scenario, where Normalization axiom is violated. By Normalization axiom, a 'measure of inequality' should be zero when everyone earns the same income.

Background to Our Paper

(Venkatasubramanian *et al.*, 2015) (VLS henceforth) had argued that some amount of income inequality is fair. By their way of fairness, they measure inequality by paying people according to their productivity and since people inherently differ in their productivity, they get paid differently. The resulting inequality is 'fair' according to them, which moves away from a leftist measure of being egalitarian in paying everyone equally, the result of which according to them will lead to bifurcations in pay-outs to the employees in the presence of perfectly competitive employees and firms.

Even when people and firm's actions seem purposeful from epistemic exterior stand point with their underlying behavior of selfishness being in contrast to the teleologically random movements exhibited by a set of molecules in a closed container, VLS argue that the thermodynamic measure of 'entropy' can be a measure of 'fair' inequality. We refer the reader to VLS paper for a convincing argument on why the thermodynamic entropy can be a measure of a 'fairness' imbibed inequality.

(Marjit *et al.*, 2019) gives an empirical illustration of their SPS allocation by using different measures of inequality, namely., absolute and relative measures. However, the result of an inequality-preserving fiscal redistribution does not seem to hold for the VLS measure of 'fairness' imbibed inequality. When we proceed along the lines of (Marjit et al., 2019), and introduce a transfer amount of 'T' from a person having high income post- growth to a person with low income post- growth, then for this entropy as a measure of inequality, there simply does not exist a transfer amount that preserves inequality post-distribution, casting doubt on whether an SPS-like allocation hold for other measures of inequality, which are not the traditional leftists or rightist measures. However, as (Marjit et al., 2019) show, the transfer amount do exist for the generalized entropy measure, which is an often cited measure of inequality in the literature.

Our claim:

The VLS measure of entropy is given by $S = \frac{1}{2} + \frac{\ln(2 * \sigma^2 * \Pi)}{2} + \mu;$

where, μ and σ are the mean and standard deviation of the income distribution.

In the following, we take 'a' and 'b' as two income levels and calculate y_1 = ln(a) and y_2 = ln(b); since the income distribution is a log-normal distribution for the VLS measure.

Given: an income profile of (y_1, y_2), which becomes $(g_1 + y_1, g_2 + y_2)$ post- growth. (Without loss of generality, we assume $y_1 > y_2$, $g_1 > g_2$). Let 'T' be the transfer amount from person 1 to person 2, hence, after redistribution the incomes profiles are $(g_1 + y_1 - T, g_2 + y_2 + T)$.

Since we want the inequality to remain same after transfer, we have $S_1 = S_2$

$$\frac{1}{2} + \frac{\ln(2 * \sigma 1^2 * \Pi)}{2} + \mu 1 = \frac{1}{2} + \frac{\ln(2 * \sigma 2^2 * \Pi)}{2} + \mu 2 \qquad (1)$$

$$\Rightarrow \ln(\sigma 1/\sigma 2) = \mu 2 - \mu 1 \tag{2}$$

For the two given income profiles post- growth, for a transfer amount 'T' to exist, this boils down to the requirement of, (refer appendix)

$$\frac{(y1-y2)^2}{(g1+y1-g2-y2)^2} > \exp(g1+y1+g2+y2-y1-y2)$$

Since the denominator of the LHS is a relatively big number, and since the RHS is raised to the power of an exponential, this inequality does not hold, at least for some incomes, thereby making it a proposition against the chance of the existence of a distribution-neutral transfer amount for the 'fair' measure of thermodynamic inequality. For instance, try $y_1 = 5$, $y_2 = 2$, $g_1 = 3$, $g_2 = 2$.

This is equivalent to saying that either the (Marjit & Sarkar, 2017) result does not hold when one or more of the axioms are violated, for which theoretical reasoning is required; or that a 'fairness' imbibed income inequality measure may never require transfers. We assume the latter not to be the case because there's a change in inequality post- growth, which necessitates transfers.

We state the result as a proposition, whose proof is as per appendix:

The existence of distribution-neutral transfer allocation is untrue for a 'fair' measure of inequality, as measured by thermodynamic entropy.

Addressing our limitation: The limitation of our argument is that, the VLS measure of entropy doesn't fall in the relative measure of inequality or in the absolute level of inequality or anywhere in between, but outside of it. For instance, let's consider an economy with two people having income profile (10, 20). Consider the growth process in this economy to have resulted in an income profile of (100, 200). Then by the relative measure, the inequality is preserved, since relative measure of inequality is scale invariant. Now, consider the growth process to have resulted in an income profile of (100, 110). Then the absolute level of inequality is preserved since absolute measures are translation invariant. In literature, the absolute measure, which is dubbed the leftist measure happens to be a conservative estimate, however we will see now that the VLS measure of inequality as measured by entropy is far more conservative than the absolute measure itself, implying (Marjit & Sarkar, 2017) never addressed this issue. (We give further explanations in the following para.) Irrespective of whether they address this case or not, we do not have an inequality-preserving fiscal redistribution in terms of 'fairness', so to say.

For the given income profile (100, 110), the VLS inequality measured by entropy goes up from 18.03 to 108.03 compared to the initial income profile (10, 20), for which the absolute measure has to remain intact at 18.03. In this sense, the VLS measure of entropy is more conservative than the absolute measure of inequality. That is precisely why we have taken the growth factors g_1 and g_2 and the transfer amount 'T' in additive form rather than in a multiplicative form in our generalization.

For clarification, we consider the in-between case of RSPS and ASPS, which (Marjit & Sarkar, 2017) had not explicitly stated. We need to see whether an income distribution which takes the form $y_1' = g_1y_1 + b$; and $y_2' = g_2y_2 + b$ post- growth will be robust to the result of existence of SPS allocation. This is an income transformation which after an inequality-preserving transfer amount 'T', makes it a relative-absolute SPS measure.

Following (Marjit & Sarkar, 2017), we proceed as follows:

$$\frac{y1-b}{y2-b}=\frac{g1y1-b-T}{g2y2-b+T}$$

$$\Rightarrow T=\frac{[y1y2(g1-g2)+b(y1-y2)+b(g2y2-g1y1)]}{y1+y2-2b}$$

Implying, a transfer amount does exist for even a relative-absolute inequality measure.

To conclude, a 'fair' measure of thermodynamic inequality satisfy three out of the four axioms of (Marjit et al., 2019), namely, Transfer axiom, Symmetry axiom and Population Variance axiom. But a measure of inequality which violates one or more of the axioms of inequality - the Normalization axiom in our case - does not possess an inequality-preserving fiscal redistribution.

Hence, even though (Marjit & Sarkar, 2017; Marjit et al., 2019) are unpublished papers, our claim of the non-existence of a distribution-neutral fiscal policy in terms of 'fairness' - for which the theoretical foundation remains an open question - serves as an applied concept of an SPS-like allocation, and is bad news for anyone concerned about justice and fairness, including us.

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Appendix

Let (y_1, y_2) be the income profile pre-growth and let (g1 + y1, g2 + y2) be the income profile post-growth, which after transfers becomes: (g1 + y1 - T, g2 + y2 + T)

$\mu1 = (y1 + y2)/2;$

$\mu2 = (g1 + y1 + g2 + y2)/2$

$\sigma1 = (y1 - y2)/2;$

$\sigma2 = (g1 + y1 - g2 - y2)/2+T$

From the condition that $\ln(\sigma1/\sigma2) = \mu2 - \mu1$

on solving, we get:

$$\frac{y1 - y2}{g1 + y1 - g2 - y2 + 2T} = sqrt(\exp(g1 + y1 - g2 - y2))$$

For a T to exist, we require:

$$\left(\frac{y1 - y2}{g1 + y1 - g2 - y2}\right)^2 > \exp(g1 + y1 - g2 - y2 - y1 - y2)$$