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Impact of Quantitative Easing on the Long-Term Investment Values

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Abstract: Presented here are simplified mathematical models for evaluation of the long-term investment values. Three scenarios were consideredin a framework of the single product economy. The first scenario assesses an impact ofcapital investments (accruedon the product market with a constant acceleration) on an equity price on the equity market. The second scenario assesses impact of both capital investments (accruedon the product market with a constant acceleration) and quantitative easing (accruedon the equity market with a constant acceleration) on an equity price on the equity market.The third scenario assesses impact of both capital investments (accruedon the product market with a constant acceleration) and quantitative tightening (accruedon the equity market with a constant acceleration)on an equity price on the equity market.

Keywords: equity price, quantitative easing, mathematical models

JEL Classification Numbers: E22, E32, E44

1. Introduction

The article continues research, which was presented in Krouglov (2019), devoted to mathematical models explaining the equity valuations.

The research of Krouglov (2019) produced and investigated a reference point. If one evaluates the equity, then earnings delivered by a company as a going concern should be comparable with the income, which would be generatedfor a lenderby the loan.And the income generated by the loan is measured byan interest rate return.Thus, when one evaluates the equity, she calculates earnings delivered by a company, relates them to the equity value, and compares the result with aninterest rate return. Therefore, the ratio of the earnings to the equity value should produce the result, which roughly coincides with an interest rate return. This result produces the so-called intrinsic equity value, which is directly proportional to the receivedearnings (for a fixed interest rate return) and inversely proportional to the interest rate return (forfixed earnings).

Distinct from the aforesaid intrinsic equity value, inKrouglov (2019)it was also investigatedthe so-called market equity value. The market equity value was determined on the equity market where the quantities ofequity supply, demand, and price were linked through a system of ordinary differential equationsas in Krouglov (2006).

Since the equity market is a derivative market compared to the product market (in the sense that equity market is mainly driven by the product market) there is also a difference between the systems of differential equations describing the product and equity markets. The product market is described by a system of homogeneous ordinary differential equations and the equity market is described by a system of non-homogeneous ordinary differential equations where a non-homogeneous term on the equity market conveys an influence from the (basic) product market to the (derivative) equity market as in Krouglov (2006) and Krouglov (2019).

For better understandingof the market equity value,it was considered in Krouglov (2019) the following scenario. Balance on the product market was broken as a company decided to make (capital) investments. It caused an increase of the product price and generated a possible economic growth (or an economic decline in other circumstances). The product price's increase on the product market was economically justified since the investments were used to improve the product qualityas seen in Krouglov (2017).

On the equity market, the product price's increase was used as a non-homogeneous term, which caused an increase of the demand for equity on the equity market. Likewise, the increase of the demand for equity caused an increase of the price of equity on the equity market. The investments made by a company caused both the increase of product price on the product market and the increase of equity price on the equity market (where the aforesaid increases were economically justified by advances of the product quality).

It is worth to note that few economic processes were analyzed and modeled by employing mathematical models of a single-producteconomy, which simplifiesmathematical treatment and even can sometimes produce the explicitexpressionsas in Krouglov (2006), Krouglov (2017), and Krouglov (2019).

In this paper I continue a research for the assessments of equity values presented in Krouglov (2019) and examine how the quantitative easing (or the quantitative tightening) can impact the market equity values. Mathematical modeling of the quantitative easing (the quantitative tightening) is restricted to the situation when the quantitative easing/tightening is contained in the equity market (not in the product market). It is justified by the fact that on practice the quantitative easing/tightening was contained in the financial system.

2. Changes on Product MarketCaused bythe Capital Investments

In this paper I examine how quantitative easing and tightening impact the equity values. Modeling of the quantitative easing and tightening is performed for the equity market. It is justified because on practice the quantitative easing/ tightening was contained in the financial system.

Since quantitative easing (quantitative tightening) is restricted to the equity market, its influence impacts the market equity values. Since quantitative easing (quantitative tightening) doesn't surface on the product market, its influence doesn't impact the intrinsic equity values.

Now, let me use the results of Krouglov (2019) to estimate a market equity value,provided the capital investment causes an economic growth.

I deploy a mathematical model of thesingle-product-economy marketas in Krouglov (2006) and Krouglov (2017), which givesa good conceptual picture. Economic forces acting on the product market reflect the inherent market forces of demand and supply complemented with the forces caused by investment. The market interactions between the forces are expressed through a system of ordinary differential equations.

When there are no disturbing economic forces, the market is in equilibrium position, i.e., the supply of and demand for product are equal, the quantities of supply and demand are developing with a constant rate and a price of the product is fixed.

I assume the market had been in an equilibrium until time $t = t_0$, volumes of the product supply $V_S(t)$ and demand $V_D(t)$ on market were equal, and they both were developing with a constant rate r_D^0. The product price $P(t)$ at that time was fixed,

$$V_D(t) = r_D^0 (t - t_0) + V_D^0 \tag{1}$$

$$V_S(t) = V_D(t) \tag{2}$$

$$P(t) = P^0 \tag{3}$$

where $$V_D(t_0) = V_D^0 .$$

When the balance between the volumes of product supply and demand is broken, market is experiencing economic forces, which act to bring the market to a new equilibrium position.

Here I use a model of the single-product economy where the investment rate is increasing with a constant acceleration as in Krouglov (2017).

Correspondingly, I assume the amount of capital investment $S_I(t)$ on market increases since time according to the following formula,

$$S_I(t) = \begin{cases} 0, & t < t_0 \\ \delta_I (t - t_0) + \dfrac{\varepsilon_I}{2}(t - t_0)^2, & t \geq t_0 \end{cases} \tag{4}$$

where $S_I(t)$ for $t < t_0$, $d_I \geq 0$, and $\varepsilon_I > 0$.

Economic forces trying to bring the market into a new equilibrium position are described by the following ordinary differential equations with regard to the volumes of product supply $V_S(t)$, demand $V_D(t)$, and price $P(t)$ given the accumulated amount of capital investment $S_I(t)$ on the product market,

$$\frac{dP(t)}{dt} = -\lambda_P \left(V_S(t) - V_D(t) - S_I(t) \right) \tag{5}$$

$$\frac{d^2 V_S(t)}{dt^2} = \lambda_S \frac{dP(t)}{dt} \tag{6}$$

$$\frac{d^2 V_D(t)}{dt^2} = -\lambda_D \frac{d^2 P(t)}{dt^2} \tag{7}$$

In Eqs. (5) – (7) above the values $\lambda_P, \lambda_S, \lambda_D \geq 0$ are constants and they reflect the price inertness, supply inducement, and demand amortization correspondingly.

Let me use a variable $D(t) \equiv \left(V_S(t) - V_D(t) - S_I(t) \right)$ representing the volume of product surplus (or shortage) on the product market. Therefore, behavior of $D(t)$ is described by the following equation for $t > t_0$,

$$\frac{d^2 D(t)}{dt^2} + \lambda_P \lambda_D \frac{dD(t)}{dt} + \lambda_P \lambda_S D(t) + \varepsilon_I = 0 \tag{8}$$

with the initial conditions, $D(t_0) = 0$, $\dfrac{dD(t_0)}{dt} = -\delta_I$.

If one introduces another variable $D_1(t) \equiv D(t) + \dfrac{\varepsilon_I}{\lambda_P \lambda_S}$, then Eq. (8) becomes,

$$\frac{d^2 D_1(t)}{dt^2} + \lambda_P \lambda_D \frac{dD_1(t)}{dt} + \lambda_P \lambda_S D_1(t) = 0 \tag{9}$$

with the initial conditions, $D_1(t_0) = \dfrac{\varepsilon_I}{\lambda_P \lambda_S}$, $\dfrac{dD_1(t_0)}{dt} = -\delta_I$.

Similar to Eq. (8), the product price $P(t)$ is described by the following equation for $t > t_0$,

$$\frac{d^2P(t)}{dt^2}+\lambda_P\,\lambda_D\frac{dP(t)}{dt}+\lambda_P\,\lambda_S\left(P(t)-P^0-\frac{\delta_I}{\lambda_S}-\frac{\varepsilon_I}{\lambda_S}(t-t_0)\right)=0 \qquad (10)$$

with the initial conditions, $P(t_0)=P^0$, $\frac{dP(t_0)}{dt}=0$.

Let me usea variable $P_1(t)\equiv P(t)-P^0-\frac{\delta_I}{\lambda_S}-\frac{\varepsilon_I}{\lambda_S}(t-t_0)+\frac{\lambda_D}{\lambda_S^2}\varepsilon_I$ to simplify analysis of the price behavior. The behavior of variable $P_1(t)$ is described by following equation for $t>t_0$,

$$\frac{d^2P_1(t)}{dt^2}+\lambda_P\,\lambda_D\frac{dP_1(t)}{dt}+\lambda_P\,\lambda_S\,P_1(t)=0 \qquad (11)$$

with the initial conditions, $P_1(t_0)=-\frac{\delta_I}{\lambda_S}+\frac{\lambda_D}{\lambda_S^2}\varepsilon_I$, $\frac{dP_1(t_0)}{dt}=-\frac{\varepsilon_I}{\lambda_S}$.

The behavior of solutions for $D_1(t)$ and $P_1(t)$ described by Eqs. (9) and (11) depends on the roots of the corresponding characteristic equations as shown in Piskunov (1965) and Petrovski (1966). Also Eqs. (9) and (11) have the same characteristic equations.

When the roots of characteristic equation are complex-valued (i.e., $\frac{\lambda_P^2\,\lambda_D^2}{4}<\lambda_P\,\lambda_S$) both variables $D_1(t)$ and $P_1(t)$ experience damped oscillations for time $t\geq t_0$. If the roots of characteristic equation are real and different (i.e., $\frac{\lambda_P^2\,\lambda_D^2}{4}>\lambda_P\,\lambda_S$) both variables $D_1(t)$ and $P_1(t)$ don't oscillate for time $t\geq t_0$. If the roots of characteristic equation are real and equal (i.e., $\frac{\lambda_P^2\,\lambda_D^2}{4}=\lambda_P\,\lambda_S$) both variables $D_1(t)$ and $P_1(t)$ don't oscillate for time $t\geq t_0$ as well.

It takes place $D_1(t)\to 0$ and $P_1(t)\to 0$ for $t\to+\infty$ if roots of characteristic equations are complex-valued ($\frac{\lambda_P^2\,\lambda_D^2}{4}<\lambda_P\,\lambda_S$), real and different ($\frac{\lambda_P^2\,\lambda_D^2}{4}>\lambda_P\,\lambda_S$), or real and equal ($\frac{\lambda_P^2\,\lambda_D^2}{4}=\lambda_P\,\lambda_S$).

We observethe following behavior for the product surplus (shortage) $D(t)$, the product price $P(t)$, the product demand $V_D(t)$, the product supply $V_S(t)$, the amount of capital investment $S_I(t)$ when $t \to +\infty$,

$$D(t) \to -\frac{\varepsilon_I}{\lambda_P \lambda_S} \tag{12}$$

$$P(t) \to \frac{\varepsilon_I}{\lambda_S}(t - t_0) + P^0 + \frac{\delta_I}{\lambda_S} - \frac{\lambda_D}{\lambda_S^2}\varepsilon_I \tag{13}$$

$$V_D(t) \to \left(r_D^0 - \frac{\lambda_D}{\lambda_S}\varepsilon_I \right)(t - t_0) + V_D^0 - \frac{\lambda_D}{\lambda_S}\delta_I + \frac{\lambda_D^2}{\lambda_S^2}\varepsilon_I \tag{14}$$

$$V_S(t) \to \left(r_D^0 + \delta_I - \frac{\lambda_D}{\lambda_S}\varepsilon_I \right)(t - t_0) + \frac{\varepsilon_I}{2}(t - t_0)^2 + V_D^0 - \frac{\lambda_D}{\lambda_S}\delta_I - \frac{\varepsilon_I}{\lambda_P \lambda_S} + \frac{\lambda_D^2}{\lambda_S^2}\varepsilon_I \tag{15}$$

$$S_I(t) = \delta_I (t - t_0) + \frac{\varepsilon_I}{2}(t - t_0)^2 \tag{16}$$

To analyze an economic growth we use variable $E_D(t) \equiv P(t) \times r_D(t)$ where $r_D(t) \equiv \frac{dV_D(t)}{dt}$, i.e., the instantaneous rate of earnings for product on the market.

The variable $E_D(t)$ converges toward $E_D(t) \to \left(\frac{\varepsilon_I}{\lambda_S}(t - t_0) + P^0 + \frac{\delta_I}{\lambda_S} - \frac{\lambda_D}{\lambda_S^2}\varepsilon_I \right)\left(r_D^0 - \frac{\lambda_D}{\lambda_S}\varepsilon_I \right)$ for $t \to +\infty$.

If $0 < \varepsilon_I < \frac{\lambda_S}{\lambda_D} r_D^0$ then it takes place $r_D^0 - \frac{\lambda_D}{\lambda_S}\varepsilon_I > 0$. Thus, it brings unrestricted increase of the rate of earnings $E_D(t)$ for product on the product market with the passage of time, i.e., $E_D(t) \to +\infty$ for $t \to +\infty$.

We can estimate a change $e_D(t)$ of the rate of earnings $E_D(t)$ for product where $e_D(t) \equiv \frac{dE_D(t)}{dt}$.

It takes place $e_D(t) \to \frac{\varepsilon_I}{\lambda_S}\left(r_D^0 - \frac{\lambda_D}{\lambda_S}\varepsilon_I\right) > 0$ when $0 < \varepsilon_I < \frac{\lambda_S}{\lambda_D} r_D^0$ for $t \to +\infty$.

If $\frac{\lambda_S}{\lambda_D} r_D^0 < \varepsilon_I < +\infty$, then it takes place $r_D^0 - \frac{\lambda_D}{\lambda_S}\varepsilon_I < 0$. Hence, it brings unrestricted decrease of the rate of earnings $E_D(t)$ for product on the product market with the passage of time, i.e., $E_D(t) \to -\infty$ for $t \to +\infty$.

We can estimate a change $e_D(t)$ of the rate of earnings $E_D(t)$ for product.

It takes place $e_D(t) \to \frac{\varepsilon_I}{\lambda_S}\left(r_D^0 - \frac{\lambda_D}{\lambda_S}\varepsilon_I\right) < 0$ when $\frac{\lambda_S}{\lambda_D} r_D^0 < \varepsilon_I < +\infty$ for $t \to +\infty$.

In this section I presented a single-product-economy model. Ihave also observedhow capital investments affects the product price, the product demand, the product supply, and the product earnings in the long run.

In the next section I will examine how the capital investment affects the price of equity.

3. Changes on Equity Market Caused bythe Capital Investments

Now, let me use results of the previous section to estimate changes of the market equity value when capital investments producean economic growth on the product market.

I deploy a mathematical model of thesingle-product-economy market, which givesa good conceptual explanation. Economic forces acting on the product market reflect the inherent market forces of demand and supply complemented with the forces caused by capital investments. The economic interactions on the product market are expressed through a system of ordinary differential equations.

When there are no disturbing economic forces, the product market is in equilibrium position, i.e., the supply of and demand for product are equal, the quantities of supply and demand are developing with a constant rate and a price of the product is fixed.

Likewise, I consider the equity market corresponding to a company operating in single product economy. Economic forces acting on the equity market reflect the inherent market forces of demand and supply complemented with the economic forces operating on the product market and the equity market as in Krouglov (2006). Interactions on the equity market are expressed through a system of non-homogeneousordinary differential equations. When there are no disturbing economic forces, both the product market and the equity marketare in equilibrium positions, i.e., the supply of and demand for product and the supply of and demand for equity are equal, the quantities of supply and demand are developing with a constant rate and the prices of product and equity are fixed.

I assume the product market and the equity market had been in an equilibrium until time $t = t_0$. Volumes of the product supply $V_S(t)$ and demand $V_D(t)$ on the product market were equal, and they both were developing with a constant rate r_D^0. The product price $P(t)$ at that time was fixed,

$$V_D(t) = r_D^0(t - t_0) + V_D^0 \tag{17}$$

$$V_S(t) = V_D(t) \tag{18}$$

$$P(t) = P^0 \tag{19}$$

where $$V_D(t_0) = V_D^0.$$

Likewise, volumes of the equity supply $V_{SE}(t)$ and demand $V_{DE}(t)$ on the equity market were equal, and they both were developing with a constant rate r_{DE}^0. The equity price $P_E(t)$ at that time was fixed,

$$V_{DE}(t) = r_{DE}^0(t - t_0) + V_{DE}^0 \tag{20}$$

$$V_{SE}(t) = V_{DE}(t) \tag{21}$$

$$P_E(t) = P_E^0 \tag{22}$$

where $$V_{DE}(t_0) = V_{DE}^0.$$

When balance between the volume of product supply and the volume ofproduct demand is broken, the product market is experiencing economic forces, which act to bring the product market to a new equilibrium position. Likewise, when balance between the volume of equity supply and the volume ofequity demand is broken, the equity market is experiencing economic forces, which act to bring the equity market to a new equilibrium position. Moreover, there are economic forces operating on both the product market and the equity market as in Krouglov (2006).

In this section I assume balances both on the product market and on the equity market are broken by thecapital investmentsdeveloping with a rate that increases with a constant acceleration as in Krouglov (2017) and Krouglov (2019).

I assume the amount of capital investments $S_I(t)$ on the product market increases since time $t = t_0$ according to the following formula,

$$S_I(t) = \begin{cases} 0, & t < t_0 \\ \delta_I (t - t_0) + \frac{\varepsilon_I}{2}(t - t_0)^2, & t \geq t_0 \end{cases} \tag{23}$$

where $S_I(t) = 0$ for $t < t_0$, $\delta_I \geq 0$, and $\varepsilon_I > 0$.

Economic forces trying to bring the product market into a new equilibrium position are described by the following ordinary differential equations with regard to the volume of product supply $V_S(t)$, the volume of product demand $V_D(t)$, the product price $P(t)$ given the accumulated amount of capital investments $S_I(t)$ on the product market,

$$\frac{dP(t)}{dt} = -\lambda_P \left(V_S(t) - V_D(t) - S_I(t) \right) \tag{24}$$

$$\frac{d^2 V_S(t)}{dt^2} = \lambda_S \frac{dP(t)}{dt} \tag{25}$$

$$\frac{d^2 V_D(t)}{dt^2} = -\lambda_D \frac{d^2 P(t)}{dt^2} \tag{26}$$

In Eqs. (24) – (26) above the values λ_P, λ_S, $\lambda_D \geq 0$ are constants and they reflect theproduct price inertness, supply inducement, and demand amortization correspondingly.

Economic forces trying to bring the equity market into a new equilibrium position are described by the following non-homogeneousordinary differential equations with regard to the volume of equity supply $V_{SE}(t)$, the volume of equity demand $V_{DE}(t)$, the equity price $P_E(t)$ and the product price $P(t)$ on the equity market,

$$\frac{dP_E(t)}{dt} = -\lambda_{PE} \left(V_{SE}(t) - V_{DE}(t) \right) \tag{27}$$

$$\frac{d^2V_{SE}(t)}{dt^2} = \lambda_{SE}\frac{dP_E(t)}{dt} \tag{28}$$

$$\frac{d^2V_{DE}(t)}{dt^2} = -\lambda_{DE}\frac{d^2P_E(t)}{dt^2} + \lambda_E\frac{dP(t)}{dt} \tag{29}$$

In Eqs. (27) – (29) above the values λ_{PE}, λ_{SE}, λ_{DE}, $\lambda_E \geq 0$ are constants.

Thus, the equity price $P_E(t)$ is described by the following equation for $t > t_0$,

$$\frac{d^2P_E(t)}{dt^2} + \lambda_{PE}\,\lambda_{DE}\frac{dP_E(t)}{dt} + \lambda_{PE}\,\lambda_{SE}\left(P_E(t) - P_E^0\right) - \lambda_{PE}\,\lambda_E\left(P(t) - P^0\right) = 0 \tag{30}$$

with the initial conditions, $P_E(t_0) = P_E^0$, $P(t_0) = P^0$, $\frac{dP_E(t_0)}{dt} = 0$, $\frac{dP(t_0)}{dt} = 0$.

I perform in Eq. (30) the passage to the limit for $t \to +\infty$ based on results of Eq. (13),

$$\frac{d^2P_E(t)}{dt^2} + \lambda_{PE}\,\lambda_{DE}\frac{dP_E(t)}{dt} + \lambda_{PE}\,\lambda_{SE}\left(P_E(t) - P_E^0\right) - \lambda_{PE}\,\lambda_E\left(\frac{\varepsilon_I}{\lambda_S}(t - t_0) + \frac{\delta_I}{\lambda_S} - \frac{\lambda_D}{\lambda_S^2}\varepsilon_I\right) = 0 \tag{31}$$

I usea variable $P_2(t) \equiv P_E(t) - P_E^0 - \frac{\lambda_E}{\lambda_{SE}\,\lambda_S}\varepsilon_I(t - t_0) - \frac{\lambda_E\,\lambda_S\,\delta_I - \lambda_E\,\lambda_D\,\varepsilon_I}{\lambda_{SE}\,\lambda_S^2}$

to simplify analysis of the market equity values. The behavior of variable $P_2(t)$ is described by following equation for $t \to +\infty$,

$$\frac{d^2P_2(t)}{dt^2} + \lambda_{PE}\,\lambda_{DE}\frac{dP_2(t)}{dt} + \lambda_{PE}\,\lambda_{SE}\,P_2(t) = 0 \tag{32}$$

with the initial conditions, $P_2(t_0) = -\frac{\lambda_E\,\lambda_S\,\delta_I - \lambda_E\,\lambda_D\,\varepsilon_I}{\lambda_{SE}\,\lambda_S^2}$, $\frac{dP_2(t_0)}{dt} = -\frac{\lambda_E}{\lambda_{SE}\,\lambda_{SE}^2}\varepsilon_I$.

The behavior of solutions for $P_2(t)$ described by Eq. (32) depends on the roots of the corresponding characteristic equation as shown in Piskunov (1965) and Petrovski (1966).

When the roots of characteristic equation are complex-valued (i.e., $\frac{\lambda_P^2 \lambda_D^2}{4} < \lambda_P \lambda_S$) variable $P_2(t)$ experiences damped oscillations for time $t \geq t_0$. If the roots of characteristic equation are real and different (i.e., $\frac{\lambda_P^2 \lambda_D^2}{4} > \lambda_P \lambda_S$) variable $P_2(t)$ doesn't oscillate for time $t \geq t_0$. If the roots of characteristic equation are real and equal (i.e., $\frac{\lambda_P^2 \lambda_D^2}{4} = \lambda_P \lambda_S$) variable $P_2(t)$ doesn't oscillate for time $t \geq t_0$.

It may be observed that $P_2(t) \to 0$ for $t \to +\infty$ if the roots of characteristic equations are complex-valued ($\frac{\lambda_P^2 \lambda_D^2}{4} < \lambda_P \lambda_S$), real and different ($\frac{\lambda_P^2 \lambda_D^2}{4} > \lambda_P \lambda_S$), or real and equal ($\frac{\lambda_P^2 \lambda_D^2}{4} = \lambda_P \lambda_S$).

We observefollowing behavior for the equity price $P_E(t)$ given the amount of capital investments $S_I(t)$ on the product market when $t \to +\infty$,

$$P_E(t) \to \frac{\lambda_E}{\lambda_{SE} \lambda_S} \varepsilon_I (t - t_0) + P_E^0 + \frac{\lambda_E \lambda_S \delta_I - \lambda_E \lambda_D \varepsilon_I}{\lambda_{SE} \lambda_S^2} \tag{33}$$

$$S_I(t) = \delta_I (t - t_0) + \frac{\varepsilon_I}{2}(t - t_0)^2 \tag{34}$$

Thus, we have determined for a model of singleproducteconomy that capital investments on the product marketperformed with a constant acceleration causes both (a) an unrestricted increase of product price on the product market (as seen from Eq. (13)), and (b) an unrestricted increase of equity price on the equity market (as seen from Eq. (33)).

The rates at which both a product price and an equity price are changing with time are directly proportional to an acceleration of the capital investments in the long run.

4. Changes on Equity Market Caused by the Capital Investments and Quantitative Easing

Here, I use the results of previous section to estimate the changes of market equity value when the capital investments producesan economic growth on the product market and the quantitative easing increases the demand for equity on the equity market.

I also deploy a mathematical model of thesingle-product-economy market, which givesa good conceptual explanation.

When there are no disturbing economic forces, the product market is in equilibrium position, i.e., the supply of and demand for product are equal, the quantities of supply and demand are developing with a constant rate and a price of the product is fixed.

Likewise, I consider the equity market corresponding to a company operating in single product economy. Economic forces acting on the equity market reflect the inherent market forces of demand and supply complemented with the economic forces operating on the product market and the equity market. Interactions on the equity market are expressed through a system of non-homogeneous ordinary differential equations. When there are no disturbing economic forces, both the product market and the equity market are in an equilibrium positions, i.e., the supply of and demand for product and the supply of and demand for equity are equal, the quantities of supply and demand are developing with a constant rate and the prices of product and equity are fixed.

I assume the product market and equity market had been in an equilibrium until time $t = t_0$. Volumes of the product supply $V_S(t)$ and demand $V_D(t)$ on the product market were equal, and they both were developing with a constant rate r_D^0. The product price $P(t)$ at that time was fixed,

$$V_D(t) = r_D^0(t - t_0) + V_D^0 \tag{35}$$

$$V_S(t) = V_D(t) \tag{36}$$

$$P(t) = P^0 \tag{37}$$

where $$V_D(t_0) = V_D^0.$$

Likewise, volumes of the equity supply $V_{SE}(t)$ and demand $V_{DE}(t)$ on the equity market were equal, and they both were developing with a constant rate r_{DE}^0. The equity price $P_E(t)$ at that time was fixed,

$$V_{DE}(t) = r_{DE}^0(t - t_0) + V_{DE}^0 \tag{38}$$

$$V_{SE}(t) = V_{DE}(t) \tag{39}$$

$$P_E(t) = P_E^0 \tag{40}$$

where $$V_{DE}\left(t_0\right) = V_{DE}^0 .$$

When balance between the volume of product supply and the volume of product demand is broken, the product market is experiencing economic forces, which act to bring the product market to a new equilibrium position. Likewise, when balance between the volume of equity supply and the volume of equity demand is broken, the equity market is experiencing economic forces, which act to bring the equity market to a new equilibrium position. Moreover, there are economic forces operating on both the product market and the equity market.

In this section I assume balances on both the product market and the equity market are broken by the capital investments developing with a rate that increases with a constant acceleration.

I assume the amount of capital investments $S_I(t)$ on the product market increases since time $t = t_0$ according to the following formula,

$$S_I\left(t\right) = \begin{cases} 0, & t < t_0 \\ \delta_I\left(t - t_0\right) + \frac{\varepsilon_I}{2}\left(t - t_0\right)^2, & t \geq t_0 \end{cases} \tag{41}$$

where $S_I\left(t\right) = 0$ for $t < t_0$, $\delta_I \geq 0$, and $\varepsilon_I > 0$.

Economic forces trying to bring the product market into a new equilibrium position are described by the following ordinary differential equations with regard to the volume of product supply $V_S\left(t\right)$, the volume of product demand $V_D\left(t\right)$, the product price $P\left(t\right)$ given the accumulated amount of capital investments $S_I\left(t\right)$ on the product market,

$$\frac{dP\left(t\right)}{dt} = -\lambda_P\left(V_S\left(t\right) - V_D\left(t\right) - S_I\left(t\right)\right) \tag{42}$$

$$\frac{d^2V_S\left(t\right)}{dt^2} = \lambda_S\frac{dP\left(t\right)}{dt} \tag{43}$$

$$\frac{d^2V_D\left(t\right)}{dt^2} = -\lambda_D\frac{d^2P\left(t\right)}{dt^2} \tag{44}$$

In Eqs. (42) – (44) above the values $\lambda_P, \lambda_S, \lambda_D \geq 0$ are constants.

Now, I assume the quantitative easing started on the equity market (at the time $t = t_1 \geq t_0$). Also, I assume that $t_1 = t_0 + \Delta$ where $\Delta \approx 0$ or $t_1 \approx t_0$ in order to avoid calculations for an economic deviation during the time interval $t_0 \le t < t_1$, which would just obscure the calculations otherwise.

I assume the amount of quantitative easing $S_{QE}(t)$ on the equity market increases since time $t = t_1$ according to the following formula,

$$S_{QE}(t) = \begin{cases} 0, & t < t_1 \\ \delta_{QE}(t - t_1) + \frac{\varepsilon_{QE}}{2}(t - t_1)^2, & t \ge t_1 \end{cases} \quad (45)$$

where $S_{QE}(t) = 0$ for $t < t_1$, $\delta_{QE} \ge 0$, and $\varepsilon_{QE} > 0$.

Thus, the balance between the volume of equity supply and the volume of equity demand was broken. The equity market was experiencing economic forces, which were trying to bring the equity market to a new equilibrium position.

Economic forces trying to bring the equity market into a new equilibrium position are described by the following non-homogeneous ordinary differential equations with regard to the volume of equity supply $V_{SE}(t)$, the volume of equity demand $V_{DE}(t)$, the equity price $P_E(t)$ and the product price $P(t)$ given the accumulated amount of quantitative easing $S_{QE}(t)$ on the equity market,

$$\frac{dP_E(t)}{dt} = -\lambda_{PE}\left(V_{SE}(t) - V_{DE}(t) - S_{QE}(t)\right) \quad (46)$$

$$\frac{d^2V_{SE}(t)}{dt^2} = \lambda_{SE}\frac{dP_E(t)}{dt} \quad (47)$$

$$\frac{d^2V_{DE}(t)}{dt^2} = -\lambda_{DE}\frac{d^2P_E(t)}{dt^2} + \lambda_E\frac{dP(t)}{dt} \quad (48)$$

In Eqs. (46) – (48) above the values λ_{PE}, λ_{SE}, λ_{DE}, $\lambda_E \ge 0$ are constants.

Thus, the equity price $P_E(t)$ is described by the following equation for $t > t_1$ where $t_1 \approx t_0$,

$$\frac{d^2P_E(t)}{dt^2}+\lambda_{PE}\,\lambda_{DE}\,\frac{dP_E(t)}{dt}+\lambda_{PE}\,\lambda_{SE}\left(P_E(t)-P_E^0\right)-\lambda_{PE}\,\lambda_E\left(P(t)-P^0\right)$$
$$-\lambda_{PE}\,\frac{dS_{QE}(t)}{dt}=0 \qquad (49)$$

with the initial conditions $P_E(t_1)=P_E^0$, $P(t_1)=P^0$, $\frac{dP_E(t_1)}{dt}=0$, $\frac{dP(t_1)}{dt}=0$,

$\frac{dS_{QE}(t_1)}{dt}=\delta_{QE}$.

I perform the passage to the limit in Eq. (49) for $t\to+\infty$ based on results of Eq. (13) and substitute $\frac{dS_{QE}(t)}{dt}$ from Eq. (45),

$$\frac{d^2P_E(t)}{dt^2}+\lambda_{PE}\,\lambda_{DE}\,\frac{dP_E(t)}{dt}+\lambda_{PE}\,\lambda_{SE}\left(P_E(t)-P_E^0\right)$$
$$-\lambda_{PE}\,\lambda_E\left(\frac{\varepsilon_I}{\lambda_S}(t-t_0)+\frac{\delta_I}{\lambda_S}-\frac{\lambda_D}{\lambda_S^2}\varepsilon_I\right)-\lambda_{PE}\left(\varepsilon_{QE}(t-t_1)+\delta_{QE}\right)=0 \qquad (50)$$

To simplify the calculations, I replace Eq. (50) with the following one,

$$\frac{d^2P_E(t)}{dt^2}+\lambda_{PE}\,\lambda_{DE}\,\frac{dP_E(t)}{dt}+\lambda_{PE}\,\lambda_{SE}\left(P_E(t)-P_E^0-\varepsilon_\Sigma(t-t_1)-\delta_\Sigma\right)=0 \qquad (51)$$

where $\varepsilon_\Sigma=\frac{\lambda_E\,\varepsilon_I+\lambda_S\,\varepsilon_{QE}}{\lambda_{SE}\,\lambda_S}$ and $\delta_\Sigma=\frac{\lambda_E\,\lambda_S\,\delta_I-\lambda_E\,\lambda_D\,\varepsilon_I+\lambda_S^2\,\delta_{QE}}{\lambda_{SE}\,\lambda_S^2}$.

I use a variable $P_2(t)\equiv P_E(t)-P_E^0-\varepsilon_\Sigma(t-t_1)-\frac{\lambda_{SE}\,\delta_\Sigma-\lambda_{DE}\,\varepsilon_\Sigma}{\lambda_{SE}}$ to perform analysis of the market equity values. The behavior of variable $P_2(t)$ is described by following equation for $t\to+\infty$,

$$\frac{d^2P_2(t)}{dt^2}+\lambda_{PE}\,\lambda_{DE}\,\frac{dP_2(t)}{dt}+\lambda_{PE}\,\lambda_{SE}\,P_2(t)=0 \qquad (52)$$

with the initial conditions, $P_2(t_1) = \dfrac{\lambda_{DE}\,\varepsilon_\Sigma - \lambda_{SE}\,\delta_\Sigma}{\lambda_{SE}}$, $\dfrac{dP_2(t_1)}{dt} = -\varepsilon_\Sigma$.

The behavior of solutions for $P_2(t)$ described by Eq. (52) depends on the roots of the corresponding characteristic equation as shown in Piskunov (1965) and Petrovski (1966).

When the roots of characteristic equation are complex-valued (i.e., $\dfrac{\lambda_P^2\,\lambda_D^2}{4} < \lambda_P\,\lambda_S$) variable $P_2(t)$ experiences damped oscillations for time $t \geq t_0$.

If the roots of characteristic equation are real and different (i.e., $\dfrac{\lambda_P^2\,\lambda_D^2}{4} > \lambda_P\,\lambda_S$) variable $P_2(t)$ doesn't oscillate for time $t \geq t_0$. If the roots of characteristic equation are real and equal (i.e., $\dfrac{\lambda_P^2\,\lambda_D^2}{4} = \lambda_P\,\lambda_S$) variable $P_2(t)$ doesn't oscillate for time $t \geq t_0$.

It may be observed that $P_2(t) \to 0$ for $t \to +\infty$ if the roots of characteristic equations are complex-valued ($\dfrac{\lambda_P^2\,\lambda_D^2}{4} < \lambda_P\,\lambda_S$), real and different ($\dfrac{\lambda_P^2\,\lambda_D^2}{4} > \lambda_P\,\lambda_S$), or real and equal ($\dfrac{\lambda_P^2\,\lambda_D^2}{4} = \lambda_P\,\lambda_S$).

We observefollowing behavior for the equity price $P_E(t)$ given the amount of capital investment $S_I(t)$ on the product market when $t \to +\infty$,

$$P_E(t) \to \varepsilon_\Sigma(t - t_1) + P_E^0 + \frac{\lambda_{SE}\,\delta_\Sigma - \lambda_{DE}\,\varepsilon_\Sigma}{\lambda_{SE}} \tag{53}$$

$$S_I(t) = \delta_I\,(t - t_0) + \frac{\varepsilon_I}{2}(t - t_0)^2 \tag{54}$$

$$S_{QE}(t) = \delta_{QE}\,(t - t_1) + \frac{\varepsilon_{QE}}{2}(t - t_1)^2 \tag{55}$$

Thus, we have determined for a model of the singleproducteconomy the capital investments performed with a constant acceleration $\varepsilon_I > 0$ cause an unrestricted increase of product price on the product market described by a linear function with slope $\frac{\varepsilon_I}{\lambda_S} > 0$ (as seen from Eq. (13)). Also, we have determined for the model of the singleproducteconomy the capital investmentsdone with a constant acceleration $\varepsilon_I > 0$ cause an unrestricted increase of equity price on the equity market described by a linear function with slope $\frac{\lambda_E}{\lambda_{SE}\lambda_S}\varepsilon_I \geq 0$ (as seen from Eq. (33)). As well, we have determined for the model of the singleproducteconomy both the capital investments performed with a constant acceleration $\varepsilon_I > 0$ and the quantitative easing performed with a constant acceleration $\varepsilon_{QE} > 0$ cause an unrestricted increase of equity price on the equity market described by a linear equation with slope $\varepsilon_\Sigma = \frac{\lambda_E}{\lambda_{SE}\lambda_S}\varepsilon_I + \frac{1}{\lambda_{SE}}\varepsilon_{QE} > 0$ (as seen from Eq. (53)).

Rate at which the product price is changing with time is directly proportional to an acceleration of the capital investmentsin the long run.Rate at which the equity price is changing with time is directly proportional to the synthesis of an acceleration of the capital investments and an acceleration of the quantitative easing in the long run.

5. Changes on Equity Market Caused by the Capital Investments and Quantitative Tightening

Here, I estimate changes of the market equity value when the capital investments producean economic growth on the product market and the quantitative tighteningdecreases the demand for equity on the equity market.

I deploy a mathematical model of thesingle-product-economy market.

When there are no disturbing economic forces, the product market is in equilibrium position, i.e., the supply of and demand for product are equal, the quantities of supply and demand are developing with a constant rate and a price of the product is fixed.

I assume the product market and equity market had been in an equilibrium until time $t = t_0$. Volumes of the product supply $V_S(t)$ and demand $V_D(t)$ on the product market were equal, and they both were developing with a constant rate r_D^0. The product price $P(t)$ at that time was fixed,

$$V_D(t) = r_D^0(t - t_0) + V_D^0 \tag{56}$$

$$V_S(t) = V_D(t) \tag{57}$$

$$P(t) = P^0 \tag{58}$$

where $V_D(t_0) = V_D^0$.

Likewise, volumes of the equity supply $V_{SE}(t)$ and demand $V_{DE}(t)$ on the equity market were equal, and they both were developing with a constant rate r_{DE}^0. The equity price $P_E(t)$ at that time was fixed,

$$V_{DE}(t) = r_{DE}^0(t - t_0) + V_{DE}^0 \tag{59}$$

$$V_{SE}(t) = V_{DE}(t) \tag{60}$$

$$P_E(t) = P_E^0 \tag{61}$$

where $V_{DE}(t_0) = V_{DE}^0$.

When balance between the volume of product supply and the volume of product demand is broken, the product market is experiencing economic forces, which act to bring the product market to a new equilibrium position. Likewise, when balance between the volume of equity supply and the volume of equity demand is broken, the equity market is experiencing economic forces, which act to bring the equity market to a new equilibrium position. Moreover, there are economic forces operating on both the product market and the equity market.

I assume balances both on the product market and on the equity market are broken by the capital investment developing with a rate that increases with constant acceleration.

I assume the amount of capital investments $S_I(t)$ on the product market increases since time $t = t_0$ according to the following formula,

$$S_I(t) = \begin{cases} 0, & t < t_0 \\ \delta_I(t - t_0) + \dfrac{\varepsilon_I}{2}(t - t_0)^2, & t \geq t_0 \end{cases} \tag{62}$$

where $S_I(t) = 0$ for $t < t_0$, $\delta_I \geq 0$, and $\varepsilon_I > 0$.

Economic forces trying to bring the product market into a new equilibrium position are described by the following ordinary differential equations with regard to the volume of product supply $V_S(t)$, the volume of product demand $V_D(t)$, the product price $P(t)$ given the accumulated amount of capital investments $S_I(t)$ on the product market,

$$\frac{dP(t)}{dt} = -\lambda_P\left(V_S(t) - V_D(t) - S_I(t)\right) \tag{63}$$

$$\frac{d^2V_S(t)}{dt^2} = \lambda_S \frac{dP(t)}{dt} \tag{64}$$

$$\frac{d^2V_D(t)}{dt^2} = -\lambda_D \frac{d^2P(t)}{dt^2} \tag{65}$$

In Eqs. (63) – (65) above the values $\lambda_P, \lambda_S, \lambda_D \geq 0$ are constants.

Now, I assume that quantitative tightening started on the equity market (at the time $t = t_1 \geq t_0$). Also, I assume that $t_1 = t_0 + \Delta$ where $\Delta \approx 0$ or $t_1 \approx t_0$ in order to avoid calculations for an economic deviation during the time interval $t_0 \leq t < t_1$, which would obscure the calculations otherwise.

I assume the amount of quantitative tightening $S_{QT}(t)$ on the equity market increases since time $t = t_1$ according to the following formula,

$$S_{QT}(t) = \begin{cases} 0, & t < t_1 \\ \delta_{QT}(t - t_1) + \dfrac{\varepsilon_{QT}}{2}(t - t_1)^2, & t \geq t_1 \end{cases} \tag{66}$$

where $S_{QT}(t) = 0$ for $t < t_1$, $\delta_{QT} \geq 0$, and $\varepsilon_{QT} > 0$.

Thus, the balance between the volume of equity supply and the volume of equity demand was broken. The equity market was experiencing economic forces, which were trying to bring the equity market to a new equilibrium position.

Economic forces trying to bring the equity market into a new equilibrium position are described by the following non-homogeneous ordinary differential equations with regard to the volume of equity supply $V_{SE}(t)$, the volume of equity demand $V_{DE}(t)$, the equity price $P_E(t)$ and the product price $P(t)$ given the accumulated amount of quantitative tightening $S_{QT}(t)$ on the equity market,

$$\frac{dP_E(t)}{dt} = -\lambda_{PE}\left(V_{SE}(t) - V_{DE}(t) + S_{QT}(t)\right) \tag{67}$$

$$\frac{d^2V_{SE}(t)}{dt^2} = \lambda_{SE}\frac{dP_E(t)}{dt} \tag{68}$$

$$\frac{d^2V_{DE}(t)}{dt^2} = -\lambda_{DE}\frac{d^2P_E(t)}{dt^2} + \lambda_E\frac{dP(t)}{dt} \tag{69}$$

In Eqs. (67) – (69) above the values $\lambda_{PE}, \lambda_{SE}, \lambda_{DE}, \lambda_E \geq 0$ are constants.

Thus, the equity price $P_E(t)$ is described by the following equation for $t > t_1$ where $t_1 \approx t_0$,

$$\begin{aligned}&\frac{d^2P_E(t)}{dt^2} + \lambda_{PE}\,\lambda_{DE}\frac{dP_E(t)}{dt} + \lambda_{PE}\,\lambda_{SE}\left(P_E(t) - P_E^0\right) - \lambda_{PE}\,\lambda_E\left(P(t) - P^0\right)\\ &+\lambda_{PE}\frac{dS_{QT}(t)}{dt} = 0\end{aligned} \tag{70}$$

with the initial conditions $P_E(t_1) = P_E^0$, $P(t_1) = P^0$, $\dfrac{dP_E(t_1)}{dt} = 0$, $\dfrac{dP(t_1)}{dt} = 0$, $\dfrac{dS_{QT}(t_1)}{dt} = \delta_{QT}$.

I perform the passage to the limit in Eq. (70) for $t \to +\infty$ based on results of Eq. (13) and substitute $\dfrac{dS_{QT}(t)}{dt}$ from Eq. (66),

$$\begin{aligned}&\frac{d^2P_E(t)}{dt^2} + \lambda_{PE}\,\lambda_{DE}\frac{dP_E(t)}{dt} + \lambda_{PE}\,\lambda_{SE}\left(P_E(t) - P_E^0\right)\\ &-\lambda_{PE}\,\lambda_E\left(\frac{\varepsilon_I}{\lambda_S}(t - t_0) + \frac{\delta_I}{\lambda_S} - \frac{\lambda_D}{\lambda_S^2}\varepsilon_I\right) + \lambda_{PE}\left(\varepsilon_{QT}(t - t_1) + \delta_{QT}\right) = 0\end{aligned} \tag{71}$$

To simplify the calculations, I replace Eq. (71) with the following one,

$$\frac{d^2 P_E(t)}{dt^2} + \lambda_{PE}\,\lambda_{DE}\,\frac{dP_E(t)}{dt} + \lambda_{PE}\,\lambda_{SE}\left(P_E(t) - P_E^0 - \varepsilon'_\Sigma\,(t - t_1) - \delta'_\Sigma\right) = 0 \quad (72)$$

where $\varepsilon'_\Sigma = \dfrac{\lambda_E\,\varepsilon_I - \lambda_S\,\varepsilon_{QT}}{\lambda_{SE}\,\lambda_S}$ and $\delta'_\Sigma = \dfrac{\lambda_E\,\lambda_S\,\delta_I - \lambda_E\,\lambda_D\,\varepsilon_I - \lambda_S^2\,\delta_{QT}}{\lambda_{SE}\,\lambda_S^2}$.

I use a variable $P_2(t) \equiv P_E(t) - P_E^0 - \varepsilon'_\Sigma(t - t_1) - \dfrac{\lambda_{SE}\,\delta'_\Sigma - \lambda_{DE}\,\varepsilon'_\Sigma}{\lambda_{SE}}$ to perform analysis of the market equity values. The behavior of variable $P_2(t)$ is described by following equation for $t \to +\infty$,

$$\frac{d^2 P_2(t)}{dt^2} + \lambda_{PE}\,\lambda_{DE}\,\frac{dP_2(t)}{dt} + \lambda_{PE}\,\lambda_{SE}\,P_2(t) = 0 \quad (73)$$

with the initial conditions, $P_2(t_1) = \dfrac{\lambda_{DE}\,\varepsilon'_\Sigma - \lambda_{SE}\,\delta'_\Sigma}{\lambda_{SE}}$, $\dfrac{dP_2(t_1)}{dt} = -\varepsilon'_\Sigma$.

The behavior of solutions for $P_2(t)$ described by Eq. (73) depends on the roots of the corresponding characteristic equation as shown in Piskunov (1965) and Petrovski (1966).

When the roots of characteristic equation are complex-valued (i.e., $\dfrac{\lambda_P^2\,\lambda_D^2}{4} < \lambda_P\,\lambda_S$) variable $P_2(t)$ experiences damped oscillations for time $t \geq t_0$. If the roots of characteristic equation are real and different (i.e., $\dfrac{\lambda_P^2\,\lambda_D^2}{4} > \lambda_P\,\lambda_S$) variable $P_2(t)$ doesn't oscillate for time $t \geq t_0$. If the roots of characteristic equation are real and equal (i.e., $\dfrac{\lambda_P^2\,\lambda_D^2}{4} = \lambda_P\,\lambda_S$) variable $P_2(t)$ doesn't oscillate for time $t \geq t_0$.

It may be observed that $P_2(t) \to 0$ for $t \to +\infty$ if the roots of characteristic equations are complex-valued ($\dfrac{\lambda_P^2\,\lambda_D^2}{4} < \lambda_P\,\lambda_S$), real and different ($\dfrac{\lambda_P^2\,\lambda_D^2}{4} > \lambda_P\,\lambda_S$), or real and equal ($\dfrac{\lambda_P^2\,\lambda_D^2}{4} = \lambda_P\,\lambda_S$).

We observefollowing behavior for the equity price $P_E(t)$ given the amount of capital investment $S_I(t)$ on the product market when $t \to +\infty$,

$$P_E(t) \to \varepsilon'_\Sigma(t-t_1) + P_E^0 + \frac{\lambda_{SE}\,\delta'_\Sigma - \lambda_{DE}\,\varepsilon'_\Sigma}{\lambda_{SE}} \tag{74}$$

$$S_I(t) = \delta_I(t-t_0) + \frac{\varepsilon_I}{2}(t-t_0)^2 \tag{75}$$

$$S_{QT}(t) = \delta_{QT}(t-t_1) + \frac{\varepsilon_{QT}}{2}(t-t_1)^2 \tag{76}$$

Thus, we have determined for a model of the singleproducteconomy the capital investments done with a constant acceleration $\varepsilon_I > 0$ cause an unrestricted increase of equity price on the equity market described by a linear function with slope $\frac{\lambda_E}{\lambda_{SE}\,\lambda_S}\varepsilon_I \geq 0$ (as seen from Eq. (33)). As well, we have determined for the model of the singleproducteconomy the capital investments performed with a constant acceleration $\varepsilon_I > 0$ and the quantitative tightening performed with a constant acceleration $\varepsilon_{QT} > 0$ cause an unrestricted change of equity price on the equity market described by a linear equation with slope $\varepsilon'_\Sigma = \frac{\lambda_E}{\lambda_{SE}\,\lambda_S}\varepsilon_I - \frac{1}{\lambda_{SE}}\varepsilon_{QT}$ (as seen from Eq. (74)).We have determined thequantitative tightening performed with a modest constant acceleration $0 < \varepsilon_{QT} \leq \frac{\lambda_E}{\lambda_S}\varepsilon_I$ causes an unrestricted increase of equity price on the equity market described by a linear function with slope $\varepsilon'_\Sigma = \frac{\lambda_E\,\varepsilon_I - \lambda_S\,\varepsilon_{QT}}{\lambda_{SE}\,\lambda_S} > 0$. We have determined the quantitative tightening performed with a large constant acceleration $\frac{\lambda_E}{\lambda_S}\varepsilon_I < \varepsilon_{QT}$ causes an unrestricted decrease of equity price on the equity market described by a linear function with slope $\varepsilon'_\Sigma = \frac{\lambda_E\,\varepsilon_I - \lambda_S\,\varepsilon_{QT}}{\lambda_{SE}\,\lambda_S} < 0$.

Rate at which the product price is changing with time is directly proportional to an acceleration of the capital investments in the long run. Rate at which the equity price is changing with time is directly proportional to the synthesis of an acceleration of the capital investments and an acceleration of the quantitative tightening in the long run.

6. Conclusions

The article presents and examines the mathematical models of market equity valuations in the long run. The three distinct scenarios are examined – (a) the capital investments on the product market, (b) both the capital investments on the product market and the quantitative easing on the equity market, (c) both the capital investments on the product market and the quantitative tightening on the equity market.

First, the capital investments on the product market in a model of the single product economy are examined. The capital investments considered here are the ones accruing with a constant acceleration. The capital investments on the product market accrued with a constant acceleration induce an unrestricted increase of product price on the product market. Additionally, the capital investments accrued with a constant acceleration on the product market induce an unrestricted increase of equity price on the equity market. Rates at which both the product price and the equity price are changing with time in the long run are directly proportional to an acceleration of the capital investments.

Second, both the capital investments on the product market and the quantitative easing on the equity market in a model of the single product economy are examined. The capital investments on the product market accrued with a constant acceleration induce both an unrestricted increase of product price on the product market and an unrestricted increase of equity price on the equity market. The quantitative easing considered here are the ones accruing with a constant acceleration. The quantitative easing on the equity market accrued with a constant acceleration induces a supplementary unrestricted increase of equity price on the equity market. Rate at which the equity price is changing with time in the long run is directly proportional to the synthesis of an acceleration of the capital investments and an acceleration of the quantitative easing (where both accelerations are added with appropriate coefficients).

Third, both the capital investments on the product market and the quantitative tightening on the equity market in a model of the single product economy are examined. The capital investments on the product market accrued with a constant acceleration induce both an unrestricted increase of product price on the product market and an unrestricted increase of equity price on the equity market. The quantitative tightening considered here are the ones accruing with a constant acceleration. The quantitative tightening on the equity market accrued with a constant acceleration induces an unrestricted change of equity price on the equity market. The quantitative tightening performed with a modest constant acceleration causes an unrestricted increase of equity price on the equity market. The quantitative tightening performed with a large constant acceleration causes an unrestricted decrease of equity price on the equity market. Rate at which the equity price is changing with time in the long run is directly proportional to the synthesis of an acceleration of the capital investments and an acceleration of the quantitative tightening (where the latterconstant acceleration is subtracted from the former constant acceleration with the appropriate coefficients).

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