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Forecasting Gold Prices with ARIMA and GARCH Models

Ioannis Syrris[1] and Vijay Shenai[2*]

[1]*MSc Finance, University of Westminster, London W1 7BY, UK*
[2]*PhD, University of Lincoln, Lincoln LN6 7TS, UK*
* *corresponding author kvshenai11@gmail.com*

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Abstract: This paper examines the efficiency of the forecasting properties of time series models, namely the ARIMA and hybrid ARIMA-GARCH models on daily data of Gold prices for the period 2018 to 2019. First, the paper assesses the unique features of financial data, particularly volatility clustering and fat-tails of the return distribution, and addresses the limitations of using autoregressive integrated moving average (ARIMA) models in financial economics. Secondly, it examines the application of GARCH models for forecasting of both conditional means as well as the conditional variance of the returns. Moreover, using the standard model selection criteria such as AIC, BIC and SIC, the forecasting performance of various candidate ARIMA and GARCH models are examined for an out of sample period. The findings of this paper are that a hybrid ARIMA-GARCH model performs better than an ARIMA or GARCH model by itself in terms of forecasting the returns and volatility of Gold price series. In summary, while ARIMA models have shown the ability to capture the autoregressive process, GARCH models had to be utilised to capture the intense volatility of the Gold commodity. The empirical results obtained in this paper could guide investors to manage risk and return better on their investment decisions.

Keywords: ARIMA, GARCH, volatility, forecasting

JEL Classifications: C58,C52,G15

1. Introduction

1.1 Background

In recent years, an increasing concern is emerging about predicting the future prices and returns of stock prices or commodities. A basic characteristic that all financial markets have in common is the uncertainty of the short and long term price values. Individual investors whose aim is to maximize their profits are looking for methods to precisely forecast the movements of the index or stock prices. Financial institutions and investors need an effective strategy to take decisions based on everyday predictions for the markets and such forecasts are extremely difficult to achieve due to the complexity of the financial markets. To date, various models have been generated for the purpose of predicting future prices. As per the literature, two different forecasting approaches are utilised, the Artificial Intelligence Technique and the Statistical Techniques (Saini, Singh and Laxmi, 2016). The most popular models for short term prediction in the field of financial series are ARIMA models.

This paper focuses on building different price forecasting models and comparing them to identify the most accurate model for forecasting the Gold commodity price. Accuracy in the forecasting of Gold prices is vital for all market participants as this metal is very different from other assets: it is extremely liquid, it conserves its value over time, it plays a central role in international currency reserves and it also promotes the stabilization of international money market (Tripathy, 2017). An example of this distinctiveness can be found in its performance during the financial crises period 2008 -2009 when its global price increased on an average 6% while most of the other mineral prices dropped 40% (Tripathy, 2017). This behaviour is clear evidence that Gold is generally perceived as a safe haven on stock market direction, a belief that many studies support (Baur and McDermott, 2010; Baur and Lucey, 2010; Takashi and Shigeyuki, 2014). Indeed, it is not a casualty if most of the academics believe that an increase in Gold price is a premonition of a fall in the price of other financial assets: in fact, it has been observed that Gold price is not only a reflection of inflation expectations but that it is also highly correlated to other assets such as stocks, bonds, oil prices and foreign currencies (Corti and Holliday, 2010). Therefore, an accurate prediction of Gold prices is very important as it provides a glimpse of future movements in the financial markets as a whole. By developing an accurate forecast on Gold price, investors can profit or hedge to minimize their losses.

Many attempts have been made by researchers to predict future price movements with various forecasting models in the literature. Investors are also utilizing algorithmic trading with the purpose of making rational investment decisions. Although most of the companies nowadays tend to operate statistical techniques, the most common forecasting models are the ARIMA and GARCH models. Depending on data limitations and characteristics, different models are applied for the purpose of forecasting future prices. Some of these forecasting models are the Value At Risk (VAR), the autoregressive conditional heteroscedasticity (ARCH), Autoregressive Moving Average (ARMA) and the random walk model. This study focuses on the Autoregressive Integrated Moving Average (ARIMA), the generalized autoregressive conditional heteroscedasticity (GARCH) and the threshold generalized autoregressive conditional heteroscedasticity (TGARCH). These models were chosen as they represent the most widely used and accurate models for short term price series forecast such as Gold price series index.

1.2. Research Aim

The aim of this paper is to discover the most adequate model for predicting Gold returns and based on that to help financial institutions and individual investors make rational decisions on maximizing their profit margin using the Gold commodity price movements. There is a constant need for researchers and investors to estimate the future prices and their variances of financial assets and hence multiple tools were created (random walk theory, VAR, ARCH-GARCH, and ARIMA). This paper examines different ARIMA and GARCH volatility forecasting models in terms of accuracy and consistence for daily Gold price series data of 345 observations in the period 2018-2019. The findings of this research will be useful for investment decisions as the knowledge of the Gold price movement may lead to investment decisions that maximize the profits or minimize the losses, as Gold is an important commodity that reflects inflation and markets movements.

1.3. Structure

In the first section, the aim and overview of the research were provided. Section Two is a review of relevant literature. This part presents a critical review of the most relevant articles on the subject of this research. Section Three explains the design of the research, methodological choices made and data sources. Section Four presents the results of the analysis conducted with interpretations and discussions while in Section Five, the conclusions of the research and recommendations for future research are presented.

2. Literature Review

In this section, literature on the topic of the research is reviewed and critically appraised. In the last decade, an immense need to predict stock prices and returns has arisen. Investors aiming to maximize their profits, investigate every possible way to forecast market movements of prices and volatility in the stock market as precisely as they can. In the past, predicting market movements was impossible, as for prediction, complex computing on past data was needed. However, the vast developments in technology and computer science in the last twenty years have given investors the tools to compute extremely complex calculations very quickly, hence now many investors are managing their investment decisions on algorithmic trading and and volatility forecasts generated by various softwares. This study examines four different forecast volatility techniques on Gold price series index for daily data. Thus, the literature review is divided in two main sections. In the first section, previous research on Gold price forecasts and their relationship with commodities price series is reviewed. The second part of the literature review, focuses on explaining and analysing different forecasting models that are demonstrated in this report. More specifically, the history and the various findings on the ARIMA and GARCH models are described.

2.1. Gold price forecasts and relationships with relevant variables

As Gold price can be an important indicator of market conditions, many surveys have been conducted to find the relationships between Gold price and other factors such as inflation, interest rates, exchange rates, money supply etc.

A study conducted on the Thai Gold price utilizing multiple regression and the ARIMA model concluded that the American, Australian, Canadian, Japanese currencies are significantly important to the Thai Gold price (Khaemusunun, 2009). The impact of different currencies, the oil price and the interest rates on Thai Gold price were examined using the ARIMA and the multiple regression models. The ARIMA (1, 1, 1) was found to be the most suitable model for predicting Thai Gold price. Another research on Gold price series (Ismail et al., 2009) using the Multiple linear regression (MLR) model suggested that the inflation rate, money supply and the USD/Euro exchange rate have a significant impact on Gold prices. However, a study by Kuan Min (2011) found that the relation between Gold price and inflation was opposite: the return on Gold is unable to hedge against inflation either in Japan or the United states in the short or long term, when inflation hedging strategies using Gold were tested.

Evidence of the relationship between exchange rates and Gold returns in the short run and the long run was the empirical result of another article that applied the interval method to explore the relationship between the Australian dollar exchange rate and the Gold price (Ai, 2012). Many studies related to Gold prices seek to investigate which model is the most adequate to predict Gold price. The empirical findings differ depending on the data, the time period and the number of observations. On the one hand, ARIMA (0, 1, 1) was found as the most suitable model at forecasting the Gold price, when ARIMA models were examined to forecast Gold prices (Massarrat, 2013), but on the other hand, the GARCH model was found more accurate than ARIMA model for predicting the Gold prices when the two models were compared using the Gold prices from Malaysia (Pung, 2013). Another approach supported the ARMA model and 6-step-ahead forecast model for predicting the monthly adjusted closing price of Gold, when compared with the original corresponding price and the actual prices are inside the forecast interval (Rebecca, 2014). The dynamic relationship between Gold prices and the real exchange rates in Australia using the error correction model proves that the Gold price is a good indicator of the Australian dollar/USD exchange rate. This shows that Gold price information can be used to forecast the Australian dollar/USD dollar exchange rate (Nicholas, 2014). Parametric and non-parametric time series can be also used to estimate the forecast of the Gold prices. However in this study the findings are not accurate either in the short or in the long run. Consequently, the univariate models have been found to outperform the multivariate models in terms of forecasting accuracy (Hossein, 2014).

In summary, various studies on Gold prices were examined and evaluated. Many studies argue that Gold prices are related to inflation and the exchange rates and in some cases they were also found to have a predictive content. However, in terms of the better performing model for predicting Gold prices the empirical results differ a lot.

2.2. ARIMA and GARCH models

The continuous interest of investors to formulate a model to provide accurate forecasts of the price series and the variance resulted in various forecasting models with different limitations and strengths. The most commonly used models are the ARIMA and GARCH models.

The ARIMA model was first introduced by Box and Jenkins in 1970. It is also named Box and Jenkins three stage methodology that consists of a set of activities for identifying, estimating and diagnosing the model. Many researchers have found that the ARIMA model has been efficient in generating short term forecasts for financial time series. The limitation of this model is that the financial data usually present many irregularities making the calculations more complex.

To address the heteroscedasticity problem, autoregressive conditional heteroscedasticity model (ARCH) was introduced in 1982 and it is a model that describes the variance of the current error term as a function of the squared variance of previous periods (Engle, 1982). In 1986, the generalized autoregressive conditional heteroscedasticity model (GARCH) was introduced. This model included the current error terms that are also correlated with the past error terms apart from the variance (Bollerslev, 1986).

Many researchers support the ARIMA models for forecasting the price series. One study experimented on removing the difficulty in deciding the order of an ARIMA model using the minimum AIC estimation procedure which resulted in almost identical results as those of the Box and Jenkins procedure (Ozaki, 1977). A more recent study on the Nigerian stock market for the period 1985 to 2008 examined the trend on the price series data by applying the ARIMA model. The ARIMA (2, 1, 2) model performed the best using the MAPE and MAE as estimators for the forecasting error (Abdullahi & Bakari, 2014). In addition to that, an experiment on cassava monthly prices in Ghana using ARIMA model demonstrated a good performance in terms of clustering volatility and predicting power (Kwasi & Kobina, 2014). Furthermore, a survey carried out in data obtained from the New York Stock Exchange and the Nigerian Stock Exchange using the ARIMA model illustrates that there is potential in the model for predicting stock prices on a short term basis (Adebiyiet, 2014). Moreover, a survey utilizing the ARIMA and the state space modelling on India found that ARIMA and state space modelling was close to the real time yields (Vermael, 2015). Last but not least, the nine year daily data of the Indian sectoral stock prices was examined using the ARIMA model and showed that the ARIMA (1, 1, 0) is the most accurate model for forecasts. As stock prices have an upward trend, they could be a worthy investment (Manoj & Edward, 2016).

After the establishment of the GARCH model, scientists were divided between the two models for forecasting and evaluating the price series. The most influential GARCH models were Bollerslev's GARCH (1, 1) model and Nelson's EGARCH model (Engle, 2002). A research examining whether the GARCH or the implied volatility models perform better was published right after that. The result was that it depends on the data sample and the time series but the findings of some researchers indicated that implied volatility models tend to perform better than GARCH models in the out of sample forecasts (Poon & Granger, 2002). The limitation of the GARCH models is the assumption that history repeats itself in patterns so the future prices can be estimated based on the previous prices.

The theory questioning the GARCH models is called "random walks" and support that the future path of the price is independent of the past values and follow a series of random numbers and the prices have no memory therefore GARCH cannot be used as a forecasting tool of the future prices (Fama, 1965; Lock, 2007). The previous theory is based in some research where the findings show that the volatility does not follow the normal distribution but kurtosis is noticed with fat tails (Petrica, Stancu & Tindeche, 2016).

On the other hand, many empirical studies have been conducted to clarify whether random walk theory applies and in many cases the GARCH models are proven to be accurate in the forecasting results (Lo & McKinley, 1988; Chang & Ting, 2000). One possible solution for the sudden changes in volatility and the fat tails of the distribution would be to apply k-mean clustering in the model and the after normalization of the model apply the ARIMA models (Badge, 2013).

It is evident from the prior literature that many price and volatility models exist in the literature, thus selecting the appropriate model has significant contribution to the investment decisions. The main forecasting models presented in the literature are the ARIMA, GARCH, random walk and the implied volatility forecast models. The contribution of this report to the prior literature is to examine the Gold price series for the period 2018 to 2019 by estimating the ARIMA and GARCH models, generate out of sample forecasts to decide the dominant model and based on that to advise investors of the validity of these models for predicting Gold returns.

3. Data and Methodology

3.1. Data

In this section, the data used for the estimation of the different forecasting models will be examined. The data used for this analysis was obtained from Bloomberg, a private financial, software and Data Company whose software is used worldwide by traders and investors. For this report, a sample of daily data of the Gold price index from 02/01/2018 to 29/04/2019 was collected. "The clean or flat price index reflects the position when the accrued interest element is stripped out of the gross index". The incentive of daily data for this specific period is based on two different reasons. Firstly, as Gold is one of the most important commodities, the results of this research will depict the impact of different factors in the economy such as inflation or unemployment rate. Secondly, this specific period was selected as the aim of the report is to evaluate different forecasting models in terms of accuracy and make an out of sample forecast based on the findings. As prices change often, recent and daily data are considered relevant factors for testing the efficiency of prediction models. Less frequent data (weekly, monthly) is less likely to be accurate. The data series will be examined for stationarity which is a condition for estimating forecasting models.

3.2. Methodology

3.2.1 Introduction

In this section, the methods and tools used to develop models predicting Gold prices are presented. Research can be conducted using a qualitative or quantitative approach. The Qualitative approach in research is a nonstatistical method in which the data are mainly obtained from words, images and numbers and are mainly utilized in the social sciences or educational research. The Quantitative approach follows an empirical investigation of phenomena using statistical, mathematical or computational techniques. This technique is mainly utilised to test hypothesis using numerical data and to reveal the relationship and trends between variables. In this case, the researcher is testing a hypothesis in a sample of the population and trying to attain unbiased results that represent the whole population.

The quantitative approach in this project focuses on examining various models that forecast the returns and the residuals and based on that, the dominant model is identified and used for out of sample forecasts on Gold returns. Firstly, the price series will be examined to ascertain whether they follow the normal distribution and also whether they are stationary using the unit root test. The second part includes estimating the ARMA and ARIMA models for developing the price series return forecasts. Gold is a risky asset, and volatility clustering may be a common phenomenon. So in addition to the ordinary GARCH formulation TGARCH and GARCH-M models will be tested. The T-GARCH model allows for estimating the effect of good and bad news on the volatility of the series while the GARCH-M models allows for the risk to be included in the mean equation. Finally, the ARIMA and GARCH models' forecast accuracy and volatility will be compared to determine the best performing model and this model will be used to provide out-of-sample forecasts for the Gold returns.

3.2.2. ARMA and ARIMA models

A combination of two processes, the AR (p) and MA (q), to give a new time series of models called ARMA (p, q) models. The AR (p) is an autoregressive process where p represents the number of lagged variables that the model will have. The AR (p) model can be expressed by using the sum operator Ó as follows:

$$Y_t = \varphi_0 + \sum_{i=1}^{p} \varphi_i Y_{t-i} + u_t \tag{1}$$

Where Y_t is the Gold return at time t and u_t is the residuals at time t. The MA (q) is a moving average model that implicates that depends on the value of the values of past errors at time t. The MA (q) model's general form can be written using the sum operator as:

$$Y_t = \theta_0 - \sum_{i=1}^{q} \theta_i u_{t-i} + u_t \tag{2}$$

Where Y_t is the Gold return at time t, u_t are the residuals at time t and u_{t-i} are the residuals at time t-i. When the return is correlated with both the previous returns and the previous residuals a combination of the previous models is applied to capture both the autoregression process and the moving average process and it is called ARMA (p, q) model. The general form of an ARMA (p, q) model is:

$$Y_t = \psi_0 + \varphi_1 Y_{t-1} + \varphi_2 Y_{t-2} + ...\varphi_p Y_{t-p} + u_t - \theta_1 u_{t-1} - \theta_2 u_{t-2} - ... - \theta_q u_{t-q} \tag{3}$$

Which can be rewritten, using the summations as:

$$Y_t = \psi_0 + \sum_{i=1}^{p} \varphi_i Y_{t-i} + u_t - \sum_{i=1}^{q} \theta_i u_{t-i} \tag{4}$$

An ARMA (p, q) model has the assumption of stationarity which applies only in the AR (p) part of the specification only. In spite of this, most economic and financial time series shows trends over time, thus the mean returns of will be different among the years, which means the price series are not stationary so ARMA model cannot be applied. Thus, to induce stationarity, a process called differencing is used to remove the trend of the raw data. The first difference of series are given by the equation:

$$Z_t = Y_t - Y_{t-1} \tag{5}$$

If after the first differencing a series is stationary then the series is called integrated of order one and denoted I (1). If the series after the first integration is still stationary, the second difference should be estimated. In general, if the series is stationary after d differences then it is called I (d). If a process has an ARIMA (p, d, q) representation, the has an ARMA (p, q) representation, integrated d times, as presented by the equation below:

$$Y_t = \psi_0 + \sum_{i=1}^{p} \varphi_i Y_{t-i} + u_t - \sum_{i=1}^{q} \theta_i u_{t-i} \tag{6}$$

A three stage method aiming at selecting a parsimonious ARIMA model for the purpose of estimating and forecasting a univariate time series was first introduced by Box-Jenkins (1976).

The three stages of this procedure are:

- Identification: The first stage includes identifying possible ARIMA model adequate for the time series. A comparison of the sample's autocorrelation function (ACF) and partial autocorrelation function (PACF) may provide several adequate models. When the assumption of stationarity is violated, the ACF of the series will not show signs of decay. The series in this case can be converted in stationary by differencing. After stationarity is achieved, by utilizing the ACF and PACF table the p and q orders of the ARIMA model can be obtained.

- Estimation: the second step includes estimating the various models identified in the first step.
- Diagnostic checking: This part includes examining the goodness of fit and accuracy of the model. The normality of the residuals is verified using Jarque-Bera test on the residuals. Furthermore, the Akaike Information Criteria (AIC), the Schwartz Information Criteria (SBC) together with the adjusted R square decide for the correct order of the various models. The lower the AIC and SBC and the higher adjusted R square indicate better fit of the model.

3.2.3. GARCH, TGARCH and GARCH-M models

While the ARMA and ARIMA models capture the autocorrelation of the returns, the number of integrations needed to make the model stationary and the moving average process, it is critical in some occasions to capture the clustering volatility of the time series. For that purpose Engle (1982) introduced the autoregressive conditional heteroscedasticity (ARCH) models to account for the volatility clustering observed in economic or financial data, e.g., inflation, stock and exchange-rates returns. The definition of ARCH model of order p is defined as follows:

$$\begin{aligned}&\text{Conditional mean equation}: Y_t = f(\Phi, \Omega_{t-1}) + u_t, u_t \sim N(0, \sigma^2)\\ &\text{Conditional variance equation}: \sigma_t^2 = a_0 + a_1 u_{t-1}^2 + a_2 u_{t-2}^2 + \ldots + a_p u_{t-p}^2\end{aligned} \tag{7}$$

Where Y_t is the return series, f is a function of Φ number of parameters, a set of information available at a time t-1 and u_{t-p}^2 are the squared residuals at time t-p. The limitation of the ARCH model is that usually in financial data to capture the clustering volatility a large order of ARCH model need to be estimated. Thus, in 1986 Bollerslev introduced the generalized ARCH model (GARCH) which in its simplest form is equivalent to an infinite number of ARCH models, ARCH (). This model is called GARCH (1, 1) and it is easier to estimate as the parameters of the model are less than the ARCH model. The definition of an GARCH (p, q) model is as follows:

$$\begin{aligned}&\text{Conditional mean equation}: Y_t = f(\Phi, \Omega_{t-1}) + u_t, u_t \sim N(0, \sigma^2)\\ &\text{Conditional variance equation}: \sigma_t^2 = \omega + \Sigma_{i=1}^{p} a_i u_{t-i}^2 + \Sigma_{i=1}^{q} \beta_i \sigma_{t-i}^2\end{aligned} \tag{8}$$

Where u^2 are the squared residuals and σ^2 is the variance. It is important to be noted that the residuals in the GARCH models do not always follow the normal distribution.

Another model from the GARCH family developed to account for good and bad news effects in markets is called TGARCH. The simpler form of this model is TGARCH (1, 1) whose form is:

$$\sigma_t^2 = \omega + \alpha u_{t-1}^2 + \gamma u_{t-1}^2 d_{t-1} + \beta \sigma_{t-1}^2 \tag{9}$$

Where d_{t-1} is a dummy variable, that captures the impact of good and bad news in the market and takes the value 0 when good news occurs and 1 when bad news occurs in the markets.

If the conditional variance or standard deviation is introduced into the mean equation, the resultant is the GARCH-in-Mean (GARCH-M) model (Engle, Lilien and Robins, 1987) where $X'\theta$ stands for other predetermined variables which can be introduced into the mean equation, λ is the coefficient of the volatility term and ϵ_t is the error term

$$\begin{aligned} Y_t &= X_t'\theta + \lambda\sigma_t + \epsilon_t \\ Y_t &= X_t'\theta + \lambda\sigma_t^2 + \epsilon_t \\ Y_t &= X_t'\theta + \lambda \log(\sigma_t^2) + \epsilon_t \end{aligned} \tag{10}$$

The GARCH-M model is often used in financial applications where the expected return on an asset is related to the expected asset risk. The estimated coefficient on the expected risk is a measure of the risk-return trade-off.

3.2.4. Out of sample forecast evaluation

The aim of this paper is to estimate alternative models for forecasting gold returns and then to assess which models provide more accurate forecasts. As outlined in the prior part of the methodology various forecast models will be evaluated. The models under consideration are: 1) ARIMA models 2) ARIMA GARCH models 3) simple GARCH model. Each of these models have their advantages and some limitations.

The method used for the evaluation of the three different models in this paper consists of various steps. Firstly, each model is divided in two parts, the in sample period and the out of sample period that is exploited as a vector for comparing the forecasts with actual data. Secondly, using a rolling window of a constant size for each model the out of sample forecasts are computed. Afterwards, the actual data are compared with the results of each model's forecasts to identify the accuracy of each model respectively.

Theil coefficients which capture forecast accuracy in a comprehensive way are utilized (Pindyck and Rubinfeld (1997) and Makridakis et al. (1998)).

4.. Analysis and Findings

4.1. Introduction

The purpose of this section is to present the empirical results from the tests made in this research. After testing different families of ARIMA and GARCH models, the best models will be identified in terms of accuracy and goodness of fit. Then, the forecasting ability will be examined using also the GARCH, TGARCH and GARCH-M models. Statistical tools will be used to examine which of these models is more adequate to determine the future price and volatility of the Gold index.

4.2. Data plot, Descriptive statistics and unit root test

At first, a graph is plotted of the Gold price series index from 02/01/2018 to 29/04/2019 to examine its behaviour visually. This is presented below in Figure 1.

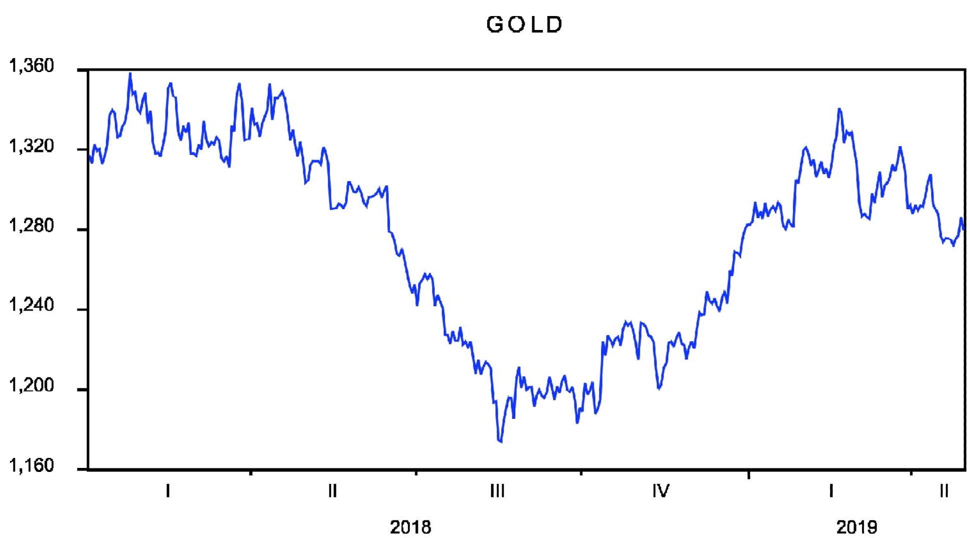

Figure 1: Daily data of Gold Index from beginning of January 2018 to the end of April 2019.

Source: Authors' work

As the graph indicates, the peak price of Gold index was at the start of 2018 around 1360 then a sudden drop is evident until the third quarter of 2018 with lowest price less than 1180. Finally, a slight up rise in the price of

Gold is clear until the first quarter of 2019. The series above exhibits the characteristics of a non-stationary series. With the purpose of identifying various forecasting models, such as ARIMA and GARCH, it is necessary to convert the daily prices into daily returns. The logarithmic return formula used is:

$$R_i = \ln\left(\frac{P_i}{P_{i-1}}\right) \tag{11}$$

where R_i is the logarithmic return the day i, P_i is the daily price of Gold the day i and P_{i-1} is the daily price of the Gold the day i-1.

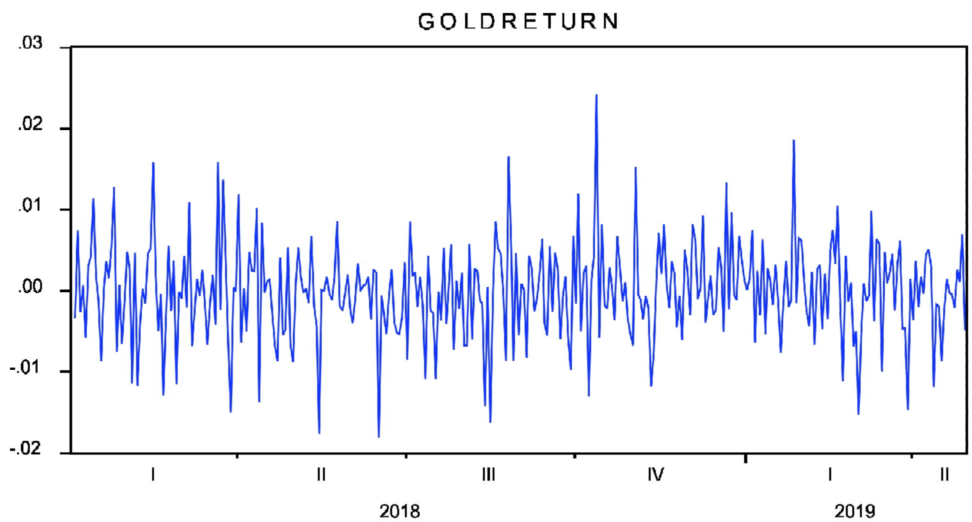

Figure 2: Daily Gold Index Returns from the beginning of January 2018 to the end of April 2019.

Source: Authors' work

The plot of the returns in Figure 2 provides more valuable information about the data as there is the phenomenon of volatility clustering as it was expected from the literature review. The return interval is between -0.02 and 0.02. As expected there is volatility clustering and in some cases big changes are followed by large changes and small changes by small changes in the indices. There is no evidence of a trend, and the series appears to show a tendency to mean reversion. However there may be autocorrelation in the data. For further understanding of the data and to test for normality and stationarity the descriptive statistics are examined. The tables below illustrates the descriptive statistics of the returns:

Figure 3 below demonstrates the distribution of the returns of Gold computed by Eviews software.

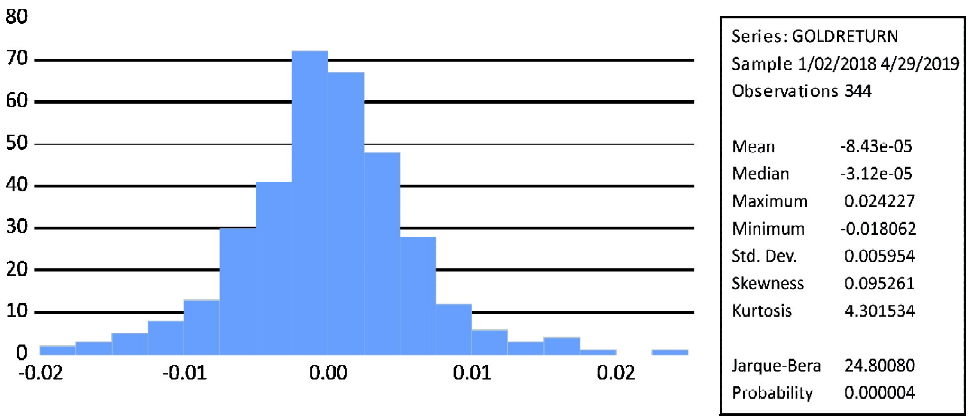

Figure 3: Plot of the daily Gold Index Returns from the beginning of January 2018 to the end of April 2019.

Source: Authors' work

Figure 3 shows that Gold return series does not follow a normal distribution: it is leptokurtic (K>3) with a positive skew (>0); the JB statistic is <0.05.

The results of the ADF test on Gold returns is given below in Table 1.

Table 1
Results of ADF test on Gold Returns from the beginning of January 2018 to the end of April 2019

Null Hypothesis: GOLDRETURN has a unit root
Exogenous: Constant
Lag Length: 0 (Automatic - based on SIC, maxlag=16)

		t-Statistic	*Prob.**
Augmented Dickey-Fuller test statistic		-20.22756	0.0000
Test critical values:	1% level	-3.449220	
	5% level	-2.869750	
	10% level	-2.571213	

*MacKinnon (1996) one-sided p-values.

Augmented Dickey-Fuller Test Equation
Dependent Variable: D(GOLDRETURN)
Method: Least Squares
Date: 29/11/20 Time: 15:38
Sample (adjusted): 4/01/2018 29/04/2019
Included observations: 343 after adjustments

Variable	*Coefficient*	*Std. Error*	*t-Statistic*	*Prob.*
GOLDRETURN(-1)	-1.091374	0.053955	-20.22756	0.0000
C	-8.13E-05	0.000321	-0.253415	0.8001
R-squared	0.545427	Mean dependent var		-4.58E-06
Adjusted R-squared	0.544094	S.D. dependent var		0.008802
S.E. of regression	0.005943	Akaike info criterion		-7.407229
Sum squared resid	0.012046	Schwarz criterion		-7.384851
Log likelihood	1272.340	Hannan-Quinn criter.		-7.398315
F-statistic	409.1541	Durbin-Watson stat		1.982093
Prob (F-statistic)	0.000000			

Source: Authors' work

Table 1 shows the results of the ADF test for Gold returns and rejects the null hypothesis of a unit root. If a unit root exists, the data is non-stationary, and this can cause problems in statistic inference. As the probability of the ADF statistic is 0, the null hypothesis is not accepted; so the Gold returns series do not have a Unit Root. After this verification, ARIMA models are identified in the next sector of the research.

4.3. ARIMA models and forecasts

To select the appropriate ARIMA model Box and Jenkins methodology is applied to the data, as explained in the methodology section. This methodology includes testing for stationarity of the series, identification of the ARIMA models, and testing these models to provide forecasts. Since

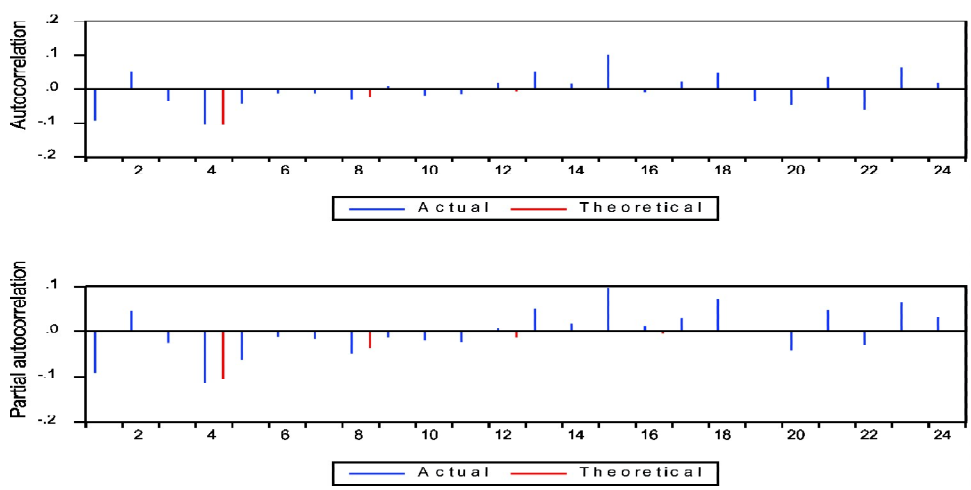

Figure 4: ACF and PACF functions on Gold Returns data from the beginning of January 2018 to the end of April 2019.

Source: Authors' work

the data for the Gold return series do not have a Unit Root, a visual representation on the ACF and PACF on the original data will verify if the returns are stationary or should be integrated.

The ACF and PACF plots show that the auto-correlation function decays, supporting the inference of stationarity. Next the correlogram is examined to identify ARIMA models.

Table 2
Correlogram on Gold Returns from the beginning of January 2018 to the end of April 2019

Date: 08/22/19 Time: 15:31
Sample: 1/02/2018 4/29/2019
Included observations: 344

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	-0.091	-0.091	2.8860	0.089
		2	0.053	0.045	3.8624	0.145
		3	-0.035	-0.026	4.2864	0.232
		4	-0.105	-0.113	8.1129	0.088
		5	-0.044	-0.062	8.7973	0.117
		6	-0.011	-0.012	8.8415	0.183
		7	-0.011	-0.015	8.8806	0.261
		8	-0.031	-0.050	9.2227	0.324
		9	0.007	-0.012	9.2410	0.415
		10	-0.018	-0.021	9.3504	0.499
		11	-0.013	-0.024	9.4064	0.584
		12	0.019	0.006	9.5341	0.657
		13	0.051	0.050	10.461	0.656
		14	0.017	0.018	10.560	0.720
		15	0.102	0.097	14.316	0.502
		16	-0.009	0.013	14.343	0.573
		17	0.023	0.029	14.529	0.629
		18	0.047	0.072	15.345	0.638
		19	-0.036	0.000	15.811	0.670
		20	-0.047	-0.041	16.640	0.676
		21	0.034	0.047	17.074	0.707
		22	-0.060	-0.030	18.392	0.682
		23	0.066	0.064	19.985	0.643
		24	0.018	0.031	20.110	0.690
		25	-0.005	-0.001	20.121	0.740
		26	0.056	0.058	21.294	0.727
		27	-0.036	-0.021	21.779	0.749
		28	-0.048	-0.062	22.643	0.751
		29	0.010	0.008	22.682	0.791
		30	0.040	0.039	23.293	0.803
		31	-0.033	-0.037	23.718	0.822
		32	0.060	0.035	25.084	0.803
		33	0.059	0.069	26.422	0.784
		34	0.007	0.030	26.439	0.819
		35	0.020	0.024	26.586	0.846
		36	0.029	0.031	26.911	0.864

Source: Authors' work

The correlogram shows that there are some spikes in the plots and the following ARIMA models are identified: (4,0,4) and ARIMA (4,0,15).

After estimating the ARIMA(4,0,4) and the ARIMA(4,0,15) models, for the period 2/1/2018 to 29/3/2019 (leaving some observations for out of sample forecast) the detailed outputs for which are included in the appendix (App 1 and 2), model criteria are examined. The coefficients of the AR and MA terms of the estimated model ARIMA (4,0,4) are not significant, but they are significant at the 10% level for ARIMA (4,0,15).

Table 3
Model characteristics of Arima (4,0,4) and Arima (4,0,15)

Model	*AIC*	*SC*	*HQ*	*Volatility*	*Adj R^2*
ARIMA (4,0,4)	-7.3729	-7.3261	-7.3542	3.59E-05	0.0053
ARIMA(4,0,15)	-7.3808	-7.3341	-7.3622	3.56E-05	0.0135

Source: Authors' work

ARIMA(4,0,15) also has a lower volatility and higher Adjusted R^2. So this is the better model.

The forecast performance of the ARIMA (4,0,15) model is attached in Appendix 3 which compares the forecast of the ARIMA model with the actual returns. The findings from this graph can be interpreted as: the ARIMA model is rotating around the zero mean but it is not capturing adequately the volatility clustering of the return series. As a result, the forecast density of the model may not be that accurate to capture the Gold volatility.

4.4. GARCH models and forecasts

Having identified the ARIMA (4,0,15) as the better model, GARCH models can now be estimated. In the methodology it was stated that Gold being a risky asset, T GARCH models (which incorporate the effect of good and bad news in the variance equation) and GARCH-M models (which capture the relationship of volatility with the return in the mean equation) would be estimated in addition to the standard GARCH (1, 1) model. The results of these estimations are attached (App 4,6 and 7).

Table 4
Comparison of the performance of alternative GARCH models, by specification.

Model	*AIC*	*SC*	*HQ*	*Adj R^2*
ARIMA (4,0,15)-GARCH(1,1)	-7.4459	-7.3751	-7.4177	0.0186
ARIMA(4,0,15)-TGARCH (1,1)	-7.3945	-7.3119	-7.3616	0.0087
ARIMA (4,0,15)-GARCH-M	-7.4198	-7.3372	-7.3869	0.0152

Source: Authors' work

Comparing the significance of the coefficients in the equation and the related AIC, SC, HQ criteria and the Adj R^2 for the models shows that ARIMA (4,0,15)-GARCH(1,1) is the best performer of the GARCH family of models tested for goodness of fit of the data. The coefficient of the bad-news term in the variance of the T GARCH model and the coefficient of the volatility term in the mean equation of the GARCH-M model are also not significant.

As the distribution of the Gold returns series is leptokurtic, it is just as well to estimate alternative ARIMA (4,0,15)-GARCH(1,1) models under all three error distributions (normal error distribution (NED), student-t error distribution (STED) and general error distribution (GED). To test whether the ARIMA-GARCH models have an explanatory power and value over the ARIMA model, by itself , forecasts needs to be estimated and evaluated. To estimate the forecasts the same procedure as for the ARIMA model is followed. First of all, the data sample is divided in two periods: 1) the in sample period (02/01/2018 to 29/03/2019) and then 2) the out of sample period which is one month (29/03/2019 to 29/04/2019). Then using the rolling windows method forecasts are estimated for these 22 days of the out of sample period using 22 rolling windows one for each observation. The forecast values from a simple GARCH (1,1) model (App 12) are also added for comparison. Finally, to evaluate the forecasting error for the out of sample period, the Theil coefficients (Pindyck and Rubinfeld (1997) and Makridakis *et al.* (1998)) are taken into account.

Table 5
Comparison of the forecast performance of the Arima (4,0,15) and Arima (4,0,15)-GARCH(1,1) models, with different error distributions and the simple GARCH(1,1) model.

Model	U^1	U^M	U^S	U^C	U^2
ARIMA (4,0,15)	0.8737	0.0146	0.6643	0.3211	1.0361
ARIMA (4,0,15) GARCH(1,1) NED	0.8603	0.0133	0.6145	0.3721	1.0550
ARIMA (4,0,15) GARCH(1,1) STED	0.8726	0.0152	0.6465	0.3382	1.0597
ARIMA(4,0,15) GARCH(1,1) GED	0.8736	0.0171	0.6590	0.3230	1.0442
Simple GARCH(1,1)	0.9908	0.0061	0.9939	0.0000	0.9563

where
U^1 Theil Inequality coefficient
U^M is the bias proportion of U^2
U^S is the variance proportion of U^2
U^C is the covariance proportion of U^2
U^2 Theil inequality coefficient
Source: Authors' work

The findings of the table illustrate that the ARIMA (4,0,15) GARCH (1, 1) model with normal error distribution (NED) has better forecasting performance than other models when the forecasts are evaluated with Theil coefficients: the Arima(4,0,15) Garch(1,1) model with (NED) has the lowest U^1 coefficient and the highest U^C coefficient. The simple GARCH (1,1) model does not have a much better forecast ability than a naïve forecast. A visual representation of the forecast also supports the findings of this paper; the graph in figure 5 seems to capture better the volatility than the ARIMA model. ARIMA (4,0,15)-GARCH(1,1) thus is the dominant forecasting model.

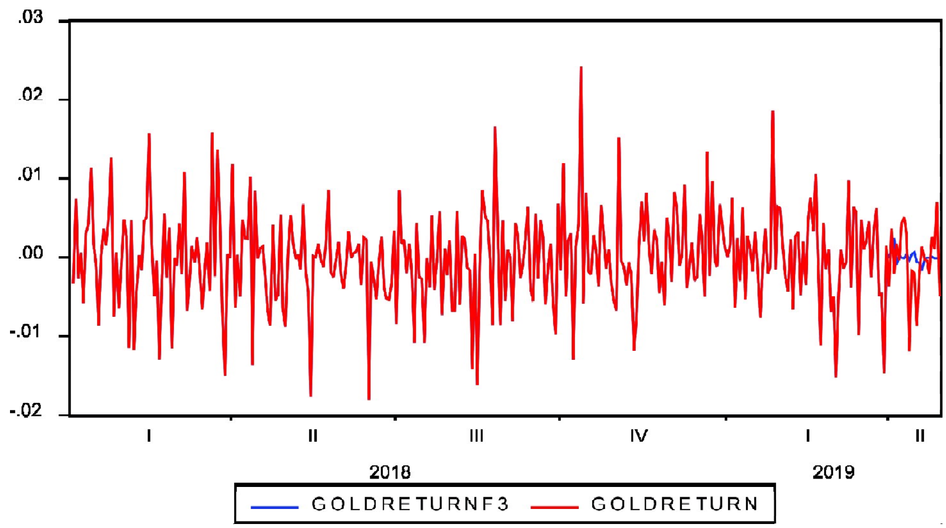

Figure 5: Gold actual returns and GARCH forecast for ARIMA(4,0,15) GARCH(1,1)
Source: Authors' work

5. Conclusion

This paper presents the extensive process of building ARIMA and GARCH models for Gold return prediction using the Box-Jenkins methodology of developing the ARIMA models for short term forecasting. The aim was to develop and test alternative ARIMA, and hybrid ARIMA-GARCH models and evaluate their forecasting abilities. Alternative ARIMA, ARIMA-GARCH, ARIMA-TGARCH and ARIMA GARCH-M models were estimated using robust testing procedures. Of the ARIMA models identified ARIMA (4, 0, 15) had the best goodness of fit whilst from the family of hybrid Arima GARCH models, ARIMA(4,0,15)-GARCH(1,1) NED had the best statistics. A simple GARCH(1,1) model had no better forecasting ability than a naïve forecast. Financial data have properties of volatility clustering and this was overall captured better by the hybrid ARIMA GARCH models. Moreover, given that Gold returns are leptokurtic, alternative error distributions were also examined. Over the period of the research from 02/01/2018 to 29/04/2019 on Gold returns data, the hybrid ARIMA-GARCH (1,1) NED model had the best out of sample forecasting performance. The analysis shows the importance of modelling both the returns and variance of financial assets for better forecasts. In summary, while ARIMA models have shown the ability to capture the autoregressive process, GARCH models had to be utilized to capture the intense volatility of the Gold commodity. The empirical results obtained in this paper could guide investors to manage risk and return better on their investment decisions.

Future research

Future research could extend to other financial assets or commodities like silver and oil.

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Appendices

Appendix 1
Estimated ARIMA(4,0,4) Model

Source: Authors' work

Dependent Variable: GOLDRETURN
Method: ARMA Maximum Likelihood (OPG - BHHH)
Date: 29/11/20 Time: 04:05
Sample: 3/01/2018 29/03/2019
Included observations: 323
Convergence achieved after 15 iterations
Coefficient covariance computed using outer product of gradients

Variable	*Coefficient*	*Std. Error*	*t-Statistic*	*Prob.*
C	-5.48E-05	0.000282	-0.194600	0.8458
AR(4)	0.275227	0.422739	0.651057	0.5155
MA(4)	-0.392308	0.402662	-0.974284	0.3307
SIGMASQ	3.59E-05	2.21E-06	16.19273	0.0000
R-squared	0.014541	Mean dependent var		-5.97E-05
Adjusted R-squared	0.005273	S.D. dependent var		0.006042
S.E. of regression	0.006026	Akaike info criterion		-7.372918
Sum squared resid	0.011584	Schwarz criterion		-7.326136
Log likelihood	1194.726	Hannan-Quinn criter.		-7.354243
F-statistic	1.568968	Durbin-Watson stat		2.203578
Prob(F-statistic)	0.196800			
Inverted AR Roots	.72	.00-.72i	-.00+.72i	-.72
Inverted MA Roots	.79	.00+.79i	-.00-.79i	-.79

Appendix 2
Estimated ARIMA(4,0,15) Model

Source: Authors' work

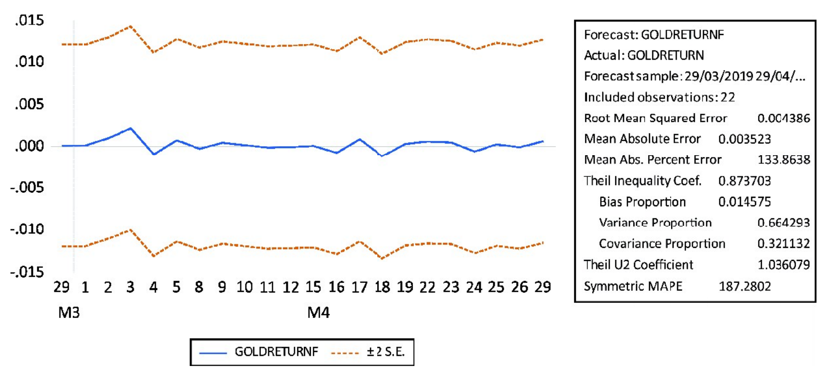
Dependent Variable: GOLDRETURN
Method: ARMA Maximum Likelihood (OPG - BHHH)
Date: 29/11/20 Time: 04:08
Sample: 3/01/2018 29/03/2019
Included observations: 323
Convergence achieved after 13 iterations
Coefficient covariance computed using outer product of gradients

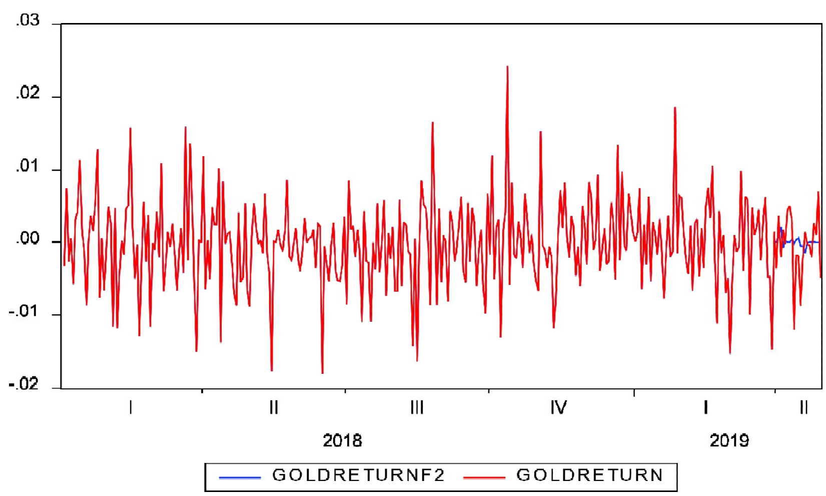
Variable	*Coefficient*	*Std. Error*	*t-Statistic*	*Prob.*
C	-4.77E-05	0.000334	-0.142709	0.8866
AR(4)	-0.111736	0.064161	-1.741501	0.0826
MA(15)	0.094404	0.051323	1.839426	0.0668
SIGMASQ	3.56E-05	2.21E-06	16.11142	0.0000

R-squared	0.022682	Mean dependent var		-5.97E-05
Adjusted R-squared	0.013491	S.D. dependent var		0.006042
S.E. of regression	0.006001	Akaike info criterion		-7.380858
Sum squared resid	0.011488	Schwarz criterion		-7.334076
Log likelihood	1196.009	Hannan-Quinn criter.		-7.362183
F-statistic	2.467831	Durbin-Watson stat		2.211924
Prob(F-statistic)	0.062082			
Inverted AR Roots	.41-.41i	.41+.41i	-.41+.41i	-.41-.41i
Inverted MA Roots	.84+.18i	.84-.18i	.69+.50i	.69-.50i
	.43-.74i	.43+.74i	.09-.85i	.09+.85i
	-.26-.81i	-.26+.81i	-.57-.63i	-.57+.63i
	-.78+.35i	-.78-.35i	-.85	

Appendix 3
Out of sample forecast outputs for ARIMA(4,0,15) Model

Source: Authors' work





ARIMA (4, 0, 15) Forecast and actual Gold returns

Appendix 4
Estimated Arima (4,0,15)-GARCH(1,1) NED

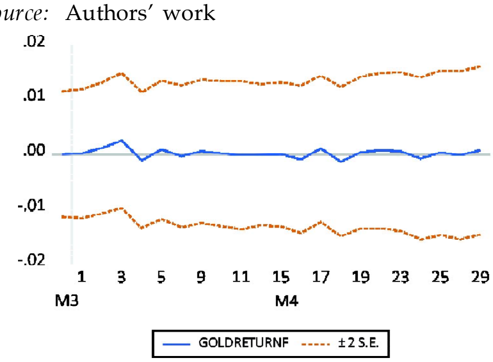
Source: Authors' work

Dependent Variable: GOLDRETURN
Method: ML ARCH - Normal distribution (BFGS / Marquardt steps)
Date: 29/11/20 Time: 04:10
Sample (adjusted): 9/01/2018 29/03/2019
Included observations: 319 after adjustments
MA Backcast: 19/12/2017 8/01/2018
Presample variance: backcast (parameter = 0.7)
GARCH = C(4) + C(5)*RESID(-1)^2 + C(6)*GARCH(-1)

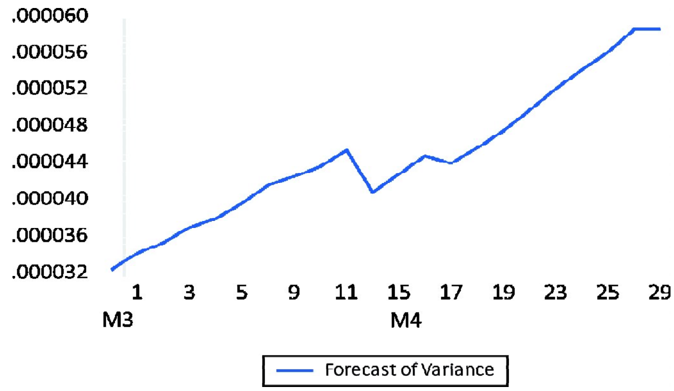
Variable	*Coefficient*	*Std. Error*	*z-Statistic*	*Prob.*
C	-0.000105	0.000301	-0.349840	0.7265
AR(4)	-0.133323	0.051743	-2.576643	0.0100
MA(15)	0.107483	0.044636	2.407965	0.0160
	Variance Equation			
C	9.16E-07	3.29E-07	2.781638	0.0054
RESID(-1)^2	-0.049707	0.010608	-4.685636	0.0000
GARCH(-1)	1.028605	0.010884	94.50265	0.0000
R-squared	0.024744	Mean dependent var		-6.72E-05
Adjusted R-squared	0.018571	S.D. dependent var		0.006061
S.E. of regression	0.006004	Akaike info criterion		-7.445947
Sum squared resid	0.011392	Schwarz criterion		-7.375129
Log likelihood	1193.629	Hannan-Quinn criter.		-7.417665
Durbin-Watson stat	2.204632			
Inverted AR Roots	.43-.43i	.43+.43i	-.43+.43i	-.43-.43i
Inverted MA Roots	.84-.18i	.84+.18i	.70-.51i	.70+.51i
	.43+.75i	.43-.75i	.09+.86i	.09-.86i
	-.27+.82i	-.27-.82i	-.58-.64i	-.58+.64i
	-.79-.35i	-.79+.35i	-.86	

Appendix 5
Out of sample forecast of estimated Arima (4,0,15)-GARCH(1,1) model.

Source: Authors' work



Forecast: GOLDRETURNF	
Actual: GOLDRETURN	
Forecast sample: 29/03/2019 29/04/...	
Included observations: 22	
Root Mean Squared Error	0.004413
Mean Absolute Error	0.003570
Mean Abs. Percent Error	137.8274
Theil Inequality Coef.	0.860319
Bias Proportion	0.013279
Variance Proportion	0.614618
Covariance Proportion	0.372104
Theil U2 Coefficient	1.054969
Symmetric MAPE	186.9071



Appendix 6
Estimated Arima (4,0,15)-TGARCH(1,1) model.

Source: Authors' work

Dependent Variable: GOLDRETURN
Method: ML ARCH - Normal distribution (BFGS / Marquardt steps)
Date: 29/11/20 Time: 04:12
Sample (adjusted): 9/01/2018 29/03/2019
Included observations: 319 after adjustments
MA Backcast: 19/12/2017 8/01/2018
Presample variance: backcast (parameter = 0.7)
GARCH = C(4) + C(5)*RESID(-1)^2 + C(6)*RESID(-1)^2*(RESID(-1)<0) + C(7)*GARCH(-1)

Variable	*Coefficient*	*Std. Error*	*z-Statistic*	*Prob.*
C	-0.000119	0.000386	-0.307336	0.7586
AR(4)	-0.021601	0.071712	-0.301218	0.7632
MA(15)	0.066168	0.057024	1.160351	0.2459
	Variance Equation			
C	1.80E-05	5.84E-06	3.088908	0.0020
RESID(-1)^2	0.010559	0.001156	9.138269	0.0000
RESID(-1)^2*(RESID(-1)<0)	-0.140454	0.018292	-7.678368	0.0000
GARCH(-1)	0.594519	0.157722	3.769424	0.0002
R-squared	0.014888	Mean dependent var		-6.72E-05
Adjusted R-squared	0.008653	S.D. dependent var		0.006061
S.E. of regression	0.006035	Akaike info criterion		-7.394553
Sum squared resid	0.011507	Schwarz criterion		-7.311931
Log likelihood	1186.431	Hannan-Quinn criter.		-7.361557
Durbin-Watson stat	2.184037			
Inverted AR Roots	.27+.27i	.27-.27i	-.27-.27i	-.27+.27i
Inverted MA Roots	.82-.17i	.82+.17i	.68-.49i	.68+.49i
	.42+.72i	.42-.72i	.09+.83i	.09-.83i
	-.26+.79i	-.26-.79i	-.56+.62i	-.56-.62i
	-.76-.34i	-.76+.34i	-.83	

Appendix 7
Estimated Arima (4,0,15)-GARCH M (1,1) model

Source: Authors' work

Dependent Variable: GOLDRETURN
Method: ML ARCH - Normal distribution (BFGS / Marquardt steps)
Date: 11/02/20 Time: 17:22
Sample (adjusted): 1/09/2018 3/29/2019
Included observations: 319 after adjustments
MA Backcast: 12/19/2017 1/08/2018
Presample variance: backcast (parameter = 0.7)
GARCH = C(5) + C(6)*RESID(-1)^2 + C(7)*GARCH(-1)

Variable	*Coefficient*	*Std. Error*	*z-Statistic*	*Prob.*
@SQRT(GARCH)	-0.223367	0.358184	-0.623610	0.5329
C	0.001143	0.001907	0.599499	0.5488
AR(4)	-0.155548	0.043325	-3.590298	0.0003
MA(15)	0.112572	0.043659	2.578457	0.0099
	Variance Equation			
C	9.68E-07	2.63E-07	3.673154	0.0002
RESID(-1)^2	-0.043233	0.007141	-6.053935	0.0000
GARCH(-1)	1.019310	7.19E-05	14171.11	0.0000
R-squared	0.024564	Mean dependent var		-6.72E-05
Adjusted R-squared	0.015274	S.D. dependent var		0.006061
S.E. of regression	0.006014	Akaike info criterion		-7.419892
Sum squared resid	0.011394	Schwarz criterion		-7.337270
Log likelihood	1190.473	Hannan-Quinn criter.		-7.386896
Durbin-Watson stat	2.211751			
Inverted AR Roots	.44+.44i	.44+.44i	-.44-.44i	-.44-.44i
Inverted MA Roots	.85-.18i	.85+.18i	.70-.51i	.70+.51i
	.43+.75i	.43-.75i	.09+.86i	.09-.86i
	-.27+.82i	-.27-.82i	-.58-.64i	-.58+.64i
	-.79-.35i	-.79+.35i	-.86	

Appendix 8
Estimated Arima (4,0,15)-GARCH (1,1) model, STED

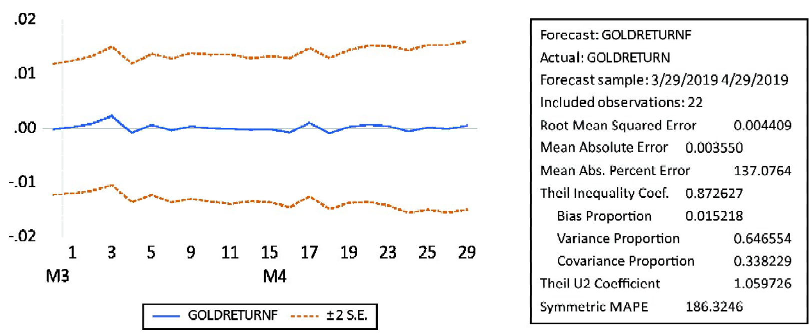
Source: Authors' work

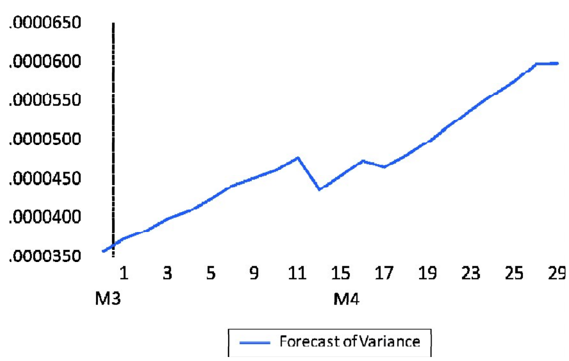
Date: 11/02/20 Time: 18:13
Sample (adjusted): 1/09/2018 3/29/2019
Included observations: 319 after adjustments
Convergence not achieved after 500 iterations
Coefficient covariance computed using outer product of gradients
MA Backcast: 12/19/2017 1/08/2018
Presample variance: backcast (parameter = 0.7)
GARCH = C(4) + C(5)*RESID(-1)^2 + C(6)*GARCH(-1)

Variable	*Coefficient*	*Std. Error*	*z-Statistic*	*Prob.*
C	-5.08E-05	0.000282	-0.179864	0.8573
AR(4)	-0.130687	0.050099	-2.608586	0.0091
MA(15)	0.078296	0.045565	1.718356	0.0857
	Variance Equation			
C	8.06E-07	3.93E-07	2.052475	0.0401
RESID(-1)^2	-0.043664	0.014691	-2.972193	0.0030
GARCH(-1)	1.025096	0.016154	63.45860	0.0000
T-DIST. DOF	10.15989	5.681108	1.788364	0.0737
R-squared	0.023781	Mean dependent var		-6.72E-05
Adjusted R-squared	0.017602	S.D. dependent var		0.006061
S.E. of regression	0.006007	Akaike info criterion		-7.447819
Sum squared resid	0.011404	Schwarz criterion		-7.365197
Log likelihood	1194.927	Hannan-Quinn criter.		-7.414823
Durbin-Watson stat	2.203038			
Inverted AR Roots	.43+.43i	.43+.43i	-.43-.43i	-.43-.43i
Inverted MA Roots	.83-.18i	.83+.18i	.68-.50i	.68+.50i
	.42+.73i	.42-.73i	.09+.84i	.09-.84i
	-.26+.80i	-.26-.80i	-.56-.63i	-.56+.63i
	-.77-.34i	-.77+.34i	-.84	

Appendix 9
Out of sample forecast for Arima (4,0,15)-GARCH (1,1) model, STED

Source: Authors' work





Appendix 10
Estimated Arima (4,0,15)-GARCH (1,1) model, with GED

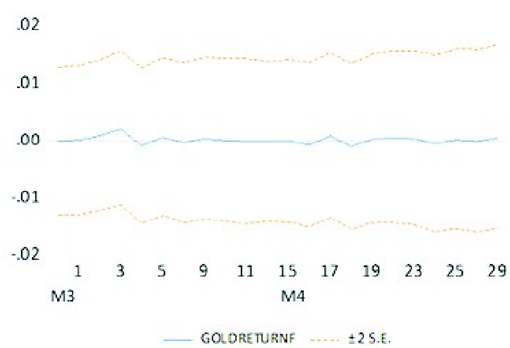
Source: Authors' work

Dependent Variable: GOLDRETURN
Method: ML ARCH - Generalized error distribution (GED) (BFGS / Marquardt steps)
Date: 11/02/20 Time: 18:23
Sample (adjusted): 1/09/2018 3/29/2019
Included observations: 319 after adjustments
MA Backcast: 12/19/2017 1/08/2018
Presample variance: backcast (parameter = 0.7)
GARCH = C(4) + C(5)*RESID(-1)^2 + C(6)*GARCH(-1)

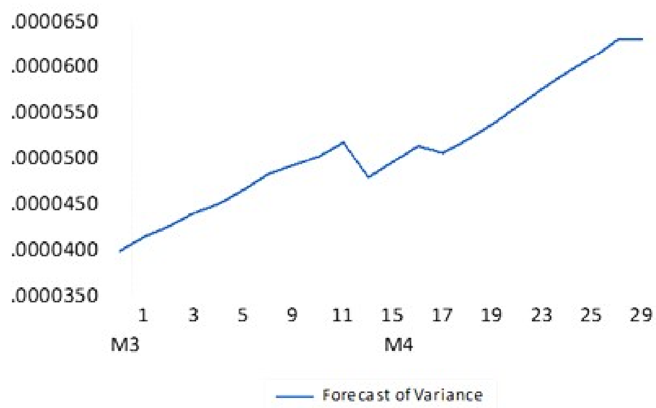
Variable	*Coefficient*	*Std. Error*	*z-Statistic*	*Prob.*
C	-8.74E-06	0.000273	-0.032054	0.9744
AR(4)	-0.118847	0.043753	-2.716296	0.0066
MA(15)	0.086765	0.043892	1.976765	0.0481
	Variance Equation			
C	9.33E-07	3.40E-07	2.746653	0.0060
RESID(-1)^2	-0.041527	0.009138	-4.544262	0.0000
GARCH(-1)	1.018951	0.000115	8851.844	0.0000
GED PARAMETER	1.498919	0.170362	8.798438	0.0000
R-squared	0.024507	Mean dependent var		-6.72E-05
Adjusted R-squared	0.018333	S.D. dependent var		0.006061
S.E. of regression	0.006005	Akaike info criterion		-7.440919
Sum squared resid	0.011395	Schwarz criterion		-7.358297
Log likelihood	1193.827	Hannan-Quinn criter.		-7.407923
Durbin-Watson stat	2.201290			
Inverted AR Roots	.42-.42i	.42+.42i	-.42+.42i	-.42-.42i
Inverted MA Roots	.83-.18i	.83+.18i	.69-.50i	.69+.50i
	.42+.74i	.42-.74i	.09+.84i	.09-.84i
	-.26+.81i	-.26-.81i	-.57-.63i	-.57+.63i
	-.78-.35i	-.78+.35i	-.85	

Appendix 11
Out of sample forecast for Arima (4,0,15)-GARCH (1,1) model, with GED

Source: Authors' work



Forecast: GOLDRETURNF	
Actual: GOLDRETURN	
Forecast sample: 3/29/2019 4/29/2019	
Included observations: 22	
Root Mean Squared Error	0.004399
Mean Absolute Error	0.003536
Mean Abs. Percent Error	136.5395
Theil Inequality Coef.	0.873690
Bias Proportion	0.017101
Variance Proportion	0.659032
Covariance Proportion	0.323867
Theil U2 Coefficient	1.044198
Symmetric MAPE	187.4176



Appendix 12
Estimated simple GARCH (1,1) model

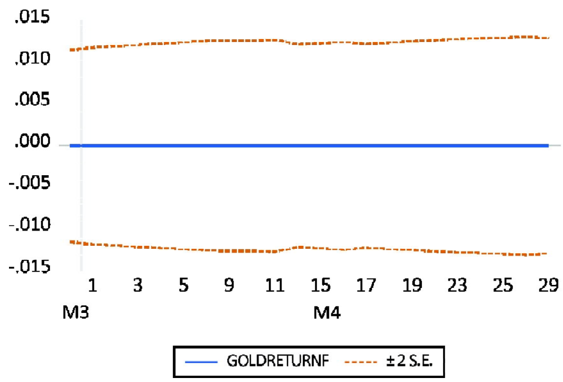
Source: Authors' work

Dependent Variable: GOLDRETURN
Method: ML ARCH - Normal distribution (BFGS / Marquardt steps)
Date: 11/03/20 Time: 07:49
Sample (adjusted): 1/03/2018 3/29/2019
Included observations: 323 after adjustments
Convergence achieved after 44 iterations
Coefficient covariance computed using outer product of gradients
Presample variance: backcast (parameter = 0.7)
GARCH = C(2) + C(3)*RESID(-1)^2 + C(4)*GARCH(-1)

Variable	*Coefficient*	*Std. Error*	*z-Statistic*	*Prob.*
C	-3.69E-05	0.000337	-0.109455	0.9128
	Variance Equation			
C	4.50E-06	3.99E-06	1.128735	0.2590
RESID(-1)^2	-0.032156	0.019467	-1.651817	0.0986
GARCH(-1)	0.909209	0.097315	9.342962	0.0000
R-squared	-0.000014	Mean dependent var		-5.97E-05
Adjusted R-squared	-0.000014	S.D. dependent var		0.006042
S.E. of regression	0.006042	Akaike info criterion		-7.368937
Sum squared resid	0.011755	Schwarz criterion		-7.322155
Log likelihood	1194.083	Hannan-Quinn criter.		-7.350262
Durbin-Watson stat	2.186949			

Appendix 13
Out of sample forecast for simple GARCH (1,1) model

Source: Authors' work



Forecast: GOLDRETURNF	
Actual: GOLDRETURN	
Forecast sample: 3/29/2019 4/29/2019	
Included observations: 22	
Root Mean Squared Error	0.004294
Mean Absolute Error	0.003244
Mean Abs. Percent Error	98.58470
Theil Inequality Coef.	0.990775
Bias Proportion	0.006092
Variance Proportion	0.993908
Covariance Proportion	0.000000
Theil U2 Coefficient	0.956282
Symmetric MAPE	192.1396