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A REVIEW OF LITERATURE ON SPECULATIVE BUBBLES

Joseph Emmanuel FANTCHO

Docteur en Sciences Economiques, Ingénieur Statisticien Economiste, Institut Supérieur des Sciences et de Technologie, Université des Montagnes - B.P. 208 Bangangté - Cameroun
E-mail : jefantcho@udm.aed-cm.org

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Abstract

This paper investigates the nature and the presence of bubbles in financial markets. We find in the existence literature the answer of the following question: what are bubbles? Are they consistent with rationality? Can they have real effects? How do they behave? What are implications of transversality condition? What methods have been used to detect them? We found that, many debates rose to know whether price "bubbles" ever existed. Financial economists and market participants often hold quite different views about the price of an asset. The "pro-bubble" side is largely supported by some hedge fund managers and some policy-makers. While on the other side, a number of academic economists are skeptical of the bubbles theory. We have also found that, bubbles likely have real effects on economy. In particular, price spikes on commodities would adversely affected the social welfare of consumers, especially those in developing countries.

Keywords: Rationality and bubble, rational bubbles, transversality condition, reel effect of bubbles, detecting bubbles.

1. INTRODUCTION

This paper is a broad study, drawing on a wide range of published research and historical evidence, on stock prices bubbles. On December 1996, Alan Greenspan, chairman of the Federal Reserve Board in Washington, used the term "irrational exuberance"[1] to describe the behavior of stock market investors. The words irrational exuberance quickly became Greenspan's most famous quote and a catch phrase for everyone who follows the market.

Financial history can be read in many respects, as a history of boom and burst bubbles. The infamous Dutch Tulip Mania (1634 - 1637), the French Mississippi Bubble (1719 - 1720), the South Sea Bubble in the United Kingdom (1720s), the first Latin American debt boom (1820s), and railway manias in the

United Kingdom (1840s) and United States (1870s) are all notable examples. In the past century, no busts have been more devastating than the Great Depression ushered in by the collapse of US stock markets in 1929. Over the past few decades, the Japanese Heisei bubble in the late 1980s, the various emerging market booms and busts in the 1980s and 1990s, and the equity mania in the late 1990s, offer others examples of speculative frenzies gone awry.

Despite such evidence, many debates persist on the existence of speculative bubbles. The existence of speculative bubbles in financial markets has been a long-standing issue under debate. Financial economists and market participants often hold quite different views about the price of an asset. On the one hand, financial economists usually believe that given the assumption of rational behavior and rational expectations, the price of an asset must simply reject market fundamentals, that is to say, the price of an asset, can only depend on information about current and future returns from this asset. Deviations from this market fundamental value are taken as prima facie signs of irrationality. On the other hand, market participants argue that strange events and self-fulfilling rumors may well influence the price, if believed by other participants to do so; "crowd psychology" becomes an important determinant of price. Rationality of behavior often does not imply that the price of an asset be equal to its fundamental value. In other words, there can be rational deviations of the price from fundamental value - rational bubbles. The word "bubble" recalls to some famous episodes in finance history in which asset price rose far higher than it could be easily explained by fundamentals, and with investors appeared to betting that other investors would drive price even higher in the future. History has too often witnessed the rise and collapse of assets price. The first recorded bubble is the "Tulip mania", in February 1637 - a period in Dutch history where prices for tulip bulbs reached extraordinarily high levels and then suddenly collapsed. Almost surely, the financial crisis caused by the burst of the U.S. housing bubble[2] is not last one[3]. Many debates rose to know whether price "bubbles" ever existed. The "pro-bubble" side is largely supported by some hedge fund managers and some policy-makers. On the other side, a number of academic economists are skeptical of the bubble theory, citing a lack of empirical evidence (e.g., Krugman, 2008).

This paper investigates the nature and the presence of bubbles in financial markets. What are bubbles? Are bubbles consistent with rationality? Can they have real effects? How do they behave? What are implications of transversality condition? What methods have been used to detect them? These are questions we answer in the following sections. The paper is organized as follows: Section 2 focus on rationality and bubbles. Section 3 gives some types of rational bubbles. Section 4 is devoted to present some empirical tests that have been used to detect bubbles. Section 5 discusses the link between transversality condition and bubbles, and presents Kamihigashi's result on the necessity of the transversality condition. Section 6 is the conclusion.

2. RATIONALITY AND BUBBLE

2.1. Description

Rationality of behavior and expectations, together with market clearing, imply that assets are voluntary held and that no agent can, given his private information and information revealed by price, increase his expected utility by reallocating his portfolio. With many other assumptions, this lead to a standard "efficient market" or "no arbitrage condition". Let us define the net simple return (Blanchard and Watson, 1982),

$$R_{t+t} = \frac{P_{t+1} + D_{t+1} - P_t}{P_t} \tag{1.a}$$

this definition is straightforward, but it uses two notations conventions that deserve emphasis. First P_{t+1} denotes the price of an asset measured at the beginning of the period $t + 1$, or equivalently an ex-dividend price: purchase of a stock at a price P_t today, gives one a claim to the next period dividend per share D_{t+1}; but not the period's dividend D_t. D_{t+1} is the direct return, one can see D_{t+1} as the dividend, although it may take depending on the asset, pecuniary or non pecuniary forms. Second, R_{t+t} denotes the return on the asset held from t to $t + 1$. R_{t+t} is the return of the holding asset, which is the sum of the dividend price ratio and the capital gain. t indicates the time. The subscript t+1 is used because the return is known at the time t+1. Let us assume that, the expected return of the asset is constant, that is:

$$E(R_{t+1} \mid F_t) = R \tag{1.b}$$

Taking the expectations[4] of the identity (1.*a*), imposing (1.*b*) and rearranging, we obtain an equation relating the current stock price to the next period expected stock price and dividend:

$$P_t = \frac{1}{1+R} E_t(P_{t+1} + D_{t+1}) \tag{1.c}$$

F_t is the information set available at time t, E_t is a short for $E(|F_t)$. The condition (1.*b*) states that the expected return on the asset is equal to the interest rate R; assumed constant. Among the assumptions needed to obtain equation (1.*c*), some are inessential and could be relaxed at a cost of increase complexity of notations.

Given assumption of rational expectations and the fact that agent do not forget, so that $F_t \subseteq F_{t+1}$, the equation (1.*c*) is solved recursively, by repeatedly substituting out future prices and using the law of iterated expectation:

$$E_t\left(E_{t+j}(P_{t+T})\right) = E_t(P_{t+T}), \quad \forall T \geq 0$$

to eliminate future-dated expectations. After solving T periods, we obtain:

$$P_t = \left[\sum_{i=1}^{T}\left(\frac{1}{1+R}\right)^i E_t(D_{t+i})\right] + E_t\left[\left(\frac{1}{1+R}\right)^T P_{t+T}\right] \tag{2.a}$$

When the horizon T increases to infinity, we have:

$$P_t = \left[\sum_{i=1}^{+\infty}\left(\frac{1}{1+R}\right)^i E_t(D_{t+i})\right] + \lim_{T \to +\infty} E_t\left[\left(\frac{1}{1+R}\right)^T P_{t+T}\right] \tag{2.b}$$

The first term of the equation (2.b) is the present value of the expected dividends and thus it's called the "market fundamental" value of the asset. This term is standard in financial markets. It was introduced in economics by Flood and Garber (1980).

The relation (2.b) is well elucidated in Campbell and Mackinlav (2000), chapter 7. The basic framework for their analysis is the discounted-cash-flow or present value model. Their model relates the price of a stock to its expected future cash flows-its dividends-discounted to the present using a constant or time-varying discount rate. Since dividends in all future periods enter the present-value formula, the dividend in any one period is only a small component of the price.

$$P_t^* = \sum_{i=1}^{+\infty}\left[\left(\frac{1}{1+R}\right)^i E_t(D_{t+i})\right] \tag{3.a}$$

P_t^* is a solution of the equation (1.c), but it is not the only solution of (1.*c*). Any P_t of the following form[5]

$$P_t = P_t^* + B_t \tag{3.b}$$

where:

$$B_t = \frac{1}{1+R} E_t(B_{t+1}) \tag{3.c}$$

is a solution as well. Thus the market price can deviate from its market fundamental value without violating the arbitrage condition (1.c). As $R>0$, this violation B_t must however be expected to grow over time. The deviation B_t embodies the popular notion of "bubble", namely the movements in the price, apparently unjustified by information available at the time. When the additional term B_t in (3.b) satisfies (3.c), it is called a "rational bubble". The adjective "rational" is used because the term B_t is consistent with rational expectations and constant return.

The second term in the right-hand size of the equation (2.b) is the present expected discounted value of the stock price as the horizon grows to infinity. When we impose:

$$\lim_{T \to +\infty} E_t \left[\left(\frac{1}{1+R} \right)^T P_{t+T} \right] = 0 \tag{TC}$$

then, P_t^* becomes the unique solution of (1.*c*). (*TC*) is called the transversality condition. The condition (*TC*) rules out the presence of bubbles. If the transversality condition (*TC*) does not hold, the general solution to (1.*c*) has the form (3.*a*-*c*).

2.2. Real effects of bubbles

In the years 2000, the economic statistics being reported for the U.S. Economy have been very contradictory. The stock market has soared to record levels. Profits for the major corporations have never been higher. Meanwhile, the manufacturing and farming economies are essentially in recession, and personal bankruptcies are at record levels. Why should a part of U.S. economy is doing so well, while other parts are suffering? Henceforth, stock market expansion is associated with popular perceptions that the future is brighter or less uncertain than it was in the past. The appearance of new technologies is now named as a cause of asset price popping. Modern economies often experience large movements in asset prices that cannot be explained by changes in economic conditions or fundamentals. It is usual to refer to these episodes as asset price bubbles popping up and bursting. Do bubbles generate substantial macroeconomic effects? Blanchard and Watson (1982), Tirole (1985), Olivier (2000), Yanagawa and Grossman (1993), Caballero and Hammour (2005), have analyzed the real effects of bubbles.

Tirole (1985) show that bubbles can exist in overlapping generation model with infinite number of infinite-lived rational agents, in a dynamically inefficient equilibrium; that is, in equilibrium where too much capital is being accumulated. Consequently, bubbles crowd saving away from investment in physical capital; bubbles in Tirole's model also raise the welfare.

According to Blanchard and Watson (1982), if an asset is not reproducible, the bubbles on this asset will simply lead to rents to initial holder. Many assets subject to bubbles are partly reproducible. Blanchard and Watson consider bubbles on housing and on stock market. If a firm is initially in equilibrium, then, the marginal product of capital should be equal to the interest rate. "In absence of bubble, the value of a title to a unit capital, a share, is just equal to the replacement cost and the firm has no incentives to increase its capital". If a bubble starts on a share and increases its price by 10% above market fundamentals, one could think that the firm should add the capital stock until the marginal product of capital is reduced by 10%. The market fundamentals thus decrease by 10% and the share price is again equal to the replacement cost. Initial shareholders have made a profit on the new share issued. The story is similar for housing.

Standard neoclassical theory predicts that investment is inherently tied with the stock market through Tobin's "*q*". The essence of "*q*" theory is the following argument: if the repurchase cost of capital is less than the net present value of additional profits it will bring at the margin, then the company should invest. The only thing preventing the ratio of the two values "known as *q*" from being always equal to 1 is adjustment costs: it is expensive to install new capital and thus a deviation of q from 1 can exist, but it should diminish over time. The link between investment and the stock market is that: the value of a company is the net present value of its profits and thus, whenever one sees the stock market rising, one should simultaneously observe an increase in investment in order to bring the numerator and the denominator of the "*q*" ratio into line.

Employing an overlapping-generation endogenous growth model with a linear technology, Grossman and Yanagawa (1993) proved that asset bubbles reduce the welfare of all generations born after the bubble emerges, while they improve the welfare of the first generation. Olivier (2000) shows that in a small open economy, when the speculative bubbles arise on equity, the market value of the firm increase. Agent can get strong incentives to create new .rms. Bubbles are growth-enhancing. But, if bubbles appear on unproductive asset, then, their effects will be similar to that of Yanagawa and Grossman (1993). Bubbles on unproductive asset raise the equilibrium interest rate and lower the market value of the .rm. Hence, investments and growth fall.

A distinctive characteristic of the U.S. speculative expansion of the 1990s is that it was concentrated in the new technology sector. The stock market price of "new economy" technology and growth companies boomed, while the price of traditional "old economy" companies did not appreciate. In the same period, the share of aggregate investment that went to technology capital experienced a sharp increase. More generally, speculative growth episodes typically have been associated with the expansion of newly emerging sectors of the economy.

Caballero and Hammour (2005), propose a framework for understanding historical episodes of vigorous economic expansion accompanied by extreme asset valuations, as exhibited by the U.S. in the 1990s. They interpret this phenomenon as a "high-valuation equilibrium with a low effective cost of capital based on optimism about the future availability of funds for investment". They show that increased productivity growth provides increased future income, which fuels the key feedback from growth to saving. In Their model, a technological revolution can be considered an integral part - both as cause and consequence - of speculative growth equilibrium. Caballero and Hammour show that such feedback arises naturally when an expansion comes with technological progress in the capital producing sector, when the rest of the world has lower expansion potential,.... These ingredients were all simultaneously present in the U.S. during the 1990s. Caballero and Hammour also show that speculative growth episodes facilitate the emergence of (rational) bubbles. These bubbles can now arise even if interest rates exceed the rate of growth of the economy, and exhibit positive rather than negative co-movement with real investment.

Bubbles also seem to have existed in the commodities market. Since 2007 the world experienced dramatic swings in internationally traded food commodity prices. In June 2008, December 2010 and more recently in the autumn of 2012, food prices increased sharply and subsequently declined from their peak. The prices of many commodities experienced a spectacular run up during the period leading into the recent financial crisis. Gold prices, for example, rose by 500% between 2000 and 2011 before losing a third of their value by 2013.

Standard economic theory suggests that changes in price levels will be governed by the laws of supply and demand. But, for market participants, extreme price swings over protracted periods cannot be justified fundamentally, leading to suggestions that they may arise from speculation. Commodities are seen as an investable asset class, believed to have good diversification benefits, low correlations with stocks and bonds, and good hedging properties against inflation. As a result, many new commodity index funds were established and their activities increased trading volumes and altered the balance of transactions between hedgers and speculators (see, for example, Irwin and Sanders (2012)).

Commodities are core inputs to the production process or are consumption goods. For many media, the behavior of commodities prices, similar to that of a roller coaster, has real consequences. In particular, there have been concerns that price spikes have adversely affected the social welfare of consumers, especially those in developing countries (see, e.g., Leyaro, 2009)), since households there spend a relatively high proportion of their incomes on basic food and energy. Many commentators in the media explicitly laid the blame for the price rises and increased volatility squarely at the door of speculators, arguing that investment banks and funds were immoral to engage in strategies that may have pushed up food prices. In one particularly extreme example, Johann Hadri[6], writing in a blog for the Independent newspaper, argues that Goldman "gambled on starvation".

Persistent food price volatility can also have significant effects, for net food importing developing countries. Rising prices can negatively affect the balance of payments, foreign currency reserves and worsen the ability to implement social safety programs.

3. SOME TYPES OF RATIONAL BUBBLES

3.1. Deterministic bubbles

We have seen in section 2, that arbitrage does not by itself prevent bubbles. A simple kind of bubble is determinist bubbles:

$$B_{t+1} = (1 + R)\, B_t \qquad (4.a)$$

In such bubbles, the higher price is justified by the higher capital gain and deviation grows exponentially. To be rational, such an increase in price must continue forever, making so that a determinist bubble implausible. Everyone thinks that stock price will rise, rise they do, all enjoy their rationally anticipated profit.

The information sets are not present here. But, when we take into account the information sets, we notice that, after they have observed the prices, agents have the same information. Blanchard (1982) show that, bubbles can exist even if agents have different information. If agents have different information, each agent will have his own perception of the fundamental value, according to the equation (3.a), with different information set $F_{t,i}$ replacing F_t. As agents may not have the same fundamental value, they will not perceive the same bubbles. Each agent will have his own perception of bubble. These bubbles must still satisfy the equation (3.*c*).

3.2. Collapsing bubbles

Blanchard and Watson (1982) formulate a speculative bubble model in which the bubble component continues to grow with explosive expectations in the next time period with probability π, or crashes to zero with probability $1 - \pi$. If the bubble collapses, then the actual price will be equal to the asset's fundamental value. In their model, the explosive behavior of bubble returns compensates the investor for the increased risk of a bubble crash, as the bubble grows in size. According to Blanchard and Watson (1982), the expected bubble in period $t + 1$ will be generated by the following stochastic process:

$$B_{t+1} = \begin{cases} \dfrac{(1+R)}{\pi} B_t + \varepsilon_{t+1} & \textit{with the probability}, \ \pi > 0 \\ \varepsilon_{t+1} & \textit{with the probability}, \ 1-\pi > 0 \end{cases} \tag{4.b}$$

$0 < \pi < 1$, B_t obeys to the restriction (3.c). ε_{t+1} is a shock satisfying: $E_t(\varepsilon_{t+1}) = 0$. The Blanchard and Watson bubble's has a constant probability to burst $1 - \pi$, in any period. If the bubble does not burst, it grows at a rate $\dfrac{1+R}{\pi} - 1$, faster than R, in order to compensate the probability of bursting.

From (4.b) we note that if the bubble term at period t crashes to zero, then it cannot regenerate since the expected bubble is equal to zero. This implies that, there can be only one observed bubble in any financial time series (Diba & Grossman, 1988a)). Furthermore, in (4.b) it is assumed that the bubble crashes immediately to its collapsing state value. These are restrictive assumptions, since it is plausible that there could be several bubble episodes in a financial time series or that a bubble could slowly debate for several time periods or it might stop growing and remain at an approximately constant level for some time and then collapse or start growing again. Moreover, the explosive nature of bubbles leads Diba and Grossman (1988b) to conclude that under rational expectations, negative bubbles cannot exist since investors cannot rationally expect the value of a stock to become negative in finite time. This arises since if a negative rational speculative bubble exists, the bubble will grow geometrically causing the stock price to decrease without bound and become negative in finite time. However, Blanchard and Fisher (1989) claim that the arguments against the possibility of negative bubbles rely on a very strict form of rationality. Although the probability that the stock price will become zero or arbitrarily large is positive, this probability might be too small or the event may happen in the too distant future and thus investors decide to ignore it. Finally, in the original model of Blanchard (1979) and Blanchard and Watson (1982), the probability of the bubble continuing to exist is non-observable and assumed constant.

3.3. Intrinsic bubbles

Intrinsic bubbles were proposed by Froot and Obstfeld (1991). As we have seen, equation (1.c) can have a multiplicity solution that depends on exogenous fundamentals. Froot and Obstfeld describe how rational bubbles can arise as nonlinear solutions to linear asset-pricing model. Rational bubbles are typically view as being driven by variables extraneous to the valuation problem. However, some bubbles may only depend on the extraneous fundamental determinants of asset value. Froot and Obstfeld called such bubbles "intrinsic bubbles" because their dynamics are inherited entirely from the fundamentals. Intrinsic bubbles are constructed by finding a nonlinear function of fundamentals that satisfy (3.c). Let us consider the initial intrinsic bubbles model of Froot and Obstfeld. Suppose that the log dividends are generated by geometric martingale, that is:

$$d_{t+1} = \mu + d_t + \varepsilon_{t+1} \tag{5.a}$$

Where μ the trend growth in the dividends is, $d_t = \ln(D_t)$ is the log of the dividends at time t, ε_{t+1} is a normal random variable with mean zero and variance σ^2. Using (5.a) and assuming that period-t dividends are known when P_t is set, we obtain that, the present value of the stock price:

$$P_t^* = kD_t \tag{5.b}$$

where, $k = \dfrac{\exp\left(\mu + \dfrac{\sigma^2}{2}\right)}{1 + R - \exp\left(\mu + \dfrac{\sigma^2}{2}\right)}$. With the assumption that (3.a) converge,

we have: $1 + R > \exp\left(\mu + \dfrac{\sigma^2}{2}\right)$.

Now define $B_t = B(D_t)$ as, $B_t = c(D_t)^{\lambda}$,

where $c > 0$ is an arbitrary constant and λ is the positive root of the quadratic equation:

$$\frac{\lambda^2 \cdot \sigma^2}{2} + \lambda \cdot \mu - \ln(1 + R) = 0$$

It is easy to verify that $B(D_t)$ satisfies (3.*c*):

$$\begin{aligned}
\frac{1}{1+R} E_t[B(D_{t+1})] &= \frac{1}{1+R} E_t[c(D_t)^{\lambda} \exp(\lambda(\mu + \varepsilon_{t+1}))] \\
&= \frac{1}{1+R} c(D_t)^{\lambda} E_t[\exp(\lambda(\mu + \varepsilon_{t+1}))] \\
&= \frac{1}{1+R} c(D_t)^{\lambda} (1 + R) = B(D_t)
\end{aligned}$$

$B(D_t)$ is an example of intrinsic bubbles construct by Froot and Garber (1991). Intrinsic bubbles capture the fact that stock prices overreact to news about dividends, as argued by Shiller (2000).

$$P_t = kD_t + c(D_t)^{\lambda}$$

$$\text{and} \quad \frac{P_t}{D_t} = k + c(D_t)^{\lambda - 1} > k$$

3.4. Periodically collapsing bubbles

Evans (1991) introduces a class of rational bubbles that are always positive but periodically collapse:

$$B_{t+1} = \begin{cases} (1+R)B_t \cdot U_{t+1} & si\ B_t \leq \alpha \\ [\delta + \pi^{-1}(1+R)\theta_{t+1}(B_t - \delta(1+R)^{-1})]\ .\ U_{t+1} & si\ B_t > \alpha \end{cases} \quad (6)$$

where α, β are positive parameters with $0 < \delta < (1 + R)\alpha$; U_{t+1} is an exogenous independently identically distributed random variables; with: $E_t(U_{t+1}) = 1$. θ_{t+1} is an independently identically Bernoulli process (independent to U) which takes the value 1, with the probability π, and 0 with the probability 1 – π; 0 < π < 1. It is straightforward to verify that process (6) satisfies (3.c) and that $B_t > 0$, implies $B_s > 0$ for all $s > t$.

The Evans' model is a generalization of Blanchard's and Watson's (1982) bubble model where the size of collapse as well as their probability depends on the size of the bubble. Furthermore, Evans' periodically collapsing bubbles are always positive and burst after reaching high levels α. The model incorporates partial, rather than total collapses. Since the multiplicative random variable U_{t+1} is strictly positive, the bubble will never change sign; it remains positive and will never completely vanish. Before the bubble size reaches the level α, the probability of collapse is zero and the bubble grows at the mean rate $1+R$. A high value of the parameter α, implies bubbles with a long initial period of relatively steady slow growth. When eventually $B_t > \alpha$, the bubble erupts into a phase in which it grows faster, at the mean rate $\pi^{-1}(1+R)$ as long as the eruption continues. In this phase, the bubble collapses with the probability 1 – π per period. When the bubble collapses, it falls at value of δ and the process begins again.

4. DETECTING BUBBLES

Dating back at least to the alleged tulip mania in Holland in 1636, economists have been intrigued by the possibility of self-fulfilling price bubbles. Assert by Keynes[7], this idea has seen a revival interest as researchers have employed econometrics of rational expectations to test empirically for the existence of such bubbles. The question is: are there bubbles in stock prices? There is a large collection of literature on the empirical tests of asset price behavior. Researchers have argued that stock prices are not consistent with the fundamentals, i.e., with the discounted stream of future dividends. What methods have been used to detected bubbles?

4.1. Variance bounds tests

Variance bounds tests for equity prices were initiated by Shiller (1981) and LeRoy and Porter (1981). Shiller's test only generates point estimates of variances, so statistical significance cannot be tested, whereas LeRoy and Porter treat equity prices and dividends as a bivariate process, constructing estimates of variances with standard errors. LeRoy and Porter's test is essentially a VAR based test of the market fundamental prices, and in this sense is close to the work of Campbell and Shiller (1989). The null hypothesis is that the "market Fundamental" P_t^* defined in (3.a) is the only solution to equation (1.*c*). Under the null hypothesis, P_t^* t is the perfect foresight of P_t^* defined as:

$$P_t = \sum_{i=1}^{+\infty}\left[\left(\frac{1}{1+R}\right)^i D_{t+i}\right] \tag{7.a}$$

Comparing (3.a) and (7.a) we have:

$$P_t^* = E_t\,(P_t) \tag{7.b}$$

which form the basis of variance bounded test:

$$var(P_t^*) \leq var(P_t) \tag{7.c}$$

The variance of the ex-post rational price and the actual price are related by:

$$\text{var}(P_t) = \text{var}(P_t^*) + \frac{1}{1-\beta^2}\text{var}(U_{t+1}) \tag{7.d}$$

where:

$$\beta \frac{1}{1+R} \; and \; U_{t+1} = E_t(P_{t+1} + D_{t+1}) - P_{t+1} - D_{t+1}$$

$(U_{t+1})_{t\geq 0}$ are supposed to be independent random variables, with constant variance over the time. The ideas behind the bound is simple, the variance of a conditional mean of a distribution is less than the variance of that distribution itself. Since P_t^* is the foresight of P_t, the variance of P_t^* should be less than the variance of P_t. P_t is computed from the following recursion:

$$P_t = \frac{P_{t+1} + D_{t+1}}{1+R} \tag{7.e}$$

subject to the condition that the terminal P_T^* is the terminal[8] price P_T. For empirical applications, the process P_t is approximated by assuming a terminal value of P_T where T is today, the last data point, and constructing the P_t series recursively, using observed values of dividends. For the terminal price Shiller (1981)[9] uses the sample average of detrended real price. Although a violation of the variance bound constructed as above might be due to the presence of bubbles, these test have problems with implementation that makes them unsuitable for bubble detection. Some of these are broad problems that are present when variance bounds tests are used to evaluate the present value model, and are not specific to testing for bubbles. Kleidon (1986) assert that, the variances in question are cross-section variances at a point in time, but in estimation time-series variances are used shows that data constructed from the net present value model violates the variance bound when non-stationary time series variances are used. In general, variance bounds tests are tests of the present value model and rejection (even when there are no econometric problems) may be due to any assumption of the model failing. Flavin (1983) made two criticisms of Shiller econometric test. First, the both variance of P_t and of P_t^* are estimated with downward bias in small sample. Furthermore, the effect is more severe for P_t than for P_t^*. The second is that Shiller procedure of calculating an observable version of P_t^* also induces a bias toward rejection.

4.2. West's tests of bubbles

West (1987) has developed a specific test for rational speculative bubbles that is based on estimating the underlying equilibrium model. West's insight was to observe that, in the absence of bubbles, the Euler equation that forms the basis of no-arbitrage asset pricing can be estimated alone, which provides information about the discount rate. Then, if dividends can be represented as an autoregressive process, knowing the discount rate and the parameters of the AR process that governs dividends provides enough information to pin down the relationship between dividends and the market fundamental stock price. The actual relationship between stock prices and dividends can be directly estimated by regressing the stock prices on dividends. Under the null hypothesis, that there are no bubbles, the "actual" relationship should not differ from the "constructed" one. West test begins with the Euler equation (1.c). Equation (1.c) can be estimated in a regression form using observable variables[10]:

$$P_t = \frac{P_{t+1} + D_{t+1} + U_{t+1}}{1+R} \tag{8.a}$$

Where, U_{t+1} is given by the equation (7.d). West uses the Instrumental Variable (IV) estimation of (8.a) to provide an estimate of the discount rate. Notice that this intertemporal relationship between P_t and P_{t+1} is independent of the presence of a bubble. It only asserts that there are no arbitrage opportunities, with or without a bubble. West also assumes that dividends are exogenous and follow a stationary Autoregressive AR(1) process of the form:

$$D_{t+1} = \varphi D_t + U_{t+1} \tag{8.b}$$

The autoregressive parameter is easily recovered by an OLS regression. Given this setup, the market fundamental stock price should be:

$$P_t^* = \delta D_t$$

$$\delta = \frac{\beta \cdot \varphi}{1 - \beta \cdot \varphi}, \quad \beta = \frac{1}{1 + R}$$

The actual stock price, on the right hand, may contain a bubble. P_t is the sum of the market fundamental price P_t^* and possibly a bubble component B_t, which the null hypothesis sets to zero. If the null hypothesis is true, estimating the stock price equation give:

$$P_t = \alpha D_t + B_t \tag{8.c}$$

without taking into consideration a bubble (regressing P_t on D_t) will provide the correct estimate of α. If, however, there exists bubbles in data, and if the bubble is correlated with dividends, the estimate of α in equation (8.c), $\hat{\alpha}$, will be biased. Note that in this setup, $\hat{\alpha}$ will only be biased if the bubble is correlated with dividends and thus the test will detect only this kind of bubble. West's test is able to estimate α in two ways. If the estimated Euler equation in (1.c) correctly characterizes intertemporal asset pricing, and an autoregressive dividend process can be estimated, one estimate of the relationship between dividends and market fundamental stock prices is given by α. The second estimate, $\hat{\alpha}$, is expected to be the same as this in the absence of bubbles, but will differ from δ if bubbles are present in the data. Comparing these two estimates is the essence of West's test of speculative bubbles.

Using a Hausman coefficient restriction test, West strongly rejects the equality of δ and $\hat{\alpha}$ coefficients, indicating the presence of a bubble. Under the null hypothesis of no bubble, the two sets of estimates should be equal. If the null hypothesis is rejected, it indicates the presence of bubbles. The drawback of this test, is that it requires detailed to the specification of underlying equilibrium model. The rejection of the null hypothesis, may not be due to bubbles, but may be instead due to the imposition of a wrong model.

4.3. Cointegration test

An alternative test of bubbles that does not require detailed of the underlying equilibrium model was suggested by Diba and Grossman (1988.b). The ideas behind this test is that, rational bubbles made prices explosive, so if fundamentals are integrated of finite order, then price and fundamentals will not be cointegrated in presence of bubbles. Thus, Diba and Grossman suggest to test for bubbles, by examining the order of integration of price and fundamentals, in particular, whether prices are explosive, by testing if prices and fundamentals are cointegrated.

To understand the cointegration test of Diba and Grossman, we return to the equation (3.a) that defined the fundamentals value of a stock price. Now, consider the market fundamental component of stock price, and assume that the process generating the dividends D_t is non-stationary in level, but, that the first difference of D_t and ε_{t+1} are stationary,

$$\varepsilon_{t+1} = B_{t+1} - (1 + R)\, B_t \qquad (9.a)$$

Where, ε_{t+1} is a random variable that satisfies: $E_{t-j}\,(\varepsilon_{t+1}) = 0$, for all $j \geq 0$. If the bubbles do not exist, then stock prices would be nonstationary in levels, but stationary in first difference. If, however, stock prices contain rational bubbles, then for simple specification of the process generating ε_{t+1}, differencing stock price a finite number of time would not yield a stationary process. Specifically, from (9.a), first difference rational bubbles would have generating process:

$$[1 - (1 + R)L]\,(1 - L)\, B_{t+1} = 1 - L)\, \varepsilon_{t+1} \qquad (9.b)$$

where L denote the lag operator. Diba and Grossman note that for standard simple processes for ε_{t+1} (such as white noise) the first difference of the bubble is generated by a nonstationary and noninvertible process. Indeed, the bubble process is nonstationary regardless of how many differences are taken and this is a property that can be tested econometrically.

But, it is by now well documented that tests for unit roots and cointegration may fail to detect the presence of explosive rational bubbles that collapse periodically. In an important paper, Evans (1991) highlighted the problem by demonstrating that standard unit-root and cointegration tests for asset prices and underlying fundamentals can erroneously lead to acceptance of the no-bubble hypothesis when prices contain an explosive stochastic bubble which collapses from time to time. The essence of the problem lies with the fact that collapsing bubbles only exhibit characteristic explosive bubble behavior during their expansion phase and hence unit-root and cointegration tests are powerful enough to detect the bubble only when its expansion phase lasts for most of the sample period under investigation. Since, therefore, the problem is essentially one of identifying the expanding periods from the collapsing ones, Hall and al. (1998) proposed using a unit-root test based on an autoregressive model with Markov switching parameters (see also, Funke al. al., 1993). Such a test was shown to have considerable power to detect the presence of periodically collapsing rational bubbles in asset prices (see also, Van Norden & Vigfusson, 1998]. The difficulty, however, is that the test procedure is very computer-intensive, since simulation methods need to be used to obtain critical values for the test.

4.4. Regime-switching

Evans' criticism of unit root tests of rational bubbles led to a number of papers trying to overcome the difficulty of detecting collapsing bubbles. Nothing in the above model described by equation (3.a-c), has any implications for regime-switching. Regime-switching come from the descriptions of asset market behavior (for example, those surveyed in Kindleberger, 1989) to which the above model of bubbles is often applied. The first example of regime-switching in the rational speculative bubble framework is Blanchard (1979), who proposes a bubble that moves randomly between two states, C and S. In state C, the bubble collapses, so[11]

$$E_t B_{t+1} \mid C) = 0 \tag{10.a}$$

The state S, the bubble survives and continues to grow. This state occurs with a fixed probability π; [see equation (4.b)]. Since

$$E_t (B_{t+1}) = (1 - \pi) \cdot E_t (B_{t+1} \mid C) + \pi \cdot E_t (B_{t+1}) \mid S) \tag{10.b}$$

It follows from (3.c) that:

$$E_t(B_{t+1} \mid S) = \frac{1+R}{\pi} B_t$$

Later on, this model was generalized by Evans (1991) and Van Norden and Schaller (1993) to consider the case where both the size of collapses and their probability were functions of the size of the bubble. In the Hall and Sola test, this probability is assumed to follow a first-order Markov process, where the probability of remaining in a given regime is constant. Van Norden (1996) modifies the Blanchard model to allow for the possibility that the bubble is expected to collapse only partially in state C by replacing (10.a) with

$$E_t (B_{t+1}) \mid C) = u(B_t) \quad (10.c)$$

Where u( ) is a continuous and everywhere differentiable function such that and $0 \leq u' \leq 1$. Hence, the expected size of collapse will be a function of the relative size of the bubble. They also suggest that the probability, q of the bubble continued growth, falls as the bubble grows, so that

$$q = q(B_t) \;\; and \;\; \frac{dq}{dB_t} < 0$$

Van Norden (1993) show that a .rst-order Taylor series approximation of this process gives the following two-state switching regression system:

$$E_t (B_{t+1}) \mid C) = \alpha_C + \beta_C \cdot B_t$$

$$E_t (B_{t+1}) \mid S) = \alpha_S + \beta_S \cdot B_t$$

$$Prob(State_{t+1} = S) = \Phi(\lambda + \gamma \cdot B_t)$$

$$Prob(State_{t+1}) = C) = 1 - \Phi(\lambda + \gamma \cdot B_t)$$

where the model implies that [$\beta_C > 0$, $\beta_C < 0$ and $\gamma < 0$]. The model can be estimated using the maximum likelihood method. Van Norden and Vigfusson (1998) study the regime switching bubbles tests of Hall and Sola (1993) and Van Norden (1996) and conclude that .even with several hundred observations, the tests show sometimes considerable size distortion.. In their application, the Hall and Sola test, which has constant switching probabilities, suggests the existence of bubbles in the S&P500, but the Van Norden test, which models the switching probabilities as functions of the size of the bubble, does not indicate the presence of a bubble in the same data set. Van Norden and Vigfusson's comparison of Van Norden's test and Sola's test, seems to suggest that the exact choice of the process to be tested does matter. Markov switching tests of collapsing bubbles allow the bubble to switch between two states, but the fundamentals do not change.

It is difficult to distinguish bubbles from regime-switching fundamentals. In many small sample econometrics' problems, bubble tests remain unresolved. There is a large number of papers that propose methods to detect "rational" bubbles. But, the econometrics' tests of detecting bubbles are not very efficient. They combine the null hypothesis of no bubbles with a simple model of fundamentals. Thus, rejection of the present value model that are interpreted by some as indicating the presence of bubbles. But, this rejection can still be explained by alternative structures for the fundamentals. For each paper that finds evidence of bubbles, there is another paper that fits the data as well, without allowing a presence of bubbles. Bubbles are difficult to detect, because they can take many forms and specifying a class general enough to include most of them, make the detection a difficult task. The evidence of rejection the "no-bubble" hypothesis is a strategy to explore for bubbles. But, the rejection the rejection of the null hypothesis of "no-bubble" may be due to other phenomena than bubbles.

5. TRANSVERSALITY CONDITIONS AND BUBBLES

5.1. Theoretical argument

In the section 2, we have only shown that arbitrage condition does not prevent bubbles by itselft. Are there some conditions under which bubbles can be ruled out? Blanchard and Watson (1982) made a discussion on transversality conditions. In Blanchard and Watson's model, bubbles should have the following property:

$$\lim_{T \to +\infty} E_t(B_{t+T}) = \begin{cases} +\infty & if \;\; B_t > 0 \\ -\infty & if \;\; B_t < 0 \end{cases} \tag{11}$$

The condition (11) implies that, there cannot be negative bubbles in Blanchard and Watson's model. A negative value of B_t today implies that there is a positive probability, possibly very small, that at some date $t + T$, B_{t+T} becomes large and negative enough to make the price becomes negative. If the asset cannot be disposed at no cost, its price cannot become negative. Then rationality implies that bubble cannot be negative today for an exchangeable asset. Although the stochastic bubbles have attracted considerable attention, there are both theoretical and empirical arguments that can be used to rule out bubble solution to (1.c). Theoretical arguments may be divided into partial-equilibrium and general-equilibrium arguments.

In partial equilibrium, the first point to note is that there can never be a negative bubble on an asset. If negative bubbles existed, it would imply a negative expected asset price at some date in the future, and this would be inconsistent with the limited liability. A second important point follow from this: A bubble on a limited liability cannot start within an asset pricing model. If bubble exist today, it must has exist since asset trading began. Diba and Grossman (1988.b) argue that any rational bubble that starts after the first date of trading has an expected initial value of zero. The reason is that, if bubble ever has a zero value, its expected future value is zero by the condition (3.c). Third, a bubble cannot exist if there is any upper limit to the price of an asset. Finally, bubble cannot exist on asset such as bonds which has fix value on a terminal date.

General-equilibrium considerations also limit the possibilities for rational bubbles. Tirole (1982) has shown that bubbles cannot exist in a model with finite number of infinite-lived rational agents. His argument is easy to understand when short sales are allowed. If positive bubbles exist in an asset, infinite-live agent could sell the asset short, invest some of proceeds to pay the dividends streams, and have positive wealth left over. This arbitrage opportunity rules out bubbles. Tirole (1985) has studied the possibility of bubbles within the Diamond overlapping-generations model. In his model, there is an infinite number of finite-lived agents. Tirole shows that even here, bubble cannot rise when interest rate exceeds the growth rate of the economy, because bubble would eventually become very large relative to the wealth of the economy. This would violate some agent's budget constraint. Thus the bubbles can exist only in dynamically inefficient overlapping-generations economy that have over accumulated private capital, driving the interest rate down below the growth rate of the economy.

5.2. Technical argument on transversality condition

Transversality conditions together with Euler equation are sometime used to characterize the optimal solution of dynamic models. Stockey and Lucas (1989, p.102) provide the sufficiency of the transversality condition. Michel (1990), for a concave optimal control problem, studies more general transversality conditions that an optimal path has to satisfy. Luigi Montrucchio and Fabio Privileggi (2001) study the existence of bubbles for pricing equilibriums in a pure exchange economy "à la Lucas", with infinitely lived homogeneous agents. They prove that the pricing equilibrium is unique as long as the agents exhibit uniformly bounded relative risk aversion. They also give generic uniqueness result regardless of agent's preferences. Kamihigashi (2002) give a simple proof of the necessity of the transversality conditions. Kamihigashi (1998), Montrucchio and Privileggi (2001) construct a few "pathological" examples of economies exhibiting pricing equilibriums with bubble components. We give here the Kamihigashi's necessary of transversality condition.

Kamihigashi consider the following maximization problem:

$$\begin{cases} \max_{\{x_t\}_{t=0}^{\infty}} \sum_{t=0}^{+\infty} v_t(x_t, x_{t+1}) \\ s.t.\ x_0 = \overline{x}_0, \forall t \geq 0, (x_t, x_{t+1}) \in X_t \end{cases} \tag{12.a}$$

Kamihigashi's result: Under the some assumptions, any optimal interior path $\{x_t^*\}_{t=0}^{\infty}$ satisfies:

$$\lim_{T\to+\infty} -v_{T,2}(x_T^*, x_{T+1}^*)\cdot x_{T+1}^* = 0 \qquad (12.b)$$

Since an interior path satisfies the Euler equation:

$$v_{t,2}(x_t^*, x_{t+1}^*) + v_{t,1}(x_t^*, x_{t+1}^*) = 0 \qquad (12.c)$$

(12.b) is equivalently expressed as:

$$\lim_{T\to+\infty} v_{T,1}(x_T^*, x_{T+1}^*)\cdot x_T^* = 0 \qquad (12.d)$$

When we read this result we do not directly see how transversality conditions rule out asset price bubbles. To be more explicit, one can consider a deterministic version of Lucas (1978) asset pricing model. There are many homogeneous agents, a single good, and a single asset that pays a dividend of D_t of good in each period *t*. The population and the supply of asset are normalized to one. Each agent solves the following problem:

$$\begin{cases} \max\limits_{\{c_t, x_{t+1}\}_{t=0}^{\infty}} \sum_{t=0}^{+\infty} \beta^t \cdot u(c_t) \\ s.t.\ x_0 = 1,\ \forall t \geq 0,\ x_{t+1} \geq 0, c_t + P_t x_{t+1} = (P_t + D_t)x_t \end{cases} \qquad (P)$$

where c_t is consumption, *pt* is the price of the asset, *Dt* is the dividend, and xt is the shares in the asset at the beginning of the period t: In equilibrium, $c_t^* = D_t,\ x_{t+1}^* = 1,\ \forall t \geq 0.$ The Euler equation and the transversality condition for an interior equilibrium are:

$$u'(c_t^*)\cdot P_t = \beta \cdot u'(c_{t+1}^*)\cdot(D_{t+1} + P_{t+1}) \qquad (13.a)$$

$$\lim_{T\to+\infty} \beta^T \cdot u'(c_{t+T}^*)\cdot P_{t+T} = 0 \qquad (13.b)$$

The sequence of price $\{P_t^*\}_{t=0}^{\infty}$ given by:

$$P_t^* = \sum_{i=0}^{+\infty} \beta^i \cdot \frac{u'(c_{t+i}^*)}{u'(c_t^*)} D_{t+i} \qquad (14.a)$$

satisfies the Euler equation (13.a). The right side of (14.a) is called the fundamental value of the asset. If $\{B_t\}_{t=0}^{\infty}$ is a non negative sequence satisfying:

$$u'(c_t^*)\cdot B_t = \beta\cdot u'(c_{t+1}^*)\cdot B_{t+1} \tag{14.b}$$

then the sequence $\{P_t^* + B_t\}_{t=0}^{\infty}$ also satisfies the Euler equation. The extra-component B_t here is interpreted as a bubble. If the transversality condition is necessary, the bubble component must vanish and the price of an asset must always be equal to the fundamental value. In stochastic problems, bubbles can also be ruled out under some conditions. But there are some pathological cases in with bubbles are possible (Kamihigashi, 1998; Montrucchio & Privileggi, 2001).

Note also that, Kamihigashi (2018) establishes a simple no-bubble theorem that applies to a wide range of deterministic sequential economies with infinitely lived agents. He shows that asset bubbles never arise if at least one agent can reduce his asset holdings permanently from some period onward. His no-bubble theorem is based on the optimal behavior of a single agent, requiring virtually no assumption beyond the strict monotonicity of preferences.

6. CONCLUSION

We have seen that the existence of speculative bubbles in financial markets is a controversy problem. Some well reputed economists claim that even the most famous historical bubbles .e.g. the Dutch Tulip Mania from 1634 to 1637, as well as the worldwide new economy boom in the 1990s .can be explained by fundamentally justified expectations about future returns on the respective underlying assets. Speculative bubbles are not ruled out by rational behavior in financial markets. We have also seen that, bubbles likely have real effects on economy. In particular, price spikes on commodities would adversely affected the social welfare of consumers, especially those in developing countries, since households spend a relatively high proportion of their incomes on basic food and energy. But, testing for bubbles is not an easy task. Rational bubble can follow many types of process. This study only described the bubbles and some methods to detect them after the fact. As a next step, it would be desirable to build a model or a statistical test that identify the presence of speculative bubbles in real time.

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Notes

1. "Irrational exuberance" is the phrase used by the then-Federal Reserve Board chairman, Alan Greenspan, in a speech given at the American Enterprise Institute during the dot-com bubble of the 1990s. The phrase was interpreted as a warning that the stock market might be overvalued. Greenspan's comment was made during a televised speech on December 5, 1996.
2. From 2005 through 2008, US financial market showed a rapid increase in commodity futures prices coupled with greater overall trade volumes and larger positions held by commodity index funds. In 2009, asset prices fell to their lowest valuations in more than 20 years.
3. The Chinese housing market had experienced an unprecedented boom since 2008. The price in Shenzhen had nearly quadrupled from January 2008 to June 2017. Some have described this rise in assets price as a speculative bubble.
4. The hypothesis that the expected stock return is constant through time is sometime known as martingale model of stock prices. But a constant expected stock return does not imply a martingale for stock price itself. Recall that a martingale for price requires: $E_t(P_{t+1}) = P_t$, whereas (1.a) and (1.b) imply: $E_t(P_{t+1}) = (1 + R)\, P_t - E_t(D_{t+1})$.
5. We can notice that the second term of the equation (2.b): $B_t = \lim_{T \to +\infty} E_t \left[\left(\frac{1}{1+R} \right)^T P_{t+T} \right]$ satisfies (3.c). P_t is a solution to (1.c) if only if satisfies (3.a), (3.b) and (3.c); there is an infinite number of solutions to (1.c) depending to the form of bubble term.
6. *www.independent.co.uk/opinion/commentators/johann-hari/*.
7. Keynes in a famous passage, described the stock market as a certain type of beauty contest, in which judge try to guess the winner of the contest: speculators consecrate their "intelligence to anticipating what average opinion expects average opinion to be" [1964, p. 156].
8. Under null hypothesis, the terminal condition is: $\lim_{T \to +\infty} \left(\frac{1}{1+R} \right)^T P_{t+T} = 0$
9. Shiller simply replaced P_t by the solution of recursion equation (7.e) that satis.es the terminal condition: $P_T = \frac{1}{T} \sum_{t=1}^{T} P_t$

10. $U_{t+1} = E_t(P_{t+1} + D_{t+1}) - P_{t+1} - D_{t+1}$
11. The notation $E_t(X|C)$ denotes the expectation of X conditional on the fact that the state at t is C and on all other information available at time *t*.

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