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WHICH PLACE FOR MARKET MECHANISMS OR FOR FISCAL STABILIZATION OF PRODUCTIVITY SHOCKS IN A MONETARY UNION?

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ABSTRACT

The current paper provides precise analytical results regarding the stabilization of productivity shocks in a monetary union. We find that in case of productivity shocks, monetary policy should beinactive; their stabilization is mainly the burden of the governments. A productivity shock which reduces relative producer prices and increases the price competitiveness of a given country, can be progressively removed by market mechanisms, but at the cost of higher macro-economic fluctuations in terms of economic activity and inflation. It can also be compensated by fiscal activism, according to the budgetary constraints of the governments; without resorption of differentials in producer prices, economic variables are then better stabilized. We also find that market adjustment to a productivity shock is quicker if the member countries of the monetary union are more homogeneous regarding their budgetary constraints, or for countries with weak budgetary constraints and more budgetary flexibility. Besides, in case of productivity shocks, monetary unification could be more painful for a country with a weaker budgetary multiplier, with a higher sensitivity of its demand to the real exchange rate, or with a weaker sensitivity of its demand to the foreign economic activity.

1. INTRODUCTION

In the framework of a monetary union, the common monetary policy may not be suitable to all member countries, and structural asymmetries may then result in asymmetric business cycles and economic conditions among countries. Asymmetric shocks, asymmetric reactions, structural parameters and various sensitivities to symmetric shocks, to the common monetary policy, to the price competitiveness… result in a higher need of active and flexible budgetary policies, whereas in the framework of the European Economic and Monetary Union (EMU) for example, budgetary constraints have been introduced by the Stability and Growth Pact and the Fiscal Compact. Besides, currently, the EMU doesn't seem to be an 'Optimum Currency Area', as production factors (especially labor) are not really mobile, and as fiscal transfers by the way of the European budget remain very limited, because the political question is too sensitive. Therefore, various asymmetries between the member countries of the monetary union may imply that monetary unification is not equally beneficial for all member countries, and that membership in the monetary union may be more painful for some countries with given structural characteristics.

Indeed, in a monetary union, member countries give up monetary autonomy, and the common monetary authority conducts the monetary policy which is the most suited to the average situation of the whole monetary union. Therefore, fiscal policies remain the only instrument for pursuing country-specific goals and stabilizing asymmetric shocks. Fiscal policies become more important, and they can implement strategic behaviours intended to achieve national goals. In these conditions, the timing and size of a common monetary policy could be difficult to define, if there are diverging interests of the member countries of the monetary union. Besides, if the relative burden of the stabilization is biased between the member countries, the relative advantages or drawbacks of membership in a monetary union could vary between the member countries.

In a monetary union, the single monetary policy can only address common shocks. In the absence of nominal interest and exchange rates as policy instruments, in order to adjust to asymmetric shocks, member countries have to resort to four remaining tools of economic policy. The first one is risk-sharing against country-specific shocks through fiscal transfers and financial integration. However, regarding the 'fiscal federalism' channel, the EMU is not a federation but a union of politically autonomous countries, where the fiscal transfers and the European budget remain very limited [see: Cimadomo *et al.* (2018), for example]. Regarding risk-sharing through 'capital market or credit channels', national income and consumption smoothing via cross-border asset holdings and access to credit market, according to Roubini *et al.* (2007), they would be still much less developed in Europe than in the United-States for the period 1980-2007, even if financial integration has increased since the creation of the EMU. Besides, according to the authors, the higher risk-sharing in the EMU was accompanied by an increase of specialization in production source of business cycle asymmetry. The second economic policy tool is labor mobility, but the latter is also limited in Europe. So, in the current paper, we will mainly consider two adjustment mechanisms which are allowable in the framework of the EMUto stabilize productivity shocks: market-driven price and output adjustment, and fiscal adjustment.

Temporary inflation and output growth differentials are likely in a common currency area since prices and output adjustment is required to absorb shocks. In the Euro Area, output growth and inflation differentials are also related to the ongoing catch-up process in some of the member countries. So, inflation differentials seem to remain important in Europe, even if they have decreased since the creation of the EMU in 1999, and if we can anticipate that they should still decrease in the future; they remain much larger than those observed in the United-States [see for example: Beck *et al.* (2006)]. Besides, these differentials appear as quite persistent, with below average inflation rates in Germany, Austria, France, Finland or Belgium, for example, and above average inflation rates in Ireland, Portugal, Greece or Spain. Productivity shocks can explain part of these inflation differentials in a monetary union.

Indeed, there are still large differences in labor costs and wages' levels among the member countries of the EMU, which have not the same capital intensity. The level of technology or the social and entrepreneurial environment differ among countries. More particularly, productivity levels appear lower in Italy, Spain or Greece, perhaps because of a lower initial level of human capital. Besides, European countries tend to be specialized in specific sectors, and productivity growth differs widely across sectors. As mentioned by Krugman, a stronger economic specialization could then conduct to accentuated productivity shocks and economic growth differentials.

Productivity shocks can have two sources [see for example Duarte and Wolman (2002)]. First, for non-traded consumption goods or services, consumption price indices can differ between countries, as they correspond to distinct baskets of goods. Moreover, productivity gains in a catching-up process tend to accrue mainly in the sector of tradable goods, which is more exposed to competition and more capital intensive. So, higher national productivity in traded goods in more developed countries leads to higher national real wages, also increasing relative prices of non-traded goods. Indeed, due to competition in goods markets and mobility of labor between sectors, nominal wages tend to equalize between the tradable and the non-tradable sectors.This is the classical Balassa-Samuelson effect, expressing the tendency for consumer prices to be systematically higher in more developed countries than in less developed countries, during the process of catching-up. As a result, less developed countries with higher productivity growth in the wake of real income convergence would also be expected to experience higher inflationary tensions than more developed economies with lower rates of productivity growth. This effect is particularly accentuated in case of greater price stickiness. Indeed, nominal price and wage rigidities then avoid the necessary adjustment to the deteriorating competitiveness, and a temporary shock and advantage in terms of relative price competitiveness for less developed countries may then be considered as more persistent.

Second, regarding traded goods, price convergence may be faster in a framework of increased market integration and cross border price transparency. Productivity shocks and imbalances in a monetary union can then be compensated by market mechanisms, according to the efficiency and speed of prices and wages adjustment, which depends on their flexibility. However, even producers of traded goods can price discriminate across markets, and the Law of One Price then doesn't hold. If they choose different prices for various markets, if they don't fix prices at the same mark-up over marginal costs, this implies inflation differentials. They can do so, in particular, if the elasticity of substitution differ across countries, for example because of different tastes. Non-price competitiveness is also a structural component essential to understand trade developments in the Euro area; nevertheless, we will focus on price competiveness in the current paper.

Persistent inflation differentials can last, have negative effects on incomes and investment, and result in divergent competitiveness and cyclical conditions in the member countries of a monetary union. As mentioned by Traistaru-Siedschlag (2006), they imply persistent disparities in real interest rates. So, the latter have then consequences on real return on saving and investment, on nominal incomes and wage setting. Indeed, the real interest rate channel affects domestic demand: expected inflation higher (lower) than Euro Area inflation results in lower (higher) real interest rates which may foster (depress) domestic demand. In parallel, for tradable goods, the competitiveness channel affects the external demand through competitiveness losses (gains): inflation higher (lower) than Euro Area inflation results in real exchange rate appreciation (depreciation), lower (higher) price competitiveness and lower (higher) net exports and external demand. Furthermore, productivity shocks can be asymmetrical, as technological and institutional evolutions are different in the member countries of a monetary union. However, asymmetries may also be related to asymmetric responses to common technological shocks, because of diverging preferences between the governments, or because of structural heterogeneities between the member countries.

In this framework, the contribution of the current paper is to provide an analytical modelling and precise analytical results regarding the stabilization of productivity shocks in a monetary union. We find that in case of productivity shocks, monetary policy should be inactive, at least as long as the member countries of the monetary union are homogeneous; the stabilization of productivity shocks is mainly the burden of the governments. A productivity shock, which reduces relative producer prices and increases the competitiveness of a given country, is beneficial to the net exports of this country, and it increases its global demand. However, inflationary tensions in this country progressively reduce the differential in price competitiveness, and the productivity shock is progressively eliminated by market mechanisms, even if it is at the cost of higher macro-economic fluctuations in terms of economic activity and inflation. However, the budgetary policy can also compensate with internal demand the variation in exports and in external demand, in order to stabilize productivity shocks. In this case, fiscal activism can avoid the necessity to eliminate differentials in producer prices, as long as the budgetary policy is not excessively constrained. Our modelling also provides important and precise analytical results regarding the consequences of structural heterogeneities between the member countries of a monetary union. Indeed, market adjustment to a productivity shock is quicker if the member countries of the monetary union are more homogeneous regarding their budgetary constraints. Otherwise, if the governments remain heterogeneous regarding their preferences, monetary unification could be made easier for countries with weak budgetary constraints and preferences for stabilizing the budgetary deficit.On the contrary, in case of productivity shocks, monetary unification could be made more difficult for a country with a weak budgetary multiplier, with a higher sensitivity of its demand to the real exchange rate, or with a weaker sensitivity of its demand to the foreign economic activity.

The second section of the paper recalls the results of the economic literature regarding the stabilization of productivity shocks in a structurallyheterogeneous monetary union. The third section describes our analytical model. The fourth section studies monetary and budgetary policies intended to stabilize productivity shocks, in case of a homogeneous monetary union, and if we introduce divergences between the preferences of the member countries of the monetary union, or between the structural characteristics of these countries. The fifth section analyses the efficiency of the exchange rate channel and the speed of adjustment of the relative price competiveness of the member countries of a monetary union, as well as the consequences of productivity shocks and of the former economic policies on economic activity and inflation. It also studies the implications of various heterogeneities on economic stabilization, in order to derive implications for the advantages and drawbacks of monetary unification for the various member countries of the monetary union. Finally, the sixth section concludes the paper.

2. THE ECONOMIC LITERATURE

Monetary policy in EMU is conducted by the Governing Council of the ECB with the primary objective of maintaining price stability in the Euro Area as a whole. Therefore, monetary policy doesn't directly deal with differences in inflation rates or other national economic developments among EMU member States. However, some economic studies have underlined the advantages of taking into account national and decentralized information in formulating the common monetary policy. The ECB should advantageously take into account a weighted average of national economic variables and inflation rates, beyond only considering average variables in the monetary union [see for example: Brissimis and Skotida (2008), De Grauwe (2000), or Gros and Hefeker (2000)].Besides, the ECB is indirectly concerned by inflation differentials. Indeed, its goal is to target an inflation rate 'below but close to 2%' for the whole monetary union, which implies taking into account inflation differentials, and avoiding the possibility of negative inflation rates in some member countries. What are the sources of inflation differentials, and should they be taken into account in a monetary union?

First, inflation differentials can be a long run phenomenon due to structural differences between the member countries of a monetary union. For example, even if the prices of each good grow at the same rate in all countries, the resulting aggregate inflation may differ across countries owing to the differing composition of the baskets (composition effect). Structural differences may be related to differences between households' preferences regarding consumption in the various countries, to member countries' exposure to changes in the exchange rate of the euro and in prices of raw materials (openness to trade with non-euro area partner countries), or to rigidities in wages or prices setting (especially in the sector of services). For example, Andrés *et al.* (2003) find that in a monetary union like the EMU, the mechanism of price discrimination by producers (because countries have distinct elasticities of demand in the goods markets) is quantitatively more important than differences in price inertia. Moreover, according to the author, under asymmetric shocks, differences in the degree of openness (preference for foreign goods in consumption) as the ones observed within the EMU can have sizeable effects on the dispersion of inflation rates.Regarding these structural heterogeneities, for a better adjustment of production and labor markets, economic policies may consist in improving the mobility of production factors, and the flexibility of prices and wages setting behavior. However, regarding these long term rigidities, we must observe that obstacles are still important in Europe: structural unemployment is still high, whereas the labor force is weakly and very insufficiently mobile.

Inflation differentials can also be a short run phenomenon due to convergence process, a concern which is more the subject of the current paper. We have mentioned above the classical Balassa-Samuelson effect, expressing the tendency for consumer prices to be systematically higher whereas inflationary tensions and productivity growth are weaker in more developed countries than in less developed countries, during the process of catching-up. However, economic studies differ widely regarding the capacity of productivity shocks to explain inflation differentials, as predicted by the Balassa-Samuelson effect. Indeed, De Grauwe and Skudelny (2002) study empirically a panel data going from 1971 to 1995 for the first EU member States in order to estimate the long run effect of bilateral differences in productivity growth between the traded and non-traded goods sector on bilateral inflation differentials. Their regressions indicate a significant effect of a productivity shock on the inflation differential, as proposed by the Balassa-Samuelson effect, going up to an 8% increase in the inflation differential.

On the contrary, MacDonald and Wójcik (2004) show that for Estonia, Hungary, the Slovak Republic and Slovenia, between 1995 and 2001, administrated (regulated) prices have been a powerful force behind price and real exchange rate developments, much more important than the emphasis often placed on productivity effects, which seem to be relatively benign. So, according to the authors, the inflationary implications of the Balassa-Samuelson effect would remain very small. In the same way, Altissimo *et al.* (2005) observe that between 1990 and 2004, inflation differentials in EMU are mainly concentrated in the services sector. They find that regional asymmetric productivity shocks in the traded sector explain most of the volatility in terms of trade, and variability of the output differential across countries. These shocks are largely absorbed by terms of trade movements in the regional economies. However, they assume that relative variations in productivity in the non-traded sector (and not in the traded sector as for the Balassa-Samuelson effect) are the primary cause of price and inflation differentials, of movements in the real exchange rate, and of variability of real wages.

Besides, Arnold and Kool (2003) analyze stabilization mechanisms in a monetary union, and study regional data for the United-States between 1979 and 2000. They find that, following an increase in the regional inflation rate, in the short run, the pro-cyclical effect through the real interest rate and wealth channels is strongest. In particular, lower regional real interest rates increase regional real growth rates (consumption and investment), whereas higher real wealth accentuates this booming effect on real and nominal housing prices. Nevertheless, after a period of about 3-4 years, the cumulative worsening of the competitive position asserts its influence, and the real exchange rate appreciation can have a contra-cyclical and convergence effect in the long run. However, this effect can take some time, as the elimination of the adjustment through the nominal exchange rate reduces the size and speed with which the real exchange rate adjusts.Gilchrist *et al.* (2018) show that in response to a financial shock, firms in financially weak countries (the periphery) maintain cash flows by raising markups, in order to preserve internal liquidity, while firms in financially strong countries (the core) reduce markups, undercutting their financially constrained competitors to gain market share. When the two regions are experiencing different shocks, common monetary policy then results in a substantially higher macroeconomic volatility in the periphery, compared with a flexible exchange rate regime; this translates into a welfare loss for the union as a whole, but borne entirely by the periphery. The pricing behavior of firms in the core in response to an asymmetric financial shock implies a real exchange rate appreciation for the periphery, which causes an export-driven boom in the core countries and a deepening of the recession in the periphery.

Furthermore, Toroj (2009) underlines that in order to stabilize symmetric or asymmetric shocks in a monetary union, market flexibility is a fundamental parameter. Indeed, variability in real economic variables could be significantly reduced when product and labor markets are flexible enough to allow a quick adjustment of prices to excess demand. According to the author, the adjustment to adverse supply shocks is long and painful when the initial loss in competitiveness is not quickly offset by disinflation due to falling output gap. The greater the rigidities in goods and labor markets, the more demand pressures will tend to be passed through to prices and wages.Besides, in case of aggregate supply disturbances, with sectoral productivity shocks, Sanchez (2007) shows that the stabilization costs of renouncing monetary autonomy diminish with a flatter output-inflation tradeoff, with a larger country size, with more homogeneous supply slopes, and a with higher preference for price stability.

To sum up, in a monetary union, inflation differentials need not necessarily have adverse effects and may even be interpreted as a sound signal, provided they are associated with productivity-based growth. Nevertheless, they also frequently reflect negative factors such as market rigidities that exacerbate the inflationary effects of demand pressures. Inflation differentials across countries can be viewed as a normal feature and as an integral part of the adjustment mechanism of relative prices in a single currency area and, as such, they are not only unavoidable, but also desirable. At the same time, such re-equilibrating mechanisms sometimes appear slow to operate in a common currency area, and some of the persistent divergences observed may indeed be harmful if not seriously addressed by policy-makers.Budgetary situations must then be sufficiently sane to allow the operating of discretionary fiscal policies, able to stabilize asymmetric shocks; budgetary policies mustn't be excessively constrained and shouldn't act in a pro-cyclical way. Indeed, in the framework of a monetary union, the common monetary policy can address the question of average inflation rate in the monetary union, but the governments of the member countries and fiscal policies can only cope with regional inflation divergences.

In this framework, Deroose *et al.* (2004) show that the interaction between real exchange rate adjustment and real interest rate developments may contribute to periods of overheating and overcooling in a monetary union, during which output might be for a number of years either above or below potential. If external demand is the main source of demand imbalances, a price adjustment (deviation of domestic inflation from the euro-average) is the natural instrument to return to equilibrium. A passive market-based real exchange rate adjustment is efficient. However, if the disequilibrium is primarily due to domestic imbalances, the authors assume that an active fiscal policy action might be required. In the same way, Duarte and Wolman (2002) find that from 1995 to 2001, productivity shocks alone were enough to explain the observed volatility in the French-German inflation differential, and that the volatility of the model's inflation differential was little affected by the addition of government spending shocks. Varying country size, they find that smaller countries experience higher variability of their inflation differential in response to shocks to productivity growth. Nevertheless, the authors also underline that regional governments could suppress inflation differentials due to shocks to productivity growth by letting the income tax rate respond negatively to inflation differentials. So, a looser fiscal stance is associated with lower inflation differentials.

Krugman (1993) has widely underlined the difficulties of stabilization of a productivity shock in a monetary union. Indeed, he mentions that economic integration decreases transaction costs and reinforces specialization and geographical concentration. For example, Massachusetts was highly specialized in several narrow high technology sectors; but a demand and technological shock, a decrease in exports and capital outflows (production factors are highly mobile in the US) at the end of the 1990's start the decline of employment in the region. In the same way, according to the author, American style differentiated growth rates could imply specific regional recessions in Europe, if monetary unification is not also accompanied by more fiscal federalism. In this context, the goal of the current paper is to define, with a precise analytical model, the respective roles of market mechanisms and fiscal policies in the stabilization of productivity shocks, and the influence of structural heterogeneities between the member countries of a monetary union on these stabilization mechanisms.

3. THE MODEL

We consider a monetary union made of two countries: (i) and (j). Therefore, this analytical modelling can capture a two-country model; but we can also consider a larger monetary union, where the country (i) faces various partner countries in a monetary union globally represented and named as 'country (j)'. We supposea common monetary policy and a common nominal interest rate fixed by the common central bank in the monetary union, whereas budgetary policies are decentralized: each fiscal policy is defined by the autonomous government of each member country.

All variables are expressed in logarithms, except the interest rate which is in deviation from its long run equilibrium value, normalized to zero for simplicity. Economic variables converge towards their long run equilibrium values, where variation of output is null. We consider global macro-economic demand and supply equations, which could potentially be derived from micro-economic foundations that we will avoid to precise in the current paper. For simplicity, we also abstract from studying external interactions between the member countries of the monetary union and the rest of the world. The real exchange rate dynamics towards its equilibrium value is obviously driven by the well-known Balassa-Samuelson effect; but it is also driven by financial variables like the net foreign assets position. Nevertheless, we will avoid financial variables in the current paper, in order to simplify our model.

3.1. Demand equations

In the countries (*i*) and (*j*), demand equations can take the following expressions:

$$y_{i,t} = \delta_i \left(p^j_{j,t} - p^i_{i,t} \right) + \rho_i y_{j,t} + \gamma_i g_{i,t} - \sigma_i (i_t - \pi_{i,t}) \tag{1}$$

$$y_{j,t} = -\delta_j \left(p^j_{j,t} - p^i_{i,t} \right) + \rho_j y_{i,t} + \gamma_j g_{i,t} - \sigma_j (i_t - \pi_{j,t}) \tag{2}$$

With, in the country (*i*) in period (*t*): $\left(p^i_{i,t} \right)$: producer prices; ($y_{i,t}$): realeconomic activity; ($g_{i,t}$): real budgetary deficit; ($\pi_{i,t}$): inflation rate; (i_t): common nominal interest rate.

Demand increases with public expenditure, and thus with the budgetary deficit. So, (γ) is the sensitivity of economic activity to the fiscal deficit, a parameter which is high in the Keynesian literature (budgetary multiplier), but much weaker in the non-Keynesian tradition. Demand decreases with the real interest rate, favoring sparing and decreasing private consumption. So, (σ) is the sensitivity of economic activity to the real interest rate, to the common monetary policy of the central bank, a negative externality. Indeed, the monetary policy of the common central bank has a direct effect on output through the interest rate channel: a higher nominal interest rate increases the real interest rate in the presence of short-run rigidities in prices. This higher real interest rate then discourages consumption and investment.

Besides, demand also increases with exports, and therefore, with the price-competitiveness of a country. So, (δ) measures the sensitivity of demand to the real exchange rate; it is a measure of the price-competitiveness channel. The real exchange rate is the difference between the home and foreign producer price level. The competitiveness channel (or real exchange rate channel) measures the fact that weaker prices increase the competitiveness of exports of the national country. Demand increases with exports, and therefore, demand also increases with imports'demand from the partner countries in the monetary union. So, (ρ) is a measure of the foreign output channel: a higher economic activity translates to other countries through higher imports, according to the degree of openness of the countries.Toroj (2009) underlines that the output-gap response to price competitiveness is a fundamental factor for economic adjustment. Indeed, he finds that an open and highly trade-integrated economy would be more resistant to asymmetric demand shocks, and that the output-gap would then be less volatile. This resistance would be a combination of both a high degree of trade openness and of a high share of trade volume with the partner countries in the monetary union.

Therefore, by combining equations (1) and (2), demand equations are as follows:

$$(1-\rho_i\rho_j)y_{i,t} = (\delta_i - \delta_j\rho_i)\left(p^j_{j,t} - p^i_{i,t}\right) + \gamma_i g_{i,t} + \rho_i\gamma_j g_{j,t} - \left(\sigma_i + \rho_i\sigma_j\right)i_t + \sigma_i\pi_{i,t} + \rho_i\sigma_j\pi_{j,t} \tag{3}$$

$$(1-\rho_i\rho_j)y_{j,t} = -(\delta_j - \delta_i\rho_j)\left(p^j_{j,t} - p^i_{i,t}\right) + \rho_j\gamma_i g_{i,t} + \gamma_j g_{j,t} - \left(\sigma_j + \rho_j\sigma_i\right)i_t + \rho_j\sigma_i\pi_{i,t} + \sigma_j\pi_{j,t} \tag{4}$$

3.2. Supply equations

We make the hypothesis that in a given country (i), consumers consume a share ($1-\eta_i$) of domestic goods, ($1-\eta_i$) being a measure of home bias in consumption decisions, increasing with the relative size of the national country (i) in the monetary union. In parallel, they consume a share (η_i) of imported goods produced in the foreign countries, (η_i) being an indicator of the degree of openness of the national country, and also of the relative smallness of its size in the monetary union.Therefore, we obtain the following level of consumer prices in the country (i):

$$p_{i,t} = (1-\eta_i)p^i_{i,t} + \eta_i p^j_{j,t} = p^i_{i,t} + \eta_i\left(p^j_{j,t} - p^i_{i,t}\right) \tag{5}$$

With: ($p_{i,t}$): consumer pricesin the country (i) in period (t);$\left(p_{j,t}^{j}-p_{i,t}^{i}\right)$: competitiveness of the country (i) in comparison with country (j), linked to their relative producer prices.

We suppose that the 'law of one price' prevails: 'pricing to market' and discrimination between domestic or foreign producers is not possible. Prices of goods consumed are the same in a given country, whatever the place where they have been produced; there is perfect substitutability between domestic and foreign goods. We can mention that if$\left(\eta_i=\eta_j=\frac{1}{2}\right)$, we have two symmetrical economies, whereas if ($\eta_i = 1$), the country (i) is too small to influence macro-economic variables in the monetary union.

Furthermore, the supply equation in the country (i), the Phillips curve relating national inflation, foreign prices and national output, takes the following expression:

$$\pi_{i,t}=p_{i,t}-p_{i,t-1}=\xi_i y_{i,t}+\zeta_i\pi_{j,t} \tag{6}$$

A positive surprise inflation increases economic activity and production (Lucas function). Or we can also consider that there is a demand-pull inflation, when output increases beyond its potential level and when there is a positive output-gap. Besides, contrary to Engwerda *et al.* (2002) for example, in our model, national prices are influenced by foreign prices, and the supply function is then more complex than a simple linear relation between inflation and national economic activity. Indeed, cost-push inflation can be caused by the foreign inflation spillover. Higher foreign inflation brings about higher prices of imported goods such as raw materials (e.g. oil), intermediate and final goods used in domestic production, and it can also have inflationary consequences on national wage negotiations.

Equation (6)then implies the following supply equations for the countries (i) and (j):

$$(1-\zeta_i\zeta_j)\pi_{i,t}=\xi_i y_{i,t}+\zeta_i\xi_j y_{j,t} \tag{7}$$

$$(1-\zeta_i\zeta_j)\pi_{j,t}=\xi_j y_{j,t}+\zeta_j\xi_i y_{i,t} \tag{8}$$

So, by combining equations (3), (4), (7) and (8), we obtain the following levels of inflation in the countries (i) and (j):

$$\begin{aligned}&\left[(1-\zeta_i\zeta_j)(1-\rho_i\rho_j)+\sigma_i\sigma_j\xi_i\xi_j-(\sigma_i+\sigma_j\rho_i\zeta_j)\xi_i-(\sigma_j+\sigma_i\rho_j\zeta_i)\xi_j\right]\pi_{i,t}\\&\quad=\left[\xi_i(\delta_i-\delta_j\rho_i)-\zeta_i\xi_j(\delta_j-\delta_i\rho_j)-\sigma_j\xi_i\xi_j\delta_i\right]\left(p_{j,t}^{j}-p_{i,t}^{i}\right)+\gamma_j(\zeta_i\xi_j+\xi_i\rho_i)g_{j,t}\\&\quad-\left[\xi_i(\sigma_i+\rho_i\sigma_j)+\zeta_i\xi_j(\sigma_j+\rho_j\sigma_i)-\sigma_i\sigma_j\xi_i\xi_j\right]i_t+\gamma_i(\xi_i+\zeta_i\xi_j\rho_j-\sigma_j\xi_i\xi_j)g_{i,t}\end{aligned} \tag{9}$$

$$\begin{aligned}&\left[(1-\zeta_i\zeta_j)(1-\rho_i\rho_j)+\sigma_i\sigma_j\xi_i\xi_j-(\sigma_i+\sigma_j\rho_i\zeta_j)\xi_i-(\sigma_j+\sigma_i\rho_j\zeta_i)\xi_j\right]\pi_{j,t}\\&=-\left[\xi_j(\delta_j-\delta_i\rho_j)-\zeta_j\xi_i(\delta_i-\delta_j\rho_i)-\sigma_i\xi_i\xi_j\delta_j\right](p^j_{j,t}-p^i_{i,t})+\gamma_j(\xi_j+\zeta_j\xi_i\rho_i-\sigma_i\xi_i\xi_j)g_{j,t}\\&\quad-\left[\xi_j(\sigma_j+\rho_j\sigma_i)+\zeta_j\xi_i(\sigma_i+\rho_i\sigma_j)-\sigma_i\sigma_j\xi_i\xi_j\right]i_t+\gamma_i(\zeta_j\xi_i+\xi_j\rho_j)g_{i,t}\end{aligned} \tag{10}$$

Therefore, putting these equations (8) and (9) in equations (3) and (4), we obtain the following levels of economic activity in the countries (*i*) and (*j*):

$$\begin{aligned}&\left[(1-\zeta_i\zeta_j)(1-\rho_i\rho_j)+\sigma_i\sigma_j\xi_i\xi_j-(\sigma_i+\sigma_j\rho_i\zeta_j)\xi_i-(\sigma_j+\sigma_i\rho_j\zeta_i)\xi_j\right]y_{i,t}\\&=\left[(\delta_i-\rho_i\delta_j)(1-\zeta_i\zeta_j)-(\delta_i\sigma_j+\sigma_i\delta_j\zeta_i)\xi_j\right](p^j_{j,t}-p^i_{i,t})+\gamma_j(\rho_i-\rho_i\zeta_i\zeta_j+\sigma_i\zeta_i\xi_j)g_{j,t}\\&\quad-\left[(\sigma_i+\sigma_j\rho_i)(1-\zeta_i\zeta_j)-\sigma_i\sigma_j\xi_j(1-\zeta_i)\right]i_t+\gamma_i(1-\zeta_i\zeta_j-\sigma_j\xi_j)g_{i,t}\end{aligned} \tag{11}$$

$$\begin{aligned}&\left[(1-\zeta_i\zeta_j)(1-\rho_i\rho_j)+\sigma_i\sigma_j\xi_i\xi_j-(\sigma_i+\sigma_j\rho_i\zeta_j)\xi_i-(\sigma_j+\sigma_i\rho_j\zeta_i)\xi_j\right]y_{j,t}\\&=-\left[(\delta_j-\rho_j\delta_i)(1-\zeta_i\zeta_j)-(\delta_j\sigma_i+\sigma_j\delta_i\zeta_j)\xi_i\right](p^j_{j,t}-p^i_{i,t})+\gamma_j(1-\zeta_i\zeta_j-\sigma_i\xi_i)g_{j,t}\\&\quad-\left[(\sigma_j+\sigma_i\rho_j)(1-\zeta_i\zeta_j)-\sigma_j\sigma_i\xi_i(1-\zeta_j)\right]i_t+\gamma_i(\rho_j-\rho_j\zeta_i\zeta_j+\sigma_j\zeta_j\xi_i)g_{i,t}\end{aligned} \tag{12}$$

3.3. Preferences of the monetary and budgetary authorities

Various goals of the fiscal authorities are to stabilize inflation, economic activity and budgetary deficits. Therefore, the loss function of the government in country (i) is as follows:

$$L^G_{i,t}=E_t\sum_{k=t}^{\propto}\beta^{k-t}\left\{\alpha^{Gi}_\pi\pi^2_{i,k}+\alpha^{Gi}_y y^2_{i,k}+\alpha^{Gi}_g g^2_{i,k}\right\} \tag{13}$$

With: (β): time discount factor; (α^{Gi}_π),(α^{Gi}_y) and (α^{Gi}_g): respective weights given to stabilizing inflation, economic activity and the budgetary deficit.

The parameter (α^{Gi}_g), the necessity to stabilize the fiscal instrument, the budgetary deficit, represents the fact that a high budgetary deficit increases the public debt to be serviced in the future, which is harmful as it increases taxation rates or lowers public spending. The public debt can be a factor increasing inequalities regarding inter and intra-generations income redistribution, increasing tax distortions, or implying crowding-out effects on capital accumulation and private investment. Besides, budgetary deficits and public debt levels have been constrained, in the framework of the Economic and Monetary Union (EMU), by institutional rules of fiscal discipline, and they could potentially lead to financial sanctions in the framework of the Stability and Growth Pact and of the Fiscal Compact.

We also make the hypothesis that there is no systematic incentive to deviate from the long run equilibrium values, no systematic deficit bias; so, we suppose that the long term inflation, economic activity and deficit targets in terms of deviation are null. We limit ourselves to the systematic stabilization of productivity shocks.

In Europe, the main goal of the ECB is to ensure price stability $\left(\alpha_{\pi}^{M}\right)$. However, the ECB can also try to sustain economic activity $\left(\alpha_{y}^{M}\right)$, in the respect of this main goal. Finally, we also consider the empirical goal of interest rate smoothing $\left(\alpha_{i}^{M}\right)$ of any central bank. So, the loss function of the common central bank is as follows:

$$L_t^M = E_t \sum_{k=t}^{\infty} \beta^{k-t} \left\{ \alpha_\pi^M \left(\alpha_i \pi_{i,k} + \alpha_j \pi_{j,k} \right)^2 + \alpha_y^M \left(\alpha_i y_{i,k} + \alpha_j y_{j,k} \right)^2 + \alpha_i^M i_k^2 \right\} \qquad (14)$$

Where (α_i) and (α_j) are the respective weights given to countries (*i*) and (*j*) in the monetary union (economic weights, for example in terms of GDP, or due to a political balance of power).

3.4. Calibration of the parameters of the model

The sensitivity of economic activity to the real interest rate has been calibrated at (σ = 0.2) in Van Aarle *et al.* (2001), at (σ=0.4) in Engwerda *et al.* (2002), and at (σ=0.5) in Beetsma *et al.* (2001).Recent empirical studies differ regarding the precise value of this parameter, even if the results of our paper strongly depend on our calibration. Therefore, in this paper, we will consider (σ=0.4) as basic calibration, but we will allow a large variation of this parameter ($0<\sigma<0.8$) in order to analyze the sensitivity of our results to a variation of this parameter. We use very large variations of all parameters of our model to cover quite extreme potential variations of these parameters (see Table 1), in order to avoid the dependence of our results on this calibration. Regarding divergences between European countries,many econometric studies analyze the various channels of monetary transmission in individual European countries and examine country characteristics that may explain divergences, such as the structure of the financial system (financial stability and depth, banks' concentration, availability of alternative financing, development of capital markets and non-bank financial intermediaries, lending maturities), openness, price and wage rigidity (barriers to entrepreneurship, employment protection legislation), interest rate sensitivity to demand (industrial structure) and households' and firms' portfolio composition. The main conclusion is that there is considerable dispersion across countries. However, the studies do not give clear results with respect to the ranking of countries on the basis of monetary policy effectiveness.

Indeed, for example, Mojon and Peersman (2003) assess that according to various VAR models across economic studies, differences of the effects of monetary policy on GDP are not significantly robust between European countries. Nevertheless, over the period 1980-1998, they estimate that the GDP response to monetary policy would be (σ=–0.45) in the Netherlands, (-0.44) in Finland, (-0.32) in Belgium or in Ireland, (-0.25) in Austria, (-0.20) in Germany or in France, (-0.14) in Spain, (-0.12) in Italy, (-0.08) in Portugal. In the same way, Angeloni *et al.* (2003) report that for the period 1971-2000, after one year, the GDP response to monetary policy based on the Euro-system macro-econometric models would be on average (σ=-0.19) in the Euro Area, mainly due to the decrease in investment. It would be (-0.54) in Germany, (-0.51) in Italy or in Greece, (-0.43) in Finland, (-0.33) in Belgium, (-0.29) in Austria, (-0.28) in the Netherlands, (-0.27) in Ireland, (-0.26) in Spain, (-0.24) in France, (-0.13) in Portugal, and (-0.07) in Luxembourg.

The sensitivity of economic activity to the budgetary deficit has been calibrated at (γ=0.5) in Beetsma *et al.* (2001), and at (γ=1) in Van Aarle *et al.* (2001), Van Aarle *et al.* (2004), or in Engwerda *et al.* (2002). In this paper, we will consider (γ=1) as basic calibration, but we will allow a large variation of this parameter ($0<\gamma<1.5$), in order to analyze the sensitivity of our results to variations of this parameter (see Table 1). According to the European Commission (2002, p.41), Interlink simulations give values between (γ=0.6) in France, (γ=0.9) in Italy, and (γ=1) in Germany or in the United-Kingdom. In the same way, the European Commission (2012, p.139) tries to estimate fiscal multipliers (γ), the impact of budgetary changes in revenues and expenditures on real GDP, with a VAR analysis, for the period 1985-2010. It finds that it would only be around (γ=0.3) for Italy after one year, but around (γ=1.2) for Germany and Spain, while the average value would be around (γ=1.4) for the whole Euro Area.

Regarding variations of the budgetary multiplier between European countries, Kilponen *et al.* (2015) provide estimates of output multipliers for alternative fiscal instruments obtained by simulating fifteen structural models of the European System of Central Banks. They find that country-specific short-run fiscal multipliers are in general negative but smaller than one in absolute value. More precisely, without Zero Lower Bound constraint for the interest rate, the first year short-run fiscal multipliers related to a temporary reduction in government consumption of 1% of baseline (before shock) GDP, would be as follows: Belgium (-0.93), France (-0.92), Greece (-0.90), Estonia (-0.83), Italy (-0.79), Finland (-0.78),Portugal (-0.76), Netherlands (-0.74), Malta (-0.73), Slovenia (-0.66), Sweden (-0.60), Czech Republic (-0.54), Germany (-0.52), Spain (-0.50). Besides, temporary reductions in government consumption are typically associated with larger short-run GDP effects than temporary increases in tax rates. Furthermore, the existence of a Zero Lower Bound for monetary policy (constant interest rate for two years) only strongly increases the size of multipliers if the fiscal tightening is simultaneously implemented in the euro area as a whole; otherwise, it remains below one.

The sensitivity of national economic activity to the foreign economic activity has been calibrated at ($\rho=0.4$) in Engwerda *et al.* (2002) or in Van Aarle *et al.* (2001), and at ($\rho=0.5$) in Beetsma *et al.* (2001) or in Van Aarle *et al.* (2004). In this paper, we will consider ($\rho=0.4$) as basic calibration, but we will allow a large variation of this parameter ($0<\rho<0.8$) in order to analyze the sensitivity of our results to a variation of this parameter (see Table 1).

The sensitivity of the demand to the real exchange rate has been calibrated at ($\delta=0.2$) in Engwerda *et al.*(2002), ($\delta=0.25$) in Van Aarle *et al.* (2004), ($\delta=0.3$) in Van Aarle *et al.* (2001), and at ($\delta=0.5$) in Beetsma *et al.* (2001). In this paper, we will consider ($\delta=0.3$) as basic calibration, but we will allow a large variation of this parameter ($0<\delta<0.6$) in order to analyze the sensitivity of our results to a variation of this parameter (see Table 1). Regarding divergences between European countries, according to Bussière *et al.* (2016, p.38) for example, the estimates of the effect of a 1% depreciation of the exchange rate on net exports over GDP (δ) are ranged between: only 0.03 in the United-Kingdom, 0.06 in Greece, 0.07 in Italy, 0.08 in France, 0.11 in Germany, in Finland or in Portugal,0.12 in Spain, 0.14 in Austria, 0.22 in Denmark,0.24 in Poland, 0.30 in the Czech Republic, 0.32 in Belgium, 0.42 in the Netherlands or inHungary, and until 0.68 in Ireland where openness to trade is very high. For the period between 1998 and 2008, Toroj (2009) finds that the influence of price-competitiveness on the output-gap would be quite insignificant in Italy, in Portugal, in Spain, in France or in Finland. It would be 0.03 in Austria or in the Netherlands, 0.04 in Greece, 0.11 in Luxembourg, 0.12 in Germany, 0.19 in Belgium, but it would reach 0.58 in Ireland.

The parameter (ξ) measures the slope of the Phillips curve, and reflects the rigidities in the prices adjustment dynamics. The sensitivity of prices to the national economic activity has been calibrated at ($\xi=0.2$) in Van Aarle *et al.* (2004), and at ($\xi=0.25$) in Engwerda *et al.* (2002) or in Van Aarle *et al.* (2001).In this paper, we will consider ($\xi=0.25$) as basic calibration, but we will allow a large variation of this parameter ($0< \xi <0.5$) in order to analyze the sensitivity of our results to a variation of this parameter (see Table 1). Regarding divergences between European countries, Brissimis and Skotida (2008) find, between 1965 and 1998, that this sensitivity would be much higher in France (around $\xi=0.25$) than in Germany (around $\xi=0.04$), where the degree of price stickiness is higher. Dyne *et al.* (2009, p.36) also observe that, for a period between 1996 and 2003 for most countries, the monthly frequency of prices changes are much higher for energy prices (oil products) than for services. Besides, they find that monthly prices changes would be more frequent in Luxembourg (23%), in Portugal (21.1%), in France (20.9%), in Finland (20.3%), in Belgium (17.6%), in the Netherlands (16.2%), in Austria (15.4%) and less frequent in Germany (13.5%), in Spain (13.3%) or in Italy (10%).

The sensitivity of national prices to foreign prices has been calibrated at (ζ=0.1) in Van Aarle *et al.* (2004) and at (ζ=0.8) in Van Aarle *et al.* (2001). In this paper, we will consider (ζ=0.5) as basic calibration, but we will allow a large variation of this parameter (0< ζ <1) in order to analyze the sensitivity of our results to a variation of this parameter. Regarding divergences between European countries, according to Bussière *et al.* (2016, p.32) for example, the average elasticity of import prices with respect to the exchange rate would be (ζ=0.48). It ranges between incomplete pricing to market strategy and 0.28 in Belgium, 0.35 in Spain, 0.37 in Denmark, 0.38 in Germany, 0.41 in Austria or in Italy, 0.44 in France or in Poland, 0.46 in the Czech Republic, 0.47 in Ireland, 0.48 in Finland, in Netherlands or in the United-Kingdom, 0.49 in Portugal, 0.62 in Greece, and until 0.71 and incomplete pass- through in Hungary.

The share of foreign goods consumed in the national country (η) increases with the degree of openness and decreases with the size of the country. In this paper, we will consider (η=0.3) as basic calibration, but we will allow a large variation of this parameter (0< η<0.6) in order to analyze the sensitivity of our results to a variation of this parameter (see Table 1).

Table 1: Calibration of the parameters of our model

	Basic calibration	*Potential variation*
Sensitivity of demand to the real interest rate	$\sigma=0.4$	$0 < \sigma < 0.8$
Sensitivity of demand to the fiscal deficit	$\gamma=1$	$0 < \gamma < 1.5$
Sensitivity of demand to the foreign activity	$\rho=0.4$	$0 < \rho < 0.8$
Sensitivity of demand to the real exchange rate	$\delta=0.3$	$0< \delta <0.6$
Sensitivity of national prices to national activity	$\xi=0.25$	$0 < \xi < 0.5$
Sensitivityof national to foreign prices	$\zeta=0.5$	$0 < \zeta < 1$
Share of foreign goods consumed	$\eta=0.3$	$0 < \eta < 0.6$
Budgetary preference for stabilizing inflation	$\alpha_\pi^G = 2$	$0<\alpha_\pi^G<\infty$
Budgetary preference for stabilizing activity	$\alpha_y^G = 5$	$0<\alpha_y^G<\infty$
Budgetary preference for stabilizing public finances	$\alpha_g^G = 2.5$	$0<\alpha_g^G<\infty$
Monetary preference for stabilizing inflation	$\alpha_\pi^M = 2.5$	$\alpha_y^M<\alpha_\pi^M<\infty$
Monetary preference for stabilizing activity	$\alpha_y^M = 1$	$0<\alpha_y^M<\alpha_\pi^M$
Monetary preference for stabilizing the interest rate	$\alpha_i^M = 2.5$	$0<\alpha_i^M<\sim$

Regarding the preferences of the European Central Bank (ECB), the main goal mentioned in its statutes is to stabilize inflation, whereas empirical studies show that interest rate smoothing would be quite significant. So, in Engwerda *et al.* (2002), the common central bank cares more about inflation than about output stabilization $\left(\alpha_{\pi}^{M}=2.5>\alpha_{y}^{M}=1\right)$, and it has also an interest rate smoothing objective $\left(\alpha_{i}^{M}=2.5\right)$. Beetsma *et al.* (2001) also consider: $\left(\alpha_{i}^{M}=\alpha_{\pi}^{M}\right)$. We will retain these values in our own basic calibration (see Table 1). In Van Aarle *et al.* (2001), the relative shares of the former objectives are: $\left(\frac{\alpha_{y}^{M}}{\alpha_{\pi}^{M}}=0.6\right)$, whereas in Van Aarle *et al.* (2004), they are: $\left(\frac{\alpha_{y}^{M}}{\alpha_{\pi}^{M}}=\frac{1}{3}=0.33\right)$. Besides, in our basic calibration, we will consider that each country is equally weighted by the common central bank: ($\alpha_i = \alpha_j = 0.5$).

Nevertheless, Debortoli *et al.* (2020) show that simple loss functions should feature a high weight on measures of economic activity, sometimes even larger than the weight on inflation; the optimal value of α_{y}^{M} would be slightly above 1. Indeed, they show that the output-gap summarizes all the welfare-relevant frictions in the goods and labor markets, and that such a goal (like for the FED) would be welfare maximizing. Furthermore, Zacek (2020) finds that the best rule in terms of the lowest value of the central banks' loss function would be a rule augmented with asset prices, even if the results are shock- and model-dependent. Therefore, we have chosen to allow of large variation of the preferences of the central bank.

Regarding the preferences of the governments, Engwerda *et al.* (2002) use the following parameters: $\left(\alpha_{\pi}^{Gi}=2\right)$, $\left(\alpha_{y}^{Gi}=5\right)$ and $\left(\alpha_{g}^{Gi}=2.5\right)$. We will retain these values in our own basic calibration (see Table 1), but we will allow a very large variation of these values in order to analyze the sensitivity of our results to these governmental preferences. In Van Aarle *et al.* (2001), the relative shares of the former objectives are: $\left(\frac{\alpha_{y}^{Gi}}{\alpha_{\pi}^{Gi}}=1.5 \textit{ and } \frac{\alpha_{g}^{Gi}}{\alpha_{\pi}^{Go}}=0.8\right)$, whereas in Beetsma *et al.* (2001), they are: $\left(\frac{\alpha_{y}^{Gi}}{\alpha_{\pi}^{Gi}}=2\right)$.

Furthermore, what are the potential heterogeneities between the European countries, regarding these preferences of the governments? With an econometrical study for the period 1979-1998, Ballabriga and Martinez-Mongay (2002) find that there would mainly be a large fiscal policy inertia associated with a weak weight given to economic activity stabilization in Ireland, in Italy or in Germany, while on the contrary, in France, in the Netherlands, in Austria or in Portugal, there would be much more fiscal policy fluctuations associated with a higher weight given to output-gap stabilization. Besides, responses to output-gap fluctuations would also be much more important in the Nordic Countries. In a more recent paper, Mohl *et al.* (2019) study the two parts of automatic stabilizers in the European countries: cyclical revenues (such as income and indirect taxes) and rather a-cyclical expenditure (such as unemployment benefits). They mention that in 2018, the fiscal semi-elasticity, which measures by how many percentage points the budget surplus increases following a 1% increase in GDP, would be: 0.3% in Bulgaria, 0.32% in Romania, 0.38% in Latvia or Slovakia, 0.40% in the Czech Republic or in Lithuania, 0.44% in Croatia, 0.45% in Hungary, 0.46% in Luxembourg, 0.47% in Slovenia, 0.48% in Malta, O.49% in Estonia, 0.5% in Poland, Cyprus or Germany, 0.52% in Ireland or Greece, 0.54% in Portugal or Italy, 0.55% (the EU28 average) in the United-Kingdom or in Sweden, 0.57% in Austria, 0.58% in Finland, 0.59% in Denmark, 0.6% in Spain, 0.61% in the Netherlands, 0.62% in Belgium and until 0.63% in France. So, it appears that the semi-elasticities of both expenditure and budget balance would be smaller in central and eastern European countries, which have on average lower expenditure to-GDP ratios.

4. MONETARY AND BUDGETARY POLICIES INTENDED TO STABILIZE PRODUCTIVITY SHOCKS

The main contribution of the current paper is to provide a very precise analytical study of the consequences of productivity shocks in a monetary union, where preferences of the governments and structural parameters can differ between member countries.

Appendix A derives the equilibrium monetary and budgetary policies in case of productivity shocks. So, we obtain:

$$\frac{\partial i_t}{\partial\left(p_{j,t}^{j}-p_{i,t}^{i}\right)}=\frac{\Delta_{4,i}}{\Delta_1} \tag{15}$$

$$\frac{\partial g_{i,t}}{\partial\left(p_{j,t}^{j}-p_{i,t}^{i}\right)}=-\gamma_i\left[\alpha_g^{Gj}\left(\delta_i\sigma_j+\sigma_i\delta_j\right)\frac{\Delta_{5,i}}{\Delta_1}+\alpha_i^M\frac{\Delta_{6,i}}{\Delta_1}\right] \tag{16}$$

$\Delta_1=\Delta_3+\alpha_i^M\Delta_2>0$; $\Delta_2>0$; $\Delta_3>0$; $\Delta_{4,i}$; $\Delta_{5,i}>0$; $\Delta_{6,i}>0$ are defined in Appendix A.

Therefore, equation (15) shows that monetary policy is quite inactive in case of productivity shocks [$\Delta_{4,i}\sim 0$ and $\frac{\partial i_t}{\partial\left(p_{j,t}^{j}-p_{i,t}^{i}\right)}\xrightarrow[homogeneity]{}0$], at least as long as the member countries of the monetary union are homogeneous. The stabilization of productivity shocks is mainly the burden of the governments and of budgetary policies. So, after such shocks, the speed of removal of differences in producer prices, and of reduction of deviations of relative competitiveness between the member countries, depends on the behavior of the governments. Indeed, by combining the previous equations, we can obtain the dynamic evolution of the differential in producer prices, in the relative competitiveness of the countries (*i*) and (*j*), between the periods (*t*–1) and (*t*) (see Appendix B). So, we obtain:

$$\left(p_{j,t}^{j}-p_{i,t}^{i}\right)=\frac{\left(1-\eta_i-\eta_j\right)\Delta_1}{\left[\left(1-\eta_i-\eta_j\right)\Delta_1+\Delta_7\right]}\left(p_{j,t-1}^{j}-p_{i,t-1}^{i}\right) \tag{17}$$

where $\Delta_7 > 0$ is defined in Appendix B.

We will first mention the stabilization of productivity shocks in case of homogeneity between the member countries of the monetary union, before analyzing the implications of the existence of various structural heterogeneities between these countries.

4.1. The case of homogeneous member countries

In case of structural homogeneity between the member countries of the monetary union, the stabilization of productivity shocks doesn't depend on preferences of the monetary authority, and on the weight given to each member state by the common central bank (for example, on their difference in terms of size and of relative GDP). The stabilization only depends on the preferences of the budgetary authorities. A productivity shock which reduces relative producer prices and increases the competitiveness of the country (i) $\left(p_{j,t}^{j}-p_{i,t}^{i}\right)$ is beneficial to the net exports of this country, and it increases its global demand. So, according to equations (9) to (12), it implies expansionary and slightly inflationary tensions in the country (i), while it implies recessionary and slightly deflationary tensions in the country (j). However, the budgetary policy can compensate with internal demand the variation in exports and in external demand, in order to stabilize productivity shocks. Indeed, we obtain:

$$\frac{\partial g_{i,t}}{\partial\left(p_{j,t}^{j}-p_{i,t}^{i}\right)}\xrightarrow[homogeneity]{} -\frac{\delta}{\left\{\gamma+\frac{\alpha_g^G[(1+\zeta)(1+\rho)-\sigma\xi][(1-\zeta^2)(1-\rho^2)-\sigma\xi(2+2\rho\zeta-\sigma\xi)]}{\gamma\left[\alpha_y^G(1+\zeta)(1-\zeta^2-\sigma\xi)+\alpha_\pi^G\xi^2(1-\xi\sigma+\zeta\rho)\right]}\right\}} \tag{18}$$

$$\left(p_{j,t}^{j}-p_{i,t}^{i}\right)\xrightarrow[homogeneity]{}\frac{1}{\left[1+\frac{2\delta\xi}{(1-2\eta)\Delta_8}\right]}\left(p_{j,t-1}^{j}-p_{i,t-1}^{i}\right) \tag{19}$$

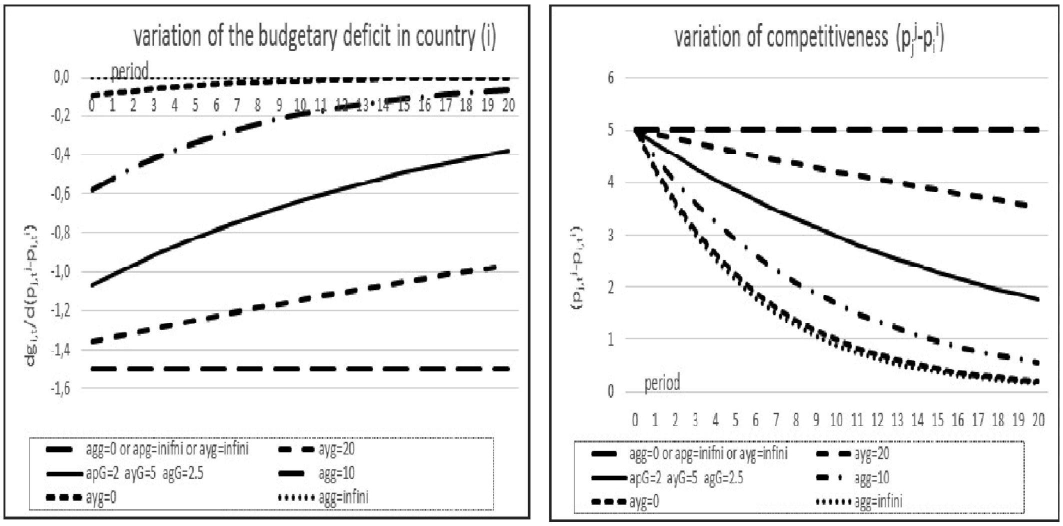
$$\Delta_8 = [(1+\zeta)(1+\rho)-\sigma\xi] + \frac{\gamma^2\left[\alpha_y^G(1-\zeta^2-\sigma\xi)(1+\zeta)+\alpha_\pi^G\xi^2(1-\xi\sigma+\zeta\rho)\right]}{\alpha_g^G[(1-\zeta^2)(1-\rho^2)-\sigma\xi(2+2\rho\zeta-\sigma\xi)]} > 0$$

So, according to equation (18), the budgetary policy is contractionary and the budgetary surplus increases (or the budgetary deficit decreases) in the country (i), where a higher price competitiveness tends to increase net exports [$\partial g_{i,t} \to -1.07\%$ with a basic productivity shock of $\left(p^j_{j,t} - p^i_{i,t}\right) = 5\%$; estimations made with EXCEL]. To the contrary, the budgetary policy must compensate for a weaker competitiveness, it is expansionary and the budgetary deficit increases in the country (*j*) where producer prices are higher. Besides, these budgetary policies are more active if the weight given to stabilizing economic variables is accentuated[$\partial g_{i,t} \xrightarrow[\alpha_\pi^G\to\infty]{} \partial g_{i,t} \xrightarrow[\alpha_y^G\to\infty]{} \partial g_{i,t} \xrightarrow[\alpha_g^G\to 0]{} -\frac{\delta_i}{\gamma_i}\left(p^j_{j,t} - p^i_{i,t}\right) = -1.5\%$ for a basic productivity shock of 5%], whereas the weight given to stabilizing the budgetary deficit and budgetary constraints are minimal (see Figure 1). If budgetary policies could fully compensate for the consequences of productivity shocks and were very active, if internal public demand could fully compensate for the variation in net exports, productivity shocks and differentials in producer prices would even have no reason to be removed [$\left(p^j_{j,t} - p^i_{i,t}\right) \xrightarrow[\alpha_\pi^G\to\infty \text{ or } \alpha_y^G\to\infty]{} \left(p^j_{j,t-1} - p^i_{i,t-1}\right)$ according to (19)].

To the contrary, budgetary policies are obviously less active if the governments give a weaker weight to stabilizing prices or economic activity[$\partial g_{i,t} \xrightarrow[\alpha_\pi^G\to 0]{} -1.06$ and $\partial g_{i,t} \xrightarrow[\alpha_y^G\to 0]{} -0.10$ with our basic calibration], whereas they give a higher weight to stabilizing the budgetary deficit[$g_{i,t} \xrightarrow[\alpha_g^G\to\infty]{} 0$]. In these conditions, the higher price competiveness of the country (i) is not sufficiently compensated by a more contractionary budgetary policy. Expansionary and inflationary tensions in the country (i) quickly deteriorate the price-competitiveness of the country, reduce net exports and the excessive global demand, and the differential in price competiveness between countries is quickly removed.

Furthermore, the inactivemonetary policy is insufficiently contractionary for the country (i) with an advantage in terms of relative price competitiveness, whereas it is insufficiently expansionary for the country (j). Therefore, budgetary policies intended to stabilize productivity shocks are positively correlated with the strength of the transmission mechanisms of monetary policy (σ). Indeed, the budgetary policy is less contractionary in the country (i) and it is less expansionary in the country (j) if transmission mechanisms of

Figure 1: Variation of competitiveness and budgetary policy according to the preferences of the governments

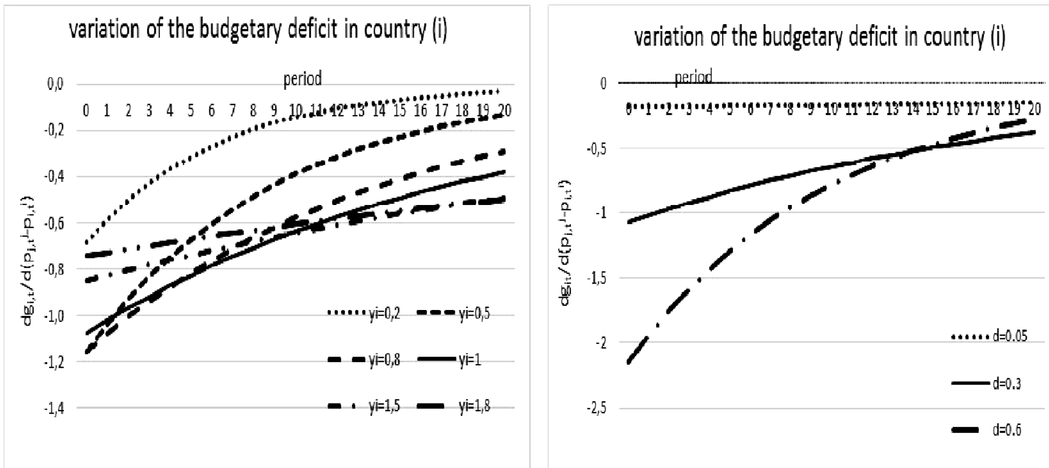


Calibration: basic calibration of structural parameters, for an initial productivity shock ($p^j_{j,0} - p^i_{i,0} = 5\%$)

monetary policy (σ) are weak; the removal of the differential in relative price competitiveness and of the productivity shock is then quicker. On the contrary, budgetary policies are more active, and the adjustment is slower, if the transmission mechanisms of monetary policy are strong.

Besides, a productivity shock destabilizes all the more economic variables as the sensitivity of demand to the real exchange rate (δ), as the sensitivity of demand to the foreign activity (ρ), or as the sensitivity of national to foreign prices (ζ) are high. Therefore, budgetary policies are positively correlated with these sensitivities: they can be less active if (δ), (ρ) or (ζ) are weak (see Figure 2). The speed of removal of a productivity shock is then slowerif (δ) is weak, whereas it isquicker if (ρ) or (ζ) are weak. Budgetary policies are also positively correlated with the home bias and with the share of national goods and services in domestic consumption (1-η): budgetary policies can be less active if (η) is high, and the speed of adjustment is then quicker. Results are more ambiguous regarding the sensitivity of national prices to national economic activity (ξ), or regarding the budgetary multipliers (γ). Regarding the impact of the fiscal policy on economic activity, the first periods are ambiguous; nevertheless, budgetary policies become after some time less active if they are less efficient to impact economic activity and if (γ) is weak (see Figure 2). The speed of adjustment and of removal of a productivity shock is then quicker if (γ) is weak.

Figure 2: Budgetary policy according to the sensitivity of demand to the real exchange rate or to the budgetary multiplier



Calibration: basic calibration of structural parameters, for an initial productivity shock $(p_{j,0}^{j} - p_{i,0}^{i} = 5\%)$

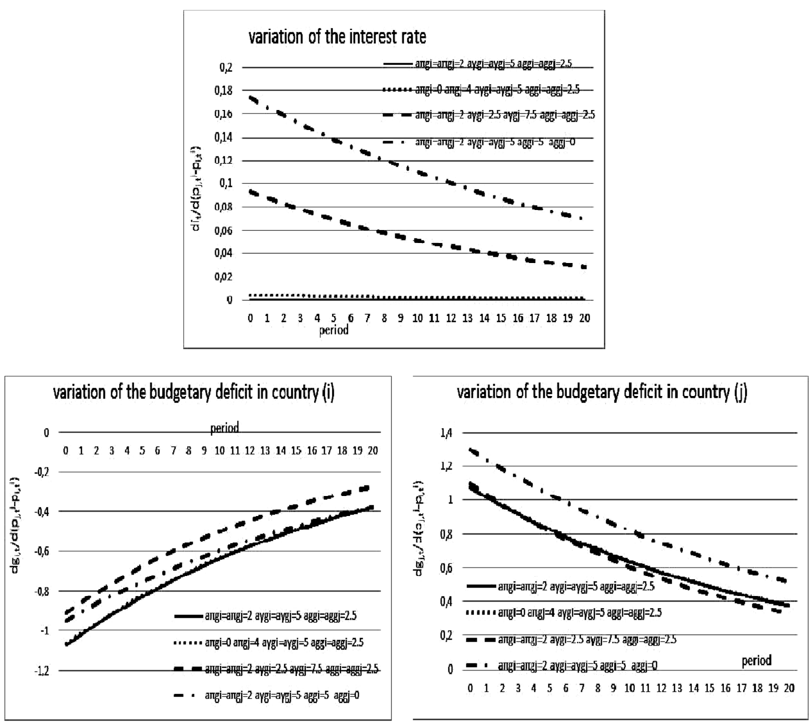
4.2. Heterogeneity in preferences ofthe governments

After a productivity shock, monetary policy is quite inactive in case of homogeneity between the member countries of the monetary union. However, in case of heterogeneity between the preferences of the member countries, monetary policy should react to such shocks [see ($\Delta_{4,i}$) in Appendix A)]. In particular, after a productivity shock, monetary policy is contractionary and the nominal interest rate increases ($i_t > 0$) if relative producer prices increase in the country (j) with the weakest preference for stabilizing the budgetary deficit$\left(\alpha_g^{Gi} > \alpha_g^{Gj}\right)$ [Then ($\Delta_{4,i} > 0$)] (see Figure 3). Indeed, if $\left(p_{j,t}^{j} > p_{i,t}^{i}\right)$, the budgetary policy is then likely to be more expansionary in the country (j) than it is contractionary in the country (i). In this context, if$\left(\alpha_g^{Gi} > \alpha_g^{Gi}\right)$, the contractionary monetary policy allows more expansionary budgetary policies. Budgetary policy is less contractionary in the country (i) [($g_{i,t}$)) decreases less, because ($\Delta_{6,i}$) decreases, even if (Δ_1) also decreases] while the budgetary policy is more expansionary and the budgetary deficit is higher in the country (j) ($g_{j,t}$ increases more) (see Figure 3).

After a productivity shock, monetary policy is also contractionary and the nominal interest rate increases ($i_t > 0$) if relative producer prices increase in the country (j) with a higher preference for stabilizing economic activity $\left(\alpha_y^{Gi} < \alpha_y^{Gj}\right)$ [Then ($\Delta_{4,i} > 0$)] (see Figure 3). If the preferences of the governments are heterogeneous $\left(\alpha_y^{Gi} \neq \alpha_y^{Gj}\right)$, part of the stabilization of the productivity shock is realized by the monetary authority, and budgetary policies can then be less active. So, the budgetary policy is slightly less contractionary in the country (i) [($g_{i,t}$) decreases less, because ($\Delta_{6,i}$) decreases, even if ($\Delta_{5,i}$) increases and if (Δ_1) decreases], while the budgetary policy can be slightly less expansionary and the budgetary deficit can be weaker in the country (*j*) ($g_{j,t}$ increases less). In these conditions, we can also underline that the budgetary policy can be less active mainly in the country (i) with the weakest preference for stabilizing economic activity $\left(\alpha_y^{Gi} < \alpha_y^{Gj}\right)$ (see Figure 3). On the contrary, monetary policy is less active if the preferences of the governments are more homogeneous $\left(\alpha_y^{Gi} = \alpha_y^{Gj}\right)$, and budgetary policies are then more active and divergent.

Besides, after a productivity shock, monetary policy is very slightly contractionary and the nominal interest rate increases ($i_t > 0$) if relative producer prices increase in the country (*j*) with the highest preference for stabilizing prices $\left(\alpha_\pi^{Gi} < \alpha_\pi^{Gj}\right)$ [Then ($\Delta_{4,i} > 0$)], even if the variation is then very limited (see Figure 3). So, if $\left(\alpha_\pi^{Gi} < \alpha_\pi^{Gj}\right)$, this contractionary

Figure 3: Variation of economic instruments according to the relative preferences of the governments in case of productivity shocks



Calibration: basic calibration of structural parameters, for an initial productivity shock $\left(p_{j,0}^j - p_{i,0}^i = 5\%\right)$

monetary policy allows very slightly more expansionary budgetary policies. Budgetary policy is then slightly less contractionary in the country (i) [($g_{i,t}$) decreases less, because ($\Delta_{5,i}$) and ($\Delta_{6,i}$) decrease in equation (16), even if (Δ_1) also decreases] while the budgetary policy is slightly more expansionary in the country (*j*), even if variations are then minimal.

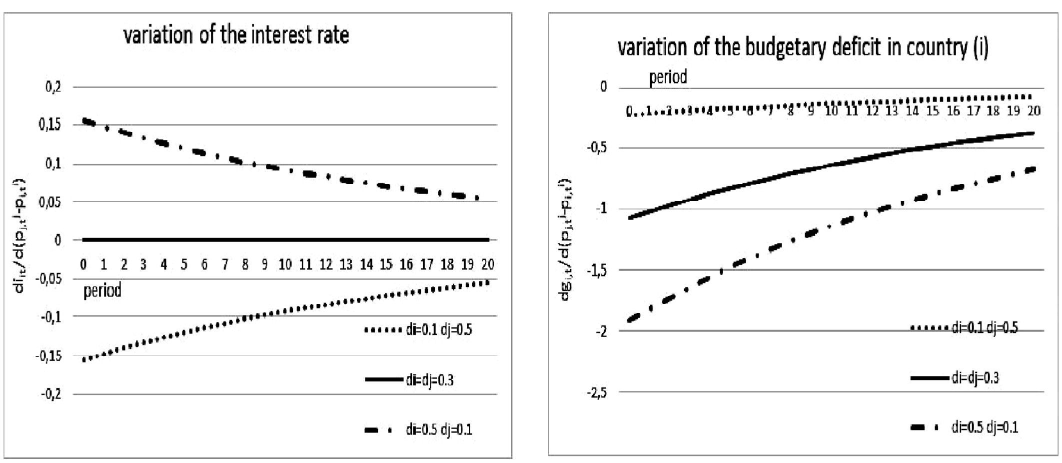
4.3. Structural heterogeneity between the member countries

What are the consequences of structural heterogeneities between the member countries of the monetary union, for economic policies intended to stabilize productivity shocks?

According to equation (15), monetary policy is expansionary ($i_t < 0$) if the productivity shock increases relative producer prices in the country (*j*) with the highest sensitivity of its demand to the real exchange rate ($\delta_i < \delta_j$) [Then ($\Delta_{4,i} < 0$)] (see Figure 4). Indeed, the negative productivity shock and the loss of relative price competitiveness of the country (j) risk to imply accentuated consequences on economic activity and strongest recessionary tensions in this country. According to equation (16), budgetary policies are then also more expansionary in both countries. Budgetary policy is strongly less contractionary in the country (i) [($g_{i,t}$) decreases less, because ($\Delta_{6,i}$) decreases], while the budgetary policy is more expansionary and the budgetary deficit is higher in the country (*j*) ($g_{j,t}$ increases more). Therefore, our modeling underlines that the budgetary policy must be more active in the country with the highest sensitivity of its demand to the real exchange rate.

According to equation (15), monetary policy is expansionary ($i_t < 0$) if the productivity shock increases relative producer prices and affects the country (*j*) with the weakest

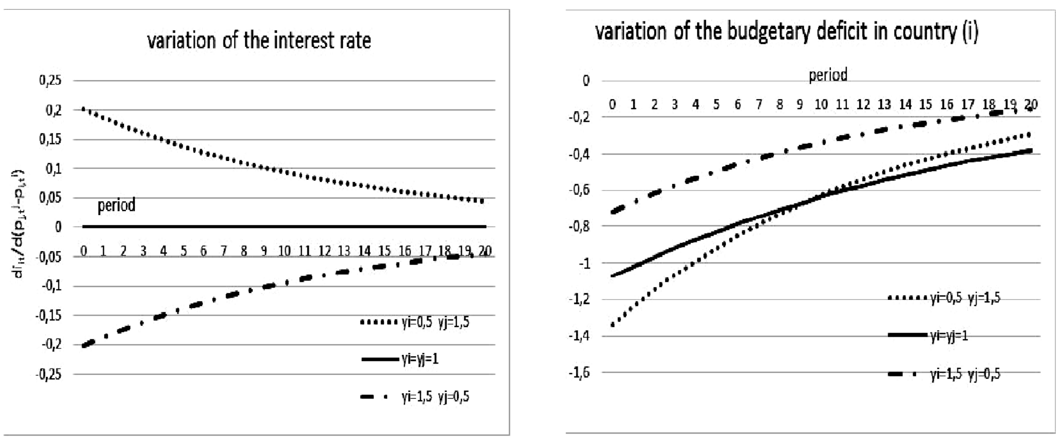
Figure 4: Variation of economic instruments according to the relative sensitivity of demand to the real exchange rate in case of productivity shocks



Calibration: basic calibration of structural parameters, for an initial productivity shock $\left(p_{j,0}^{j} - p_{i,0}^{i} = 5\%\right)$

budgetary multiplier ($\gamma_i > \gamma_j$) [Then ($\Delta_{4,i} < 0$)], with the weak estsensitivity of its demand to the budgetary deficit(see Figure 5). Indeed, the budgetary policy is then less liable to avoid recessionary risks in the country (*j*). Therefore, due to the monetary stimulus, if ($\gamma_i > \gamma_j$), the budgetary policy can be less contractionary in the country (i) [($g_{i,t}$) decreases less, because ($\Delta_{6,i}$) decreases even if (Δ_1) also decreases], as fiscal policy is also more efficient to impact economic activity in this country (i). However, the effect on the budgetary policy of the country (*j*), whichbudgetary multiplier is weaker, is more ambiguous. At the beginning, the budgetary policy is more expansionary and the budgetary deficit is accentuated in the country (j). But afterwards, the budgetary policy becomes less expansionary and the budgetary deficit becomes weaker in the country (*j*) ($g_{j,t}$ increases less) than in the case of structural homogeneity. Anyway, the stabilization of productivity shocks is mainly realized by the fiscal policy of the country (*j*) with the weakest budgetary multiplier, as this country (*j*) must compensate for the weaker efficiency of its fiscal policy with a stronger budgetary activism (see Figure 5).

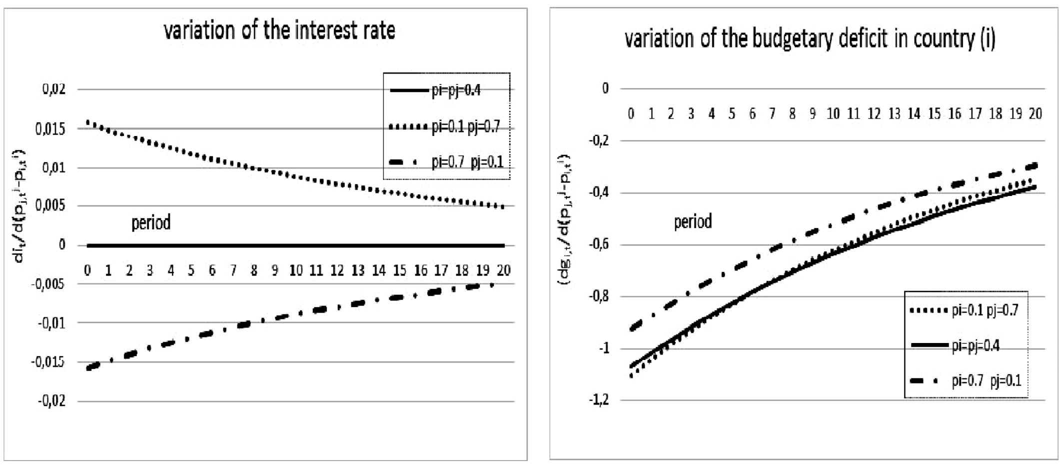
Figure 5: Variation of economic instruments according to the relative budgetary multipliers in case of productivity shocks



Calibration: basic calibration of structural parameters, for an initial productivity shock $\left(p^j_{j,0} - p^i_{i,0} = 5\%\right)$

According to equation (15), monetary policy is expansionary ($i_t < 0$) if the productivity shock increases relative producer prices and affects the country (*j*) with the weakest sensitivity of its demand to the foreign activity ($\rho_i > \rho_j$) [Then ($\Delta_{4,i} < 0$)] (see Figure 6). Indeed, the negative productivity shock and the loss of relative price competitiveness of the country (*j*) risk to imply accentuated consequences on economic activity and strongest recessionary tensions in this country [see equation (12)]. Besides, according to equation (16), budgetary policies are then also more expansionary. Budgetary policy is less contractionary in the country (i) [($g_{i,t}$) decreases less, because ($\Delta_{5,i}$) and ($\Delta_{6,i}$) decrease, while (Δ_1) increases], whereas it is more ambiguous in the country (*j*). Indeed, at the beginning, budgetary policy is more expansionary and the budgetary deficit is accentuated in the country (*j*) ($g_{j,t}$ increases more). But afterwards, the budgetary policy becomes less expansionary and the budgetary deficit becomes weaker in the country (*j*) than in the case of structural homogeneity (see Figure 6). Anyway, the stabilization of productivity shocks is mainly realized by the fiscal policy of the country (j) with the weakest sensitivity of its demand to the foreign activity.

Figure 6: Variation of economic instruments according to the relative sensitivity of demand to the foreign activity in case of productivity shocks



Calibration: basic calibration of structural parameters, for an initial productivity shock $\left(p_{j,0}^{j}-p_{i,0}^{i}=5\%\right)$

Furthermore, monetary policy is contractionary ($i_t > 0$) if the productivity shock increasesrelative producer prices in the country (*j*) with the strongest transmission mechanisms of monetary policy ($\sigma_i < \sigma_j$) [Then ($\Delta_{4,i} > 0$) in equation (15)].This allows slightly more expansionary budgetary policies in order to compensate for this monetary policy. Indeed, the budgetary policy is then less contractionary in the country (i) [($g_{i,t}$) decreases less, because ($\Delta_{5,i}$) and ($\Delta_{6,i}$) decrease, even if (Δ_1) also decreases], while the budgetary policy is more expansionary and the budgetary deficit is higher in the country (*j*) ($g_{j,t}$ increases more). However, all these variations would remain quite moderate according to our calibration.

Monetary policy is contractionary ($i_t > 0$) if the productivity shock increases relative producer prices in the country (*j*) with the highest sensitivityof national to foreign prices ($\zeta_i < \zeta_j$) [Then ($\Delta_{4,i} > 0$)]. This monetary stabilization allows less active budgetary policies, at least at the beginning of the shock [see equation (16)]. Indeed, if ($\zeta_i \neq \zeta_j$), the budgetary policy is less contractionary in the country (i) [($g_{i,t}$) decreases less, because (Δ_1) increases even if ($\Delta_{5,i}$) and ($\Delta_{6,i}$) also increase], while the budgetary policy is less expansionary and the budgetary deficit is weaker in the country (j) ($g_{j,t}$ increases less). On the contrary, the common monetary policy wouldbe inactive in case of homogeneity in sensitivities of national to foreign prices in the monetary union ($\zeta_i = \zeta_j$), and budgetary policies would then be more active and more divergent. However, these variations would remain very moderate. Furthermore, they depend on the degree of structural heterogeneity between the countries, as well as on the timing of the period considered. Indeed, if ($\zeta_i \neq \zeta_j$), budgetary policies tend to become more active than in case of structural homogeneity for more distant periods.

Monetary policy is expansionary ($i_t < 0$) if the productivity shock increases relative producer prices in the country (j) with the highest sensitivity of national prices to national economic activity ($\xi_i < \xi_j$) [Then ($\Delta_{4,i} < 0$)]. Budgetary policies are then also more expansionary. Budgetary policy is initially less contractionary in the country (i) [($g_{i,t}$) decreases less, because ($\Delta_{6,i}$) decreases and (Δ_1) increases, even if ($\Delta_{5,i}$) also increases]. However, the budgetary policy of the country (i) becomes afterwards more expansionary than in case of structural homogeneity. Besides, if ($\xi_i < \xi_j$), the budgetary policy is more expansionary and the budgetary deficit is higher in the country (j) ($g_{j,t}$ increases more). However, these variations would remain very moderate according to the calibration of our parameters.

Monetary policy is contractionary ($i_t > 0$) if the productivity shock increases relative producer prices in the country (j) with the weakest relative weight and size ($\alpha_i > \alpha_j$) in the monetary union [Then ($\Delta_{4,i} > 0$)]. Budgetary policies are then on the contrary more expansionary,in order to compensate for this monetary slowdown. The budgetary policy is initially less contractionary in the country (i) [($g_{i,t}$) decreases less, because ($\Delta_{5,i}$) decreases], whereas the budgetary policy is more expansionary and the budgetary deficit is higher in the country (j) ($g_{j,t}$ increases more). However, these variations would remain very moderateaccording to the calibration of our parameters. Finally, according to equation (17), we can also mention that heterogeneity regarding home bias and the share of foreign goods consumed in the national country ($\eta_i - \eta_j$) has no impact on monetary or budgetary instruments, or on economic activity and inflation in the member countries.

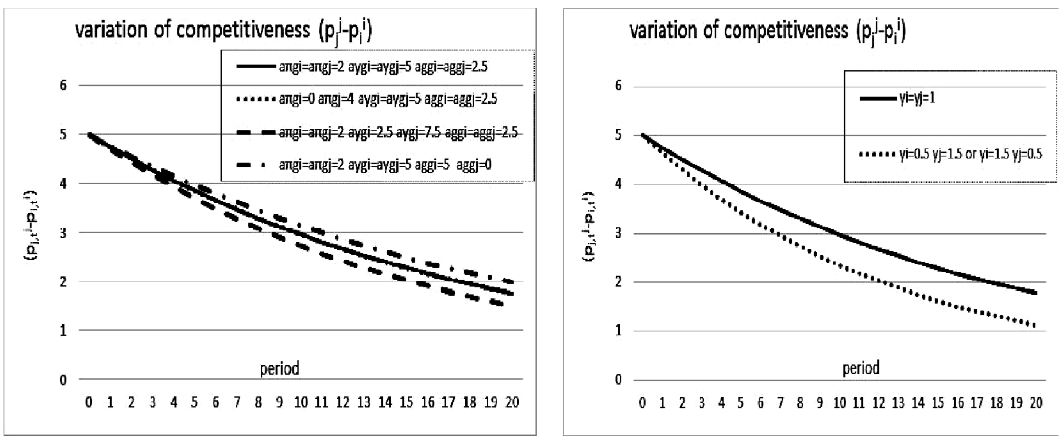
5. PRICE COMPETITIVENESS, ECONOMIC ACTIVITY AND INFLATION

5.1. Efficiency of the real exchange rate channel

According to our model and to equation (17), the real exchange rate channel is more efficient to compensate for a productivity shock, and the speed of adjustment is higher in order to quickly cancel the differential of competitiveness between producer prices of the member countries of a monetary union, for some configurations of preferences and of structural parameters of these member countries.

First, the speed of adjustment to a productivity shock is slightly higher if the member countries of the monetary union are heterogeneous regarding their preferences for stabilizing economic activity $\left(\alpha_y^{Gi} \neq \alpha_y^{Gj}\right)$[Then (Δ_1) decreases, and the right hand side of equation (17) is weaker] or inflation $\left(\alpha_\pi^{Gi} \neq \alpha_\pi^{Gj}\right)$[Then (Δ_1) decreases]. However, the speed of adjustment to a productivity shock is slightly higher if the member countries of the monetary union are homogeneous regarding their budgetary constraints, their preferences for stabilizing the budgetary deficit$\left(\alpha_g^{Gi} \neq \alpha_g^{Gj}\right)$ [Then (Δ_7) increases even if (Δ_1) also increases]. Indeed, monetary policy is then inactive and budgetary policies are less active; in particular, the budgetary policy is less expansionary in the country (j) if$\left(p_{j,t}^{j} > p_{i,t}^{i}\right)$.

Figure 7: Dynamic evolution of a productivity shock according to the relative preferences of the governments or to the relative budgetary multipliers



Calibration: basic calibration of structural parameters, for an initial productivity shock $\left(p_{j,0}^{j} - p_{i,0}^{i} = 5\%\right)$

The speed of adjustment to a productivity shock is also quicker if the member countries of the monetary union are heterogeneous regarding their budgetary multipliers $(\gamma_i \neq \gamma_j)$ [Then (Δ_1) decreases while (Δ_7) increases in equation (17)]. Indeed, budgetary policies are then less active at the global level, as monetary policy isactive and contributes to part of the stabilization of the shock (see Figure 7). In the same way, the speed of adjustment is quicker if the member countries are heterogeneous regarding their sensitivities of national to foreign economic activity $(\rho_i \neq \rho_j)$ [Then (Δ_7) increases even if (Δ_1) also increases].

To the contrary, the speed of adjustment to a productivity shock is veryslightly quicker if the member countries of a monetary union are homogeneous regarding their transmission mechanisms of monetary policy ($\sigma_i = \sigma_j$) [Then (Δ_7) increases more than (Δ_1), and the right hand side of equation (17) is weaker], their sensitivity of national prices to national economic activity ($\xi_i = \xi_j$) [Then (Δ_1) decreases while (Δ_7) increases], or their sensitivity of national to foreign prices ($\zeta_i = \zeta_j$) [Then (Δ_1) decreases even if (Δ_7) also decreases]. Indeed, a less active monetary policy then allows slightly less active budgetary policies at the global level. We can also mention that the speed of adjustment to a productivity shock doesn't depend on the relative size of the member countries (α), and on the differential in sensitivity of demand to the real exchange rate (δ) between the member countries of the monetary union [see the value of (Δ_7) in Appendix B]. Indeed, budgetary policies then compensate for each other, and they don't prevent the adjustment due to market mechanisms.

5.2. The case of homogeneous member countries

What are the consequences of equilibrium monetary and budgetary policies defined in section 4 on economic stabilization, for economic activity and inflation? According to equations (C1) and (C2) in Appendix C, we obtain:

$$\frac{\partial y_{i,t}}{\partial\left(p_{j,t}^{j}-p_{i,t}^{i}\right)} = \alpha_i^M \frac{\Delta_{8,i}}{\Delta_1} + \alpha_g^{Gi}\alpha_g^{Gj}\left(\delta_i\sigma_j + \sigma_i\delta_j\right)\frac{\Delta_{9,i}}{\Delta_1} \tag{20}$$

$$\frac{\partial \pi_{i,t}}{\partial\left(p_{j,t}^{j}-p_{i,t}^{i}\right)} = \alpha_i^M \frac{\Delta_{10,i}}{\Delta_1} + \alpha_g^{Gi}\alpha_g^{Gj}\left(\delta_i\sigma_j + \sigma_i\delta_j\right)\frac{\Delta_{11,i}}{\Delta_1} \tag{21}$$

Where: $\Delta_{8,i}$; $\Delta_{9,i}$ >0; $\Delta_{10,i}$; $\Delta_{11,i}$ > 0 are defined in Appendix C.

So, in case of structural homogeneity between the member countries of the monetary union, we obtain the following economic variables in the country (i):

$$\frac{\partial y_{i,t}}{\partial\left(p_{j,t}^{j}-p_{i,t}^{i}\right)} \xrightarrow[homogeneity]{} \frac{(1+\zeta)\delta}{\Delta_8} \tag{22}$$

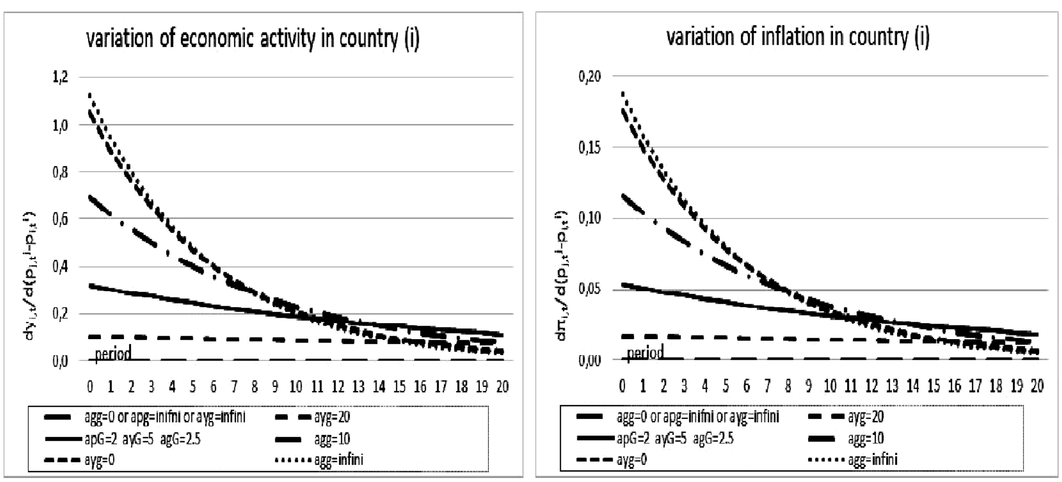
$$\frac{\partial \pi_{i,t}}{\partial\left(p_{j,t}^{j}-p_{i,t}^{i}\right)} \xrightarrow[homogeneity]{} \frac{\xi\delta}{\Delta_8} \tag{23}$$

Therefore, a productivity shock $\left(p_{j,t}^{j}-p_{i,t}^{i}>0\right)$ which reduces (increases) relative producer prices and increases (reduces) the relative competitiveness of the country (*i*) (country (*j*)), increases (reduces) net exports and can reduce (must increase) public expenditure in this country, as mentioned in section 4.1.This implies expansionary and inflationary tensions in the country (i) [$\partial y_{i,t} \to 0.32\%$ and $\partial \pi_{i,t} \to 0.05\%$with our basic calibration and with a productivity shock of $\left(p^j_{j,t} - p^i_{i,t}\right)$=5%], and on the contrary recessionary and deflationary tensions in the country (*j*).

However, if the governments favor the stabilization of economic variables and if budgetary policies were very active, economic activity and inflation could fully be stabilized [$\partial y_{i,t} \xrightarrow[\alpha^G_\pi \to \infty]{} \partial y_{i,t} \xrightarrow[\alpha^G_y \to \infty]{} \partial y_{i,t} \xrightarrow[\alpha^G_g \to 0]{} 0$] (see Figure 8). Indeed, internal public demand could then fully compensate for variations in net exports.To the contrary, if budgetary policies are less active and favor the stabilization of the budgetary deficit, economic activity and inflation increase more strongly in the country (i) [$\partial y_{i,t} \xrightarrow[\alpha^G_\pi \to 0]{} 0.33, \partial \pi_{i,t} \xrightarrow[\alpha^G_\pi \to 0]{} 0.05$; $\partial y_{i,t} \xrightarrow[\alpha^G_y \to 0]{} 1.05$, $\partial \pi_{i,t} \xrightarrow[\alpha^G_y \to 0]{} 0.18$; $\partial y_{i,t} \xrightarrow[\alpha^G_g \to \infty]{} \frac{(1+\zeta)\delta}{[(1+\zeta)(1+\rho)-\sigma\xi]} = 1.13, \partial \pi_{i,t} \xrightarrow[\alpha^G_g \to \infty]{} \frac{\xi\delta}{[(1+\zeta)(1+\rho)-\sigma\xi]} = 0.19$ with our basic calibration], whereas they decrease more strongly in the country (*j*). Indeed, the higher relative price competiveness of the country (*i*) is then not sufficiently compensated by a relative more contractionary budgetary policy. Market mechanisms then increase the speed of adjustment and of resorption of the productivity shock, but at the cost of higher short-term macro-economic fluctuations.

Furthermore, we have mentioned in section 4.1 that the budgetary policy is less contractionary in the country (*i*) and less expansionary in the country (*j*) if transmission

Figure 8: Variation of economic activity and inflation according to the preferences of the governments



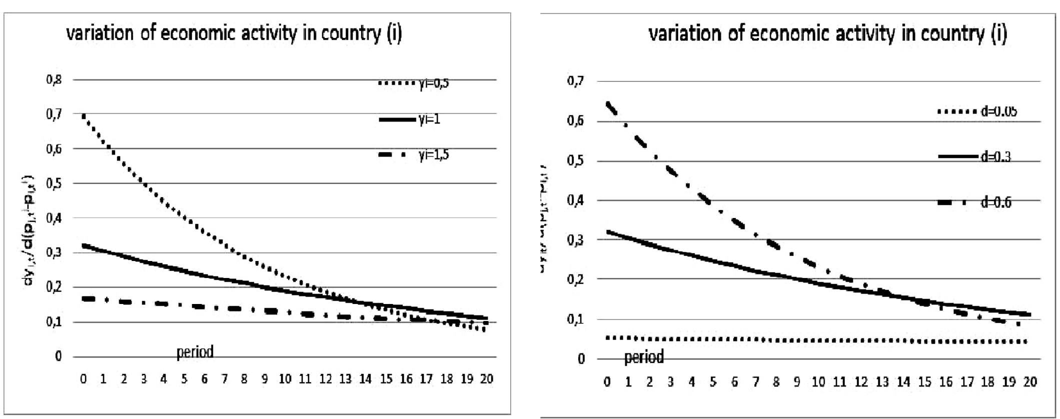
Calibration: basic calibration of structural parameters, for an initial productivity shock $\left(p^j_{j,0} - p^i_{i,0} = 5\%\right)$

mechanisms of monetary policy (σ) are weak;however, macro-economic fluctuations in terms of economic activity and inflation are then accentuated. On the contrary, budgetary policies are more active if the transmission mechanisms of monetary policy are strong, whereas short term macro-economic fluctuations are more reduced. The situation is clearly more favorable if the home bias and the share of national goods and services in domestic consumption (1–η) is weak; budgetary policies can then be less active, whereas economic activity and inflation are both more quickly stabilized.

In the same way,budgetary policies can be less active whereas economic activity and inflation are better stabilized if the sensitivity of demand to the real exchange rate (δ) is weak (see Figure 9). Regarding this parameter, Toroj (2009) also underlines that there would be some asymmetry in real exchange rate response to negative and positive output gap. Indeed, a necessary appreciation of the real exchange rate would run more quickly, whereas a depreciation would be much more heterogeneous across countries, especially in services.Furthermore, regarding the impact of the fiscal policy on economic activity, at least for all first periods immediately after the productivity shock, economic activity and inflation are better stabilized if budgetary policies are more efficient to impact economic activity and if budgetary multipliers (γ) are high (see Figure 9).

Besides, economic activity and inflation are better stabilized if the sensitivity of demand to the foreign activity (ρ) or if the sensitivity of national to foreign prices (ζ) are high, even if more active budgetary policies are then necessary. Results are still more ambiguous regarding the sensitivity of national prices to national economic activity (ξ), prices (on the product market) and wages (on the labor market) flexibility. Indeed, variations in economic activity are accentuated whereas variations in prices are slightly more limited if this sensitivity is weak.

Figure 9: Economic activity according to the sensitivity of demand to the real exchange rate or to the budgetary multiplier



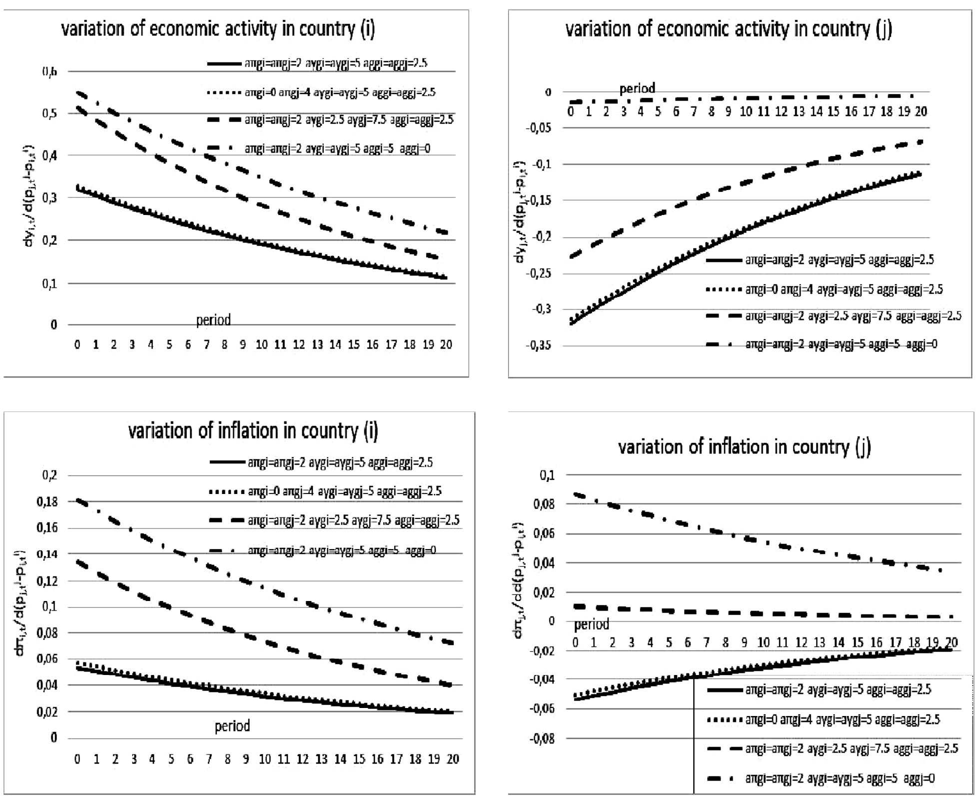
Calibration: basic calibration of structural parameters, for an initial productivity shock $\left(p_{j,0}^{j} - p_{i,0}^{i} = 5\%\right)$

5.3. Heterogeneity in preferences of the governments

We have mentioned that a productivity shock $\left(p_{j,t}^{j} - p_{i,t}^{i} > 0\right)$implies expansionary and inflationary tensions in the country (*i*), and on the contrary recessionary and deflationary tensions in the country (*j*). However, economic activity and inflation are lesswell stabilized, obviously, in the country (*i*) with a weaker preference for economic stabilization but with a higher preference for stabilizing the budgetary deficits [($\Delta_{8,i}$) or ($\Delta_{10,i}$) increase while (Δ_1) decreases if$\left(\alpha_{\pi}^{Gi} < \alpha_{\pi}^{Gj}\right)$,$\left(\alpha_{y}^{Gi} < \alpha_{y}^{Gj}\right)$ or$\left(\alpha_{g}^{Gi} > \alpha_{g}^{Gj}\right)$], whereas economic variables are then better stabilized in the country (*j*).

Therefore, monetary unification could bemade more difficult for a country with strong budgetary constraints, and with a high preference for the stabilization of budgetary deficits;

Figure 10: Variation of economic activity and inflation according to the relative preferences of the governments in case of productivity shocks



Calibration: basic calibration of structural parameters, for an initial productivity shock $\left(p_{j,0}^{j} - p_{i,0}^{i} = 5\%\right)$

for example, for a very indebted country with a weak budgetary flexibility. Indeed, the budgetary policy is then less active, but economic variables are less well stabilized. Monetary unification could also mainly be detrimental for a country with a high preference for the stabilization of economic activity and inflation. Indeed, economic variables are then better stabilized, but at the cost of a more active budgetary policy.

5.4. Structural heterogeneity between the member countries

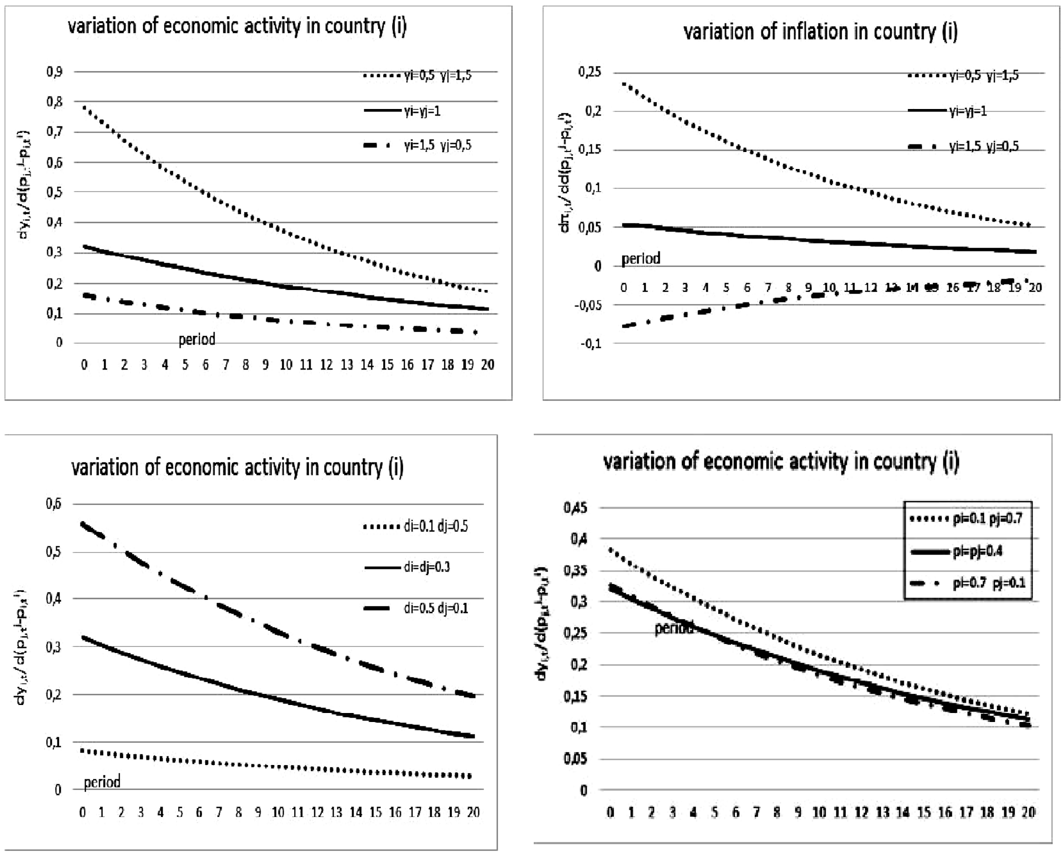
We can now conclude this section by analyzing the consequences of structural heterogeneity between the member countries of the monetary union on the efficiency of the stabilization of economic activity and inflation in case of productivity shocks.

First, our analytical model shows that in case of a productivity shock $\left(p_{j,t}^{j} - p_{i,t}^{i} > 0\right)$, economic activity and inflation are less well stabilized in the country (*i*) where the sensitivity of the demand to the budgetary deficit, where the budgetary multiplier is the weakest ($\gamma_i < \gamma_j$). Economic growth and inflationary tensions are then accentuated in this country (i)[(Δ_1) decreases, while ($\Delta_{8,i}$) or ($\Delta_{10,i}$) increase in equations (20) and (21)], despite the more contractionary budgetary policy and the higher budgetary surplus in this country. Indeed, the budgetary situation has a limited impact on economic activity in this country (i). On the contrary, recession and deflation are more limited in the country (j) with a higher budgetary multiplier, where fiscal policy is less active but more efficient to impact economic activity (see Figure 11). The global loss function is then more limited and reduced in the country (j).

According to equations (20) and (21), economic activity and inflation are also less well stabilized in the country (i) where the sensitivity of demand to the real exchange rate is the highest ($\delta_i > \delta_j$) [Then, ($\Delta_{8,i}$) and ($\Delta_{10,i}$) increase] despite the more contractionary budgetary policy, whereas recession and deflation are more limited in the country (*j*) despite the less expansionary budgetary policy (see Figure 11). Therefore, the global loss function is more limited and reduced for the country with the weakest sensitivity of its demand to the real exchange rate. Furthermore, economic activity and inflation are less well stabilized in the country (i) where the sensitivity of national demand to foreign economic activity is the weakest ($\rho_i < \rho_j$) [Then: ($\Delta_{8,i}$), ($\Delta_{9,i}$), ($\Delta_{10,i}$) and ($\Delta_{11,i}$) increase, even if (Δ_1) also increases]. On the contrary, economic variables are better stabilized in the country (*j*) where this sensitivity is the highest, despite the less active budgetary policy (see Figure 11). The global loss function is thus reduced for the country with the highest sensitivity of its demand to foreign economic activity.

In the same way, if the sensitivity of national prices to foreign prices is weaker in the country (i) ($\zeta_i < \zeta_j$), inflation and economic activity are less well stabilized in this country (i) [Then: ($\Delta_{8,i}$), ($\Delta_{9,i}$), ($\Delta_{10,i}$) and ($\Delta_{11,i}$) increase, even if (Δ_1) also increases]. On the contrary,

Figure 11: Variation of economic activityand inflation according to various relative structural parameters in case of productivity shocks



Calibration: basic calibration of structural parameters, for an initial productivity shock $\left(p_{j,0}^{j} - p_{i,0}^{i} = 5\%\right)$

recession and deflation are better stabilized in the country (j) where this sensitivity is the highest, despite the slightly less active budgetary policy. Besides, economic activity and inflation are less well stabilized and are accentuated in the country (i) where the transmission mechanisms of monetary policy are less efficient ($\sigma_i < \sigma_j$) [Then: ($\Delta 1$) decreases, while: ($\Delta_{8,i}$), ($\Delta_{9,i}$), ($\Delta_{10,i}$) and ($\Delta_{11,i}$) increase], because budgetary policy is then slightly less contractionary in this country. On the contrary, economic variables are better stabilized, recession and deflation are less accentuated in the country (j), where the budgetary policy is slightly more expansionary. Nevertheless, variations of the global loss function according to the heterogeneity of national prices to foreign prices or to the heterogeneity of the transmission mechanisms of monetary policy between the member countries of the monetary union would remain quite limited.

Furthermore, in case of a productivity shock $\left(p_{j,t}^{j}-p_{i,t}^{i}>0\right)$, economic activity and inflation are also slightly better stabilized in the biggest country of the monetary union. Indeed, if ($\alpha_i < \alpha_j$), economic growth and inflation are accentuated in the smallest country (i) [Then, ($\Delta_{9,i}$) and ($\Delta_{11,i}$) increase], despite the more contractionary budgetary policy in this country. On the contrary, recession and deflation are more limited in the biggest country (j), despite the less expansionary budgetary policy in this country.Finally, the situation is ambiguous regardingthe heterogeneityof the sensitivity of national prices to national economic activity in the monetary union, as this parameter implies a trade-off between stabilizing economic activity and inflation. Indeed, if this sensitivity is weaker in the country (i) than in the rest of the monetary union ($\xi_i < \xi_j$), inflation is better stabilized whereas economic growth is less stabilized in this country (i) [Then: ($\Delta_{8,i}$), ($\Delta_{9,i}$)and (Δ_1) increase,while ($\Delta_{10,i}$) and ($\Delta_{11,i}$) decrease]. On the contrary, deflation is accentuatedwhereas economic recession is more limitedin the country (j) where the sensitivity of national prices to national economic activity is the highest ($\xi_j > \xi_i$).

6. CONCLUSION

The contribution of the current paper is to provide an analytical modelling and precise analytical results regarding the stabilization of productivity shocks in a monetary union. We also study the consequences of heterogeneities between the preferences or between the structural parameters of the member countries of a monetary union on monetary and budgetary policies, and on the stabilization of economic activity and inflation. We find that in case of productivity shocks, monetary policy should beinactive, at least as long as the member countries of the monetary union are homogeneous. The stabilization of productivity shocks is mainly the burden of the governments and of budgetary policies. So, after such shocks, the speed of removal of differences in producer prices, and of reduction of deviations of relative competitiveness between the member countries, mostly depends on the behavior and on the preferences of the governments. A productivity shock which reduces relative producer prices and increases the competitiveness of a given country is beneficial to the net exports of this country, and it increases its global demand. However, inflationary tensions in this country progressively reduce the differential in price competitiveness, and the productivity shock is progressively eliminated by market mechanisms. Nevertheless, the budgetary policy can also compensate with internal demand the variation in exports and in external demand, in order to stabilize productivity shocks.In this case, fiscal activism can avoid the necessity to eliminate differentials in producer prices, and without any budgetary constraint, economic activity and inflation could even be fully stabilized in case of productivity shocks.

More precisely, the budgetary policy is contractionary in the country which benefits from a higher price competitiveness, while it is expansionary and the budgetary deficit increases in the other country. Nevertheless, these variations are reduced by the budgetary constraints and the weight given to stabilizing the budgetary deficit. In order to increase the pace of the adjustment to a productivity shock with market mechanisms, budgetary constraints are therefore beneficial, as with a limited budgetary activism, the higher price competitiveness of a country increases exports and inflationary tensions in this country, which naturally contributes to reduce the differential in price competitiveness. However, the quicker resorption of a productivity shock with the help of market mechanisms is then realized at the cost of higher macro-economic fluctuations in terms of economic activity and inflation. Budgetary policies are also less active, whereas the stabilization of economic activity and inflation is more quickly improved, if the share of foreign goods and services in domestic consumption is high, or if the sensitivity of demand to the real exchange rate is weak. Macro-economic stabilization is also improved if budgetary multipliers are high, whereas results are more ambiguous regarding other structural parameters.

In this context, our modelling can also provide important and precise analytical results regarding the consequences of structural heterogeneities between the member countries of a monetary union.Indeed, market adjustment to a productivity shock is quicker if the member countries of the monetary union are more homogeneous regarding their budgetary constraints. Therefore, the Fiscal Compact in the European Economic and Monetary Union could have the advantage to standardize the budgetary constraints in all member countries. However, as some heterogeneity between the preferences of the governments remains, our model shows that monetary unification could beeasier for countrieswith weak budgetary constraints and preferences for stabilizing the budgetary deficit, i.e. for countries which benefit from more budgetary flexibility, for example because they are weakly indebted. Indeed, the budgetary policymust then be more active in these countries, as a weaker part of the stabilization is realized by more constrained member countries. However, economic activity and inflation are then better stabilized in these countries with the weakest budgetary constraints.

Furthermore, in case of productivity shocks, monetary unification could be made more difficult for a country with a weak budgetary multiplier. Indeed, a country where the budgetary multiplier is weak must have a higher budgetary activism in order to stabilize productivity shocks. However, this fiscal policy has then a weak efficiency, and economic activity and inflation are less well stabilized. Monetary unification could also be more difficult for a country with a higher sensitivity of its demand to the real exchange rate, or with a weaker sensitivity of its demand to the foreign economic activity. Indeed, despite the more active budgetary policy, economic activity and inflation are then less well stabilized. Results are more ambiguous regarding other structural parameters.

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APPENDIX A: NASH EQUILIBRIUM OF THE MODEL

According to equation (13), with ($\frac{\partial g_{i,k}}{\partial g_{i,t}} = 0$ for k>t)., the budgetary deficit chosen in the country (i) verifies:

$$\frac{\partial L_{i,t}^{G}}{\partial g_{i,t}} = 2\alpha_{\pi}^{Gi}\pi_{i,t}\frac{\partial \pi_{i,t}}{\partial g_{i,t}} + 2\alpha_{y}^{Gi}y_{i,t}\frac{\partial y_{i,t}}{\partial g_{i,t}} + 2\alpha_{g}^{Gi}g_{i,t} = 0 \quad \text{(A1)}$$

Indeed, we only consider the current period, as all periods are ex ante identical. Then, using equations (9), (10), (11) and (12),we obtain a relation between budgetary deficits in both countries, and afterwards, we have:

$$g_{i,t} = f\left[i_t, \left(p_{j,t}^{j} - p_{i,t}^{i}\right)\right] \quad \text{(A2)}$$

Besides, according to equation (14), the nominal interest rate chosen by the common monetary authority verifies:

$$\frac{\partial L_t^M}{\partial i_t} = 2\alpha_{\pi}^{M}\left(\alpha_i\pi_{i,t} + \alpha_j\pi_{j,t}\right)\left(\alpha_i\frac{\partial \pi_{i,t}}{\partial i_t} + \alpha_j\frac{\partial \pi_{j,t}}{\partial i_t}\right) + 2\alpha_{y}^{M}\left(\alpha_i y_{i,t} + \alpha_j y_{j,t}\right)\left(\alpha_i\frac{\partial y_{i,t}}{\partial i_t} + \alpha_j\frac{\partial y_{j,t}}{\partial i_t}\right) + 2\alpha_i^M i_t = 0 \quad \text{(A3)}$$

Therefore, using equations (9), (10), (11) and (12), we obtain the equilibrium common nominal interest rate:

$$i_t = f\left[g_{i,t}, g_{j,t}, \left(p_{j,t}^{j} - p_{i,t}^{i}\right)\right] \quad \text{(A4)}$$

Finally, by combining this equation (A4) with the budgetary deficits ($g_{i,t}$) and ($g_{j,t}$) in equation (A2), we obtain the following equilibrium nominal interest rate:

$$i_t = \frac{\Delta_{4,i}}{\Delta_1}\left(p_{j,t}^{j} - p_{i,t}^{i}\right) \quad \text{(A5)}$$

$$\Delta_1 = \Delta_3 + \alpha_i^M \Delta_2 > 0$$

(Δ_1) decreases if ($\alpha_{\pi}^{Gi} \neq \alpha_{\pi}^{Gj}$), ($\alpha_{y}^{Gi} \neq \alpha_{y}^{Gj}$) and ($\alpha_{g}^{Gi} \neq \alpha_{g}^{Gj}$).

(Δ_1) decreases if ($\sigma_i \neq \sigma_j$) and ($\gamma_i \neq \gamma_j$), but it increases if ($\xi_i \neq \xi_j$), ($\rho_i \neq \rho_j$) and ($\zeta_i \neq \zeta_j$).

$$\Delta_2 = \alpha_g^{Gi}\alpha_g^{Gj}\left[\left(1-\zeta_i\zeta_j\right)\left(1-\rho_i\rho_j\right)+\sigma_i\sigma_j\xi_i\xi_j-(\sigma_i+\sigma_j\rho_i\zeta_j)\xi_i-(\sigma_j+\sigma_i\rho_j\zeta_i)\xi_j\right]^3$$

$$+\left[\left(1-\zeta_i\zeta_j\right)\left(1-\rho_i\rho_j\right)+\sigma_i\sigma_j\xi_i\xi_j-\left(\sigma_i+\sigma_j\rho_i\zeta_j\right)\xi_i-\left(\sigma_j+\sigma_i\rho_j\zeta_i\right)\xi_j\right]$$

$$[\alpha_y^{Gi}\alpha_g^{Gj}\gamma_i^2\left(1-\zeta_i\zeta_j-\sigma_j\xi_j\right)^2+\alpha_g^{Gi}\alpha_y^{Gj}\gamma_j^2\left(1-\zeta_i\zeta_j-\sigma_i\xi_i\right)^2$$

$$+\alpha_g^{Gi}\alpha_\pi^{Gj}\gamma_j^2\left(\xi_j-\xi_i\sigma_i\xi_j+\xi_i\zeta_j\rho_i\right)^2+\alpha_\pi^{Gi}\alpha_g^{Gj}\gamma_i^2\left(\xi_i-\xi_i\sigma_j\xi_j+\xi_j\zeta_i\rho_j\right)^2$$

$$+\alpha_y^{Gi}\alpha_\pi^{Gj}\gamma_i^2\gamma_j^2\xi_j\left(\xi_j-\xi_i\sigma_i\xi_j+\xi_i\zeta_j\rho_i\right)\left(1-\zeta_i\zeta_j-\sigma_j\xi_j\right)$$

$$+\alpha_y^{Gj}\alpha_\pi^{Gi}\gamma_i^2\gamma_j^2\xi_i\left(\xi_i-\xi_i\sigma_j\xi_j+\xi_j\zeta_i\rho_j\right)\left(1-\zeta_i\zeta_j-\sigma_i\xi_i\right)$$

$$+\alpha_y^{Gi}\alpha_y^{Gj}\gamma_i^2\gamma_j^2\left(1-\zeta_i\zeta_j-\sigma_i\xi_i\right)\left(1-\zeta_i\zeta_j-\sigma_j\xi_j\right)\left(1-\zeta_i\zeta_j\right)$$

$$+\alpha_\pi^{Gi}\alpha_\pi^{Gj}\gamma_i^2\gamma_j^2\xi_i\xi_j\left(\xi_i-\xi_i\sigma_j\xi_j+\xi_j\zeta_i\rho_j\right)\left(\xi_j-\xi_i\sigma_i\xi_j+\xi_i\zeta_j\rho_i\right)>0$$

$$\Delta_3 = \alpha_\pi^M\alpha_g^{Gi}\alpha_g^{Gj}\left[\xi_i\left(\alpha_i+\alpha_j\zeta_j\right)\left(\sigma_i+\sigma_j\rho_i\right)-\xi_i\sigma_i\sigma_j\xi_j+\xi_j\left(\alpha_j+\alpha_i\zeta_i\right)\left(\sigma_j+\sigma_i\rho_j\right)\right]^2$$

$$\left[\left(1-\zeta_i\zeta_j\right)\left(1-\rho_i\rho_j\right)+\sigma_i\sigma_j\xi_i\xi_j-\left(\sigma_i+\sigma_j\rho_i\zeta_j\right)\xi_i-\left(\sigma_j+\sigma_i\rho_j\zeta_i\right)\xi_j\right]$$

$$+\alpha_\pi^M\left[\xi_i\left(\alpha_i+\alpha_j\zeta_j\right)\left(\sigma_i+\sigma_j\rho_i\right)-\sigma_i\sigma_j\xi_j\xi_i+\xi_j\left(\alpha_j+\alpha_i\zeta_i\right)\left(\sigma_j+\sigma_i\rho_j\right)\right]$$

$$\left[\alpha_y^{Gi}\alpha_g^{Gj}\xi_j\left(\alpha_j+\alpha_i\zeta_i\right)\gamma_i^2\left(1-\zeta_i\zeta_j-\sigma_j\xi_j\right)\sigma_j+\alpha_g^{Gi}\alpha_y^{Gj}\xi_i\left(\alpha_i+\alpha_j\zeta_j\right)\gamma_j^2\left(1-\zeta_i\zeta_j-\sigma_i\xi_i\right)\sigma\right.$$

$$+\alpha_g^{Gi}\alpha_\pi^{Gj}\alpha_i\xi_i\xi_j\gamma_j^2\left(\xi_j-\xi_i\sigma_i\xi_j+\xi_i\zeta_j\rho_i\right)\sigma_i$$

$$\left.+\alpha_\pi^{Gi}\alpha_g^{Gj}\alpha_j\xi_i\xi_j\gamma_i^2\left(\xi_i-\xi_i\sigma_j\xi_j+\xi_j\zeta_i\rho_j\right)\sigma_j\right]$$

$$+\alpha_y^M\alpha_g^{Gi}\alpha_g^{Gj}\left[\left(1-\zeta_i\zeta_j\right)\left(1-\rho_i\rho_j\right)+\sigma_i\sigma_j\xi_i\xi_j-\left(\sigma_i+\sigma_j\rho_i\zeta_j\right)\xi_i-\left(\sigma_j+\sigma_i\rho_j\zeta_i\right)\xi_j\right]$$

$$[(\alpha_i\sigma_i+\alpha_i\sigma_j\rho_i+\alpha_j\sigma_j+\alpha_j\sigma_i\rho_j)\left(1-\zeta_i\zeta_j\right)-\alpha_i\sigma_i\sigma_j\xi_j(1-\zeta_i)-\alpha_j\sigma_j\sigma_i\xi_i\left(1-\zeta_j\right)]^2$$

$$+\alpha_y^M\left[\left(\alpha_i\sigma_i+\alpha_i\sigma_j\rho_i+\alpha_j\sigma_j+\alpha_j\sigma_i\rho_j\right)\left(1-\zeta_i\zeta_j\right)-\alpha_i\sigma_i\sigma_j\xi_j(1-\zeta_i)-\alpha_j\sigma_j\sigma_i\xi_i\left(1-\zeta_j\right)\right]$$

$$[\alpha_y^{Gi}\alpha_g^{Gj}\left(1-\zeta_i\zeta_j-\sigma_j\xi_j\right)\alpha_j\gamma_i^2\sigma_j\left(1-\zeta_i\zeta_j\right)+\alpha_g^{Gi}\alpha_y^{Gj}\left(1-\zeta_i\zeta_j-\sigma_i\xi_i\right)\alpha_i\gamma_j^2\sigma_i\left(1-\zeta_i\zeta_j\right)$$

$$+\alpha_g^{Gi}\alpha_\pi^{Gj}\gamma_j^2\left(\xi_j-\xi_i\sigma_i\xi_j+\xi_i\zeta_j\rho_i\right)\sigma_i\left(\alpha_i\xi_j-\alpha_j\zeta_j\xi_i\right)$$

$$+\alpha_\pi^{Gi}\alpha_g^{Gj}\gamma_i^2(\xi_i-\xi_i\sigma_j\xi_j+\xi_j\zeta_i\rho_j)\sigma_j\left(\alpha_j\xi_i-\alpha_i\xi_j\zeta_i\right)]\quad>0$$

$$\Delta_{4,i}= \alpha_\pi^M\left[\xi_i(\alpha_i+\alpha_j\zeta_j)(\sigma_i+\sigma_j\rho_i)-\sigma_i\sigma_j\xi_j\xi_i+\xi_j(\alpha_j+\alpha_i\zeta_i)(\sigma_j+\sigma_i\rho_j)\right]$$

$$[\alpha_g^{Gi}\alpha_y^{Gj}\gamma_j^2\delta_i(1-\zeta_i\zeta_j-\sigma_i\xi_i)\xi_i(\alpha_i+\alpha_j\zeta_j)-\alpha_g^{Gj}\alpha_y^{Gi}\gamma_i^2\delta_j(1-\zeta_i\zeta_j-\sigma_j\xi_j)\xi_j(\alpha_j+\alpha_i\zeta_i)$$

$$+\alpha_\pi^{Gj}\alpha_g^{Gi}\alpha_i\gamma_j^2\delta_i\xi_i\xi_j(\xi_j-\xi_i\sigma_i\xi_j+\xi_i\zeta_j\rho_i)-\alpha_\pi^{Gi}\alpha_g^{Gj}\alpha_j\gamma_i^2\delta_j\xi_i\xi_j(\xi_i-\xi_i\sigma_j\xi_j+\xi_j\zeta_i\rho_j)]$$

$$+\alpha_\pi^M\alpha_g^{Gi}\alpha_g^{Gj}\left[\xi_i(\alpha_i+\alpha_j\zeta_j)(\sigma_i+\sigma_j\rho_i)-\sigma_i\sigma_j\xi_j\xi_i+\xi_j(\alpha_j+\alpha_i\zeta_i)(\sigma_j+\sigma_i\rho_j)\right]$$

$$\left[(1-\zeta_i\zeta_j)(1-\rho_i\rho_j)+\sigma_i\sigma_j\xi_i\xi_j-(\sigma_i+\sigma_j\rho_i\zeta_j)\xi_i-(\sigma_j+\sigma_i\rho_j\zeta_i)\xi_j\right]$$

$$\left[(\delta_i-\rho_i\delta_j)\xi_i(\alpha_i+\alpha_j\zeta_j)-(\delta_j-\rho_j\delta_i)\xi_j(\alpha_j+\alpha_i\zeta_i)+(\alpha_j\sigma_i\delta_j-\alpha_i\delta_i\sigma_j)\xi_j\xi_i\right]$$

$$+\alpha_y^M\left[(\alpha_i\sigma_i+\alpha_i\sigma_j\rho_i+\alpha_j\sigma_j+\alpha_j\sigma_i\rho_j)(1-\zeta_i\zeta_j)-\alpha_i\sigma_i\sigma_j\xi_j(1-\zeta_i)-\alpha_j\sigma_j\sigma_i\xi_i(1-\zeta_j)\right]$$

$$\left[\alpha_g^{Gi}\alpha_y^{Gj}\gamma_j^2\alpha_i\delta_i(1-\zeta_i\zeta_j)(1-\zeta_i\zeta_j-\sigma_i\xi_i)-\alpha_g^{Gj}\alpha_y^{Gi}\gamma_i^2\alpha_j\delta_j(1-\zeta_i\zeta_j)(1-\zeta_i\zeta_j-\sigma_j\xi_j)\right.$$

$$+\alpha_\pi^{Gj}\alpha_g^{Gi}\gamma_j^2(\xi_j-\xi_i\sigma_i\xi_j+\xi_i\zeta_j\rho_i)\delta_i(\alpha_i\xi_j-\alpha_j\zeta_j\xi_i)$$

$$\left.-\alpha_\pi^{Gi}\alpha_g^{Gj}\gamma_i^2(\xi_i-\xi_i\sigma_j\xi_j+\xi_j\zeta_i\rho_j)\delta_j(\alpha_j\xi_i-\alpha_i\zeta_i\xi_j)\right]$$

$$+\alpha_y^M\left[(\alpha_i\delta_i-\alpha_i\rho_i\delta_j-\alpha_j\delta_j+\alpha_j\rho_j\delta_i)(1-\zeta_i\zeta_j)-\alpha_i(\delta_i\sigma_j+\sigma_i\delta_j\zeta_i)\xi_j+\alpha_j(\delta_j\sigma_i+\sigma_j\delta_i\zeta_j)\xi_i\right]$$

$$\left[(\alpha_i\sigma_i+\alpha_i\sigma_j\rho_i+\alpha_j\sigma_j+\alpha_j\sigma_i\rho_j)(1-\zeta_i\zeta_j)-\alpha_i\sigma_i\sigma_j\xi_j(1-\zeta_i)-\alpha_j\sigma_j\sigma_i\xi_i(1-\zeta_j)\right]$$

$$\alpha_g^{Gi}\alpha_g^{Gj}\left[(1-\zeta_i\zeta_j)(1-\rho_i\rho_j)+\sigma_i\sigma_j\xi_i\xi_j-(\sigma_i+\sigma_j\rho_i\zeta_j)\xi_i-(\sigma_j+\sigma_i\rho_j\zeta_i)\xi_j\right]$$

($\Delta_{4,i}>0$) if ($\alpha_\pi^{Gi}<\alpha_\pi^{Gj}$) and ($\alpha_y^{Gi}<\alpha_y^{Gj}$), but if ($\alpha_g^{Gi}>\alpha_g^{Gj}$).

($\Delta_{4,i}>0$) if ($\sigma_i<\sigma_j$), ($\zeta_i<\zeta_j$), ($\rho_i<\rho_j$), ($\gamma_i<\gamma_j$), but if ($\xi_i>\xi_j$),($\delta_i>\delta_j$)and ($\alpha_i>\alpha_j$)

So, replacing this equation (A5) in equation (A2), we obtain the following budgetary expenditure in the country (i):

$$g_{i,t}=-\gamma_i\left[\alpha_g^{Gj}(\delta_i\sigma_j+\sigma_i\delta_j)\frac{\Delta_{5,i}}{\Delta_1}+\alpha_i^M\frac{\Delta_{6,i}}{\Delta_1}\right](p_{j,t}^j-p_{i,t}^i) \qquad \text{(A6)}$$

$$\Delta_{5,i}=\alpha_\pi^M\xi_j\left[\xi_i(\alpha_i+\alpha_j\zeta_j)(\sigma_i+\sigma_j\rho_i)-\sigma_i\sigma_j\xi_j\xi_i+\xi_j(\alpha_j+\alpha_i\zeta_i)(\sigma_j+\sigma_i\rho_j)\right]$$

$$\left[\alpha_y^{Gi}(\alpha_j+\alpha_i\zeta_i)(1-\zeta_i\zeta_j-\sigma_j\xi_j)+\alpha_\pi^{Gi}\alpha_j\xi_i(\xi_i-\xi_i\sigma_j\xi_j+\xi_j\zeta_i\rho_j)\right]$$

$$+\alpha_y^M\left[(\alpha_i\sigma_i+\alpha_i\sigma_j\rho_i+\alpha_j\sigma_j+\alpha_j\sigma_i\rho_j)(1-\zeta_i\zeta_j)-\alpha_i\sigma_i\sigma_j\xi_j(1-\zeta_i)-\alpha_j\sigma_j\sigma_i\xi_i(1-\zeta_j)\right]$$

$$\left[\alpha_y^{Gi}(1-\zeta_i\zeta_j)\alpha_j(1-\zeta_i\zeta_j-\sigma_j\xi_j)+\alpha_\pi^{Gi}(\alpha_j\xi_i-\alpha_i\xi_j\zeta_i)(\xi_i-\xi_i\sigma_j\xi_j+\xi_j\zeta_i\rho_j)\right]>0$$

$$\Delta_{6,i} = \alpha_y^{Gi}\alpha_y^{Gj}\gamma_j^2\delta_i(1-\zeta_i\zeta_j-\sigma_j\xi_j)(1-\zeta_i\zeta_j-\sigma_i\xi_i)(1-\zeta_i\zeta_j)$$
$$+\alpha_g^{Gj}\alpha_y^{Gi}(1-\zeta_i\zeta_j-\sigma_j\xi_j)[(\delta_i-\rho_i\delta_j)(1-\zeta_i\zeta_j)-(\delta_i\sigma_j+\sigma_i\delta_j\zeta_i)\xi_j]$$
$$[(1-\zeta_i\zeta_j)(1-\rho_i\rho_j)+\sigma_i\sigma_j\xi_i\xi_j-(\sigma_i+\sigma_j\rho_i\zeta_j)\xi_i-(\sigma_j+\sigma_i\rho_j\zeta_i)\xi_j]$$
$$+\alpha_\pi^{Gi}\alpha_g^{Gj}(\xi_i-\xi_i\sigma_j\xi_j+\xi_j\zeta_i\rho_j)[\xi_i(\delta_i-\rho_i\delta_j)-\sigma_j\delta_i\xi_j\xi_i-\zeta_i\xi_j(\delta_j-\rho_j\delta_i)]$$
$$[(1-\zeta_i\zeta_j)(1-\rho_i\rho_j)+\sigma_i\sigma_j\xi_i\xi_j-(\sigma_i+\sigma_j\rho_i\zeta_j)\xi_i-(\sigma_j+\sigma_i\rho_j\zeta_i)\xi_j]$$
$$+\alpha_\pi^{Gi}\alpha_y^{Gj}\xi_i\delta_i\gamma_j^2(1-\zeta_i\zeta_j-\sigma_i\xi_i)(\xi_i-\xi_i\sigma_j\xi_j+\xi_j\zeta_i\rho_j)$$
$$+\alpha_\pi^{Gj}\alpha_y^{Gi}\xi_j\delta_i\gamma_j^2(1-\zeta_i\zeta_j-\sigma_j\xi_j)(\xi_j-\xi_i\sigma_i\xi_j+\xi_i\zeta_j\rho_i)$$
$$+\alpha_\pi^{Gj}\alpha_\pi^{Gi}\xi_i\xi_j\gamma_j^2\delta_i(\xi_i-\xi_i\sigma_j\xi_j+\xi_j\zeta_i\rho_j)(\xi_j-\xi_i\sigma_i\xi_j+\xi_i\zeta_j\rho_i)$$

$(\Delta_{5,i})$increases if $(\alpha_\pi^{Gi} > \alpha_\pi^{Gj})$ and $(\alpha_y^{Gi} > \alpha_y^{Gj})$

$(\Delta_{5,i})$ increases if $(\sigma_i > \sigma_j)$ and $(\zeta_i \neq \zeta_j)$, but if $(\xi_i < \xi_j)$, $(\rho_i < \rho_j)$, and $(\alpha_i < \alpha_j)$

$(\Delta_{6,i})$increases if $(\alpha_\pi^{Gi} > \alpha_\pi^{Gj})$, $(\alpha_y^{Gi} = \alpha_y^{Gj})$, but if $(\alpha_g^{Gi} < \alpha_g^{Gj})$

$(\Delta_{6,i})$ increases if $(\delta_i > \delta_j)$, $(\sigma_i > \sigma_j)$, $(\xi_i > \xi_j)$, $(\zeta_i \neq \zeta_j)$, but if $(\rho_i < \rho_j)$ and $(\gamma_i < \gamma_j)$

APPENDIX B: DYNAMIC OF THE MODEL

According to equations (5), (9) and (10), the differential between the consumer inflation rates in the countries (i) and (j) in the monetary union is as follows:

$$(\pi_{j,t+1}-\pi_{i,t+1}) = (1-\eta_i-\eta_j)[(p_{j,t+1}^j-p_{j,t}^j)-(p_{i,t+1}^i-p_{i,t}^i)]$$
$$=\frac{[\xi_i\xi_j(\sigma_i\delta_j+\sigma_j\delta_i)-\xi_j(1-\zeta_i)(\delta_j-\rho_j\delta_i)-\xi_i(1-\zeta_j)(\delta_i-\rho_i\delta_j)]}{[(1-\zeta_i\zeta_j)(1-\rho_i\rho_j)+\sigma_i\sigma_j\xi_i\xi_j-(\sigma_i+\sigma_j\rho_i\zeta_j)\xi_i-(\sigma_j+\sigma_i\rho_j\zeta_i)\xi_j]}(p_{j,t+1}^j-p_{i,t+1}^i)$$
$$+\frac{[\xi_i(1-\zeta_j)(\sigma_i+\sigma_j\rho_i)-\xi_j(1-\zeta_i)(\sigma_j+\sigma_i\rho_j)]}{[(1-\zeta_i\zeta_j)(1-\rho_i\rho_j)+\sigma_i\sigma_j\xi_i\xi_j-(\sigma_i+\sigma_j\rho_i\zeta_j)\xi_i-(\sigma_j+\sigma_i\rho_j\zeta_i)\xi_j]}i_{t+1}$$
$$-\frac{\gamma_i[\xi_i(1-\zeta_j)-\xi_j\rho_j(1-\zeta_i)-\sigma_j\xi_i\xi_j]}{[(1-\zeta_i\zeta_j)(1-\rho_i\rho_j)+\sigma_i\sigma_j\xi_i\xi_j-(\sigma_i+\sigma_j\rho_i\zeta_j)\xi_i-(\sigma_j+\sigma_i\rho_j\zeta_i)\xi_j]}g_{i,t+1} \quad \text{(B1)}$$
$$+\frac{\gamma_j[\xi_j(1-\zeta_i)-\xi_i\rho_i(1-\zeta_j)-\sigma_i\xi_i\xi_j]}{[(1-\zeta_i\zeta_j)(1-\rho_i\rho_j)+\sigma_i\sigma_j\xi_i\xi_j-(\sigma_i+\sigma_j\rho_i\zeta_j)\xi_i-(\sigma_j+\sigma_i\rho_j\zeta_i)\xi_j]}g_{j,t+1}$$

Where: $(\pi_{j,0} - \pi_{i,0})$ is the initial disequilibrium in competitiveness in the monetary union, due to past fiscal policies for example.

So, equation (B1) implies the following evolution of the differential between the production prices in the countries (i) and (j):

$$\begin{aligned}
&\big[(1-\zeta_i\zeta_j)(1-\rho_i\rho_j)(1-\eta_i-\eta_j)+\sigma_i\sigma_j\xi_i\xi_j(1-\eta_i-\eta_j)-\xi_i\xi_j(\sigma_i\delta_j+\sigma_j\delta_i)\\
&\quad-(\sigma_i+\sigma_j\rho_i\zeta_j)\xi_i(1-\eta_i-\eta_j)-(\sigma_j+\sigma_i\rho_j\zeta_i)\xi_j(1-\eta_i-\eta_j)\\
&\quad+\xi_j(1-\zeta_i)(\delta_j-\rho_j\delta_i)+\xi_i(1-\zeta_j)(\delta_i-\rho_i\delta_j)\big](p^j_{j,t}-p^i_{i,t})\\
&=\big[(1-\zeta_i\zeta_j)(1-\rho_i\rho_j)+\sigma_i\sigma_j\xi_i\xi_j-(\sigma_i+\sigma_j\rho_i\zeta_j)\xi_i-(\sigma_j+\sigma_i\rho_j\zeta_i)\xi_j\big](1-2\eta)(p^j_{j,t-1}-p^i_{i,t-1})\\
&+\gamma_i\big[\xi_j\xi_i\sigma_j-\xi_i(1-\zeta_j)+\xi_j\rho_j(1-\zeta_i)\big]g_{i,t}-\gamma_j\big[\sigma_i\xi_i\xi_j-\xi_j(1-\zeta_i)+\xi_i\rho_i(1-\zeta_j)\big]g_{j,t}\\
&\quad+\big[\xi_i(1-\zeta_j)(\sigma_i+\sigma_j\rho_i)-\xi_j(1-\zeta_i)(\sigma_j+\sigma_i\rho_j)\big]i_t
\end{aligned} \tag{B2}$$

Therefore, using equations (B2), (A5) and (A6), we obtain:

$$\begin{aligned}
&\Big\{1+\frac{[\xi_j(1-\zeta_i)(\sigma_j+\sigma_i\rho_j)-\xi_i(1-\zeta_j)(\sigma_i+\sigma_j\rho_i)]\Delta_{4,i}}{[(1-\zeta_i\zeta_j)(1-\rho_i\rho_j)+\sigma_i\sigma_j\xi_i\xi_j-(\sigma_i+\sigma_j\rho_i\zeta_j)\xi_i-(\sigma_j+\sigma_i\rho_j\zeta_i)\xi_j](1-\eta_i-\eta_j)\Delta_1}\\
&+\frac{\gamma_i^2[\xi_j\xi_i\sigma_j-\xi_i(1-\zeta_j)+\xi_j\rho_j(1-\zeta_i)][\alpha_g^{Gj}(\delta_i\sigma_j+\sigma_i\delta_j)\Delta_{5,i}+\alpha_i^M\Delta_{6,i}]}{[(1-\zeta_i\zeta_j)(1-\rho_i\rho_j)+\sigma_i\sigma_j\xi_i\xi_j-(\sigma_i+\sigma_j\rho_i\zeta_j)\xi_i-(\sigma_j+\sigma_i\rho_j\zeta_i)\xi_j](1-\eta_i-\eta_j)\Delta_1}\\
&+\frac{\gamma_j^2[\sigma_i\xi_i\xi_j-\xi_j(1-\zeta_i)+\xi_i\rho_i(1-\zeta_j)][\alpha_g^{Gi}(\delta_i\sigma_j+\sigma_i\delta_j)\Delta_{5,j}+\alpha_i^M\Delta_{6,j}]}{[(1-\zeta_i\zeta_j)(1-\rho_i\rho_j)+\sigma_i\sigma_j\xi_i\xi_j-(\sigma_i+\sigma_j\rho_i\zeta_j)\xi_i-(\sigma_j+\sigma_i\rho_j\zeta_i)\xi_j](1-\eta_i-\eta_j)\Delta_1}\\
&+\frac{[\xi_j(1-\zeta_i)(\delta_j-\rho_j\delta_i)+\xi_i(1-\zeta_j)(\delta_i-\rho_i\delta_j)-\xi_i\xi_j(\sigma_i\delta_j+\sigma_j\delta_i)]}{[(1-\zeta_i\zeta_j)(1-\rho_i\rho_j)+\sigma_i\sigma_j\xi_i\xi_j-(\sigma_i+\sigma_j\rho_i\zeta_j)\xi_i-(\sigma_j+\sigma_i\rho_j\zeta_i)\xi_j](1-\eta_i-\eta_j)}\Big\}(p^j_{j,t}-p^i_{i,t})\\
&\qquad=(p^j_{j,t-1}-p^i_{i,t-1})
\end{aligned} \tag{B3}$$

$$(p^j_{j,t}-p^i_{i,t})=\frac{(1-\eta_i-\eta_j)\Delta_1}{[(1-\eta_i-\eta_j)\Delta_1+\Delta_7]}(p^j_{j,t-1}-p^i_{i,t-1}) \tag{B4}$$

$$\Delta_7=\big[(1-\zeta_i\zeta_j)(1-\rho_i\rho_j)+\sigma_i\sigma_j\xi_i\xi_j-(\sigma_i+\sigma_j\rho_i\zeta_j)\xi_i-(\sigma_j+\sigma_i\rho_j\zeta_i)\xi_j$$

$$\{\alpha_{\pi}^{M}\alpha_{g}^{Gi}\alpha_{g}^{Gj}(\sigma_j\delta_i+\sigma_i\delta_j)\xi_i\xi_j$$
$$[\xi_i(\alpha_i+\alpha_j\zeta_j)(\sigma_i+\sigma_j\rho_i)-\sigma_i\sigma_j\xi_j\xi_i+\xi_j(\alpha_j+\alpha_i\zeta_i)(\sigma_j+\sigma_i\rho_j)]$$
$$+\alpha_{y}^{M}\alpha_{g}^{Gi}\alpha_{g}^{Gj}(\sigma_j\delta_i+\sigma_i\delta_j)[\alpha_i\xi_j(1-\zeta_i)+\alpha_j\xi_i(1-\zeta_j)]$$
$$[(\alpha_i\sigma_i+\alpha_i\sigma_j\rho_i+\alpha_j\sigma_j+\alpha_j\sigma_i\rho_j)(1-\zeta_i\zeta_j)-\alpha_i\sigma_i\sigma_j\xi_j(1-\zeta_i)-\alpha_j\sigma_j\sigma_i\xi_i(1-\zeta_j)]$$
$$+\alpha_{i}^{M}\alpha_{g}^{Gi}\alpha_{g}^{Gj}[\xi_j(1-\zeta_i)(\delta_j-\rho_j\delta_i)+\xi_i(1-\zeta_j)(\delta_i-\rho_i\delta_j)-\xi_i\xi_j(\sigma_i\delta_j+\sigma_j\delta_i)]$$
$$[(1-\zeta_i\zeta_j)(1-\rho_i\rho_j)+\sigma_i\sigma_j\xi_i\xi_j-(\sigma_i+\sigma_j\rho_i\zeta_j)\xi_i-(\sigma_j+\sigma_i\rho_j\zeta_i)\xi_j]$$
$$+\alpha_{i}^{M}\alpha_{y}^{Gi}\alpha_{g}^{Gj}\delta_j\xi_j(1-\zeta_i)\gamma_i^2(1-\zeta_i\zeta_j-\sigma_j\xi_j)+\alpha_{i}^{M}\alpha_{g}^{Gi}\alpha_{y}^{Gj}\delta_i\xi_i(1-\zeta_j)\gamma_j^2(1-\zeta_i\zeta_j-\sigma_i\xi_i)$$
$$+\alpha_{i}^{M}\alpha_{g}^{Gi}\alpha_{\pi}^{Gj}\gamma_j^2\delta_i\xi_j\xi_i(\xi_j-\xi_i\sigma_i\xi_j+\xi_i\zeta_j\rho_i)+\alpha_{i}^{M}\alpha_{\pi}^{Gi}\alpha_{g}^{Gj}\gamma_i^2\delta_j\xi_i\xi_j(\xi_i-\xi_i\sigma_j\xi_j+\xi_j\zeta_i\rho_j)\}>0$$

(Δ_7) increases if $(\alpha_g^{Gi}=\alpha_g^{Gj})$

(Δ_7) increases if $(\sigma_i=\sigma_j)$, $(\xi_i=\xi_j)$, but if $(\zeta_i\neq\zeta_j)$, $(\rho_i\neq\rho_j)$ and $(\gamma_i\neq\gamma_j)$

APPENDIX C: ECONOMIC ACTIVITY AND INFLATION

Replacing equation (A5) for (i_t) and equation (A6) for $(g_{i,t})$ and $(g_{j,t})$ in equation (11), we obtain the following economic activity in the country (i):

$$y_{i,t}=\left[\alpha_i^M\frac{\Delta_{8,i}}{\Delta_1}+\alpha_g^{Gi}\alpha_g^{Gj}(\delta_i\sigma_j+\sigma_i\delta_j)\frac{\Delta_{9,i}}{\Delta_1}\right](p_{j,t}^j-p_{i,t}^i) \qquad \text{(C1)}$$

$$\Delta_{8,i}=[(1-\zeta_i\zeta_j)(1-\rho_i\rho_j)+\sigma_i\sigma_j\xi_i\xi_j-(\sigma_i+\sigma_j\rho_i\zeta_j)\xi_i-(\sigma_j+\sigma_i\rho_j\zeta_i)\xi_j]$$
$$\{\alpha_g^{Gi}\alpha_g^{Gj}[(\delta_i-\rho_i\delta_j)(1-\zeta_i\zeta_j)-(\delta_i\sigma_j+\sigma_i\delta_j\zeta_i)\xi_j][(1-\zeta_i\zeta_j)(1-\rho_i\rho_j)+\sigma_i\sigma_j\xi_i\xi_j$$
$$-(\sigma_i+\sigma_j\rho_i\zeta_j)\xi_i-(\sigma_j+\sigma_i\rho_j\zeta_i)\xi_j]$$
$$+\alpha_y^{Gj}\alpha_g^{Gi}\gamma_j^2\delta_i(1-\zeta_i\zeta_j)(1-\zeta_i\zeta_j-\sigma_i\xi_i)+\alpha_\pi^{Gj}\alpha_g^{Gi}\gamma_j^2\delta_i\xi_j(\xi_j-\xi_i\sigma_i\xi_j+\xi_i\zeta_j\rho_i)$$
$$+\alpha_\pi^{Gi}\alpha_g^{Gj}\xi_j\zeta_i\gamma_i^2\delta_j(\xi_i-\xi_i\sigma_j\xi_j+\xi_j\zeta_i\rho_j)\}$$

$$\Delta_{8,i} = \left[(1-\zeta_i\zeta_j)(1-\rho_i\rho_j) + \sigma_i\sigma_j\xi_i\xi_j - (\sigma_i + \sigma_j\rho_i\zeta_j)\xi_i - (\sigma_j + \sigma_i\rho_j\zeta_i)\xi_j\right]$$
$$\{\alpha_g^{Gi}\alpha_g^{Gj}\left[(\delta_i - \rho_i\delta_j)(1-\zeta_i\zeta_j) - (\delta_i\sigma_j + \sigma_i\delta_j\zeta_i)\xi_j\right]\left[(1-\zeta_i\zeta_j)(1-\rho_i\rho_j) + \sigma_i\sigma_j\xi_i\xi_j - (\sigma_i + \sigma_j\rho_i\zeta_j)\xi_i - (\sigma_j + \sigma_i\rho_j\zeta_i)\xi_j\right]$$
$$+\alpha_y^{Gj}\alpha_g^{Gi}\gamma_j^2\delta_i(1-\zeta_i\zeta_j)(1-\zeta_i\zeta_j - \sigma_i\xi_i) + \alpha_\pi^{Gj}\alpha_g^{Gi}\gamma_j^2\delta_i\xi_j(\xi_j - \xi_i\sigma_i\xi_j + \xi_i\zeta_j\rho_i)$$
$$+\alpha_\pi^{Gi}\alpha_g^{Gj}\xi_j\zeta_i\gamma_i^2\delta_j(\xi_i - \xi_i\sigma_j\xi_j + \xi_j\zeta_i\rho_j)\}$$

$$\Delta_{9,i} = \left[(1-\zeta_i\zeta_j)(1-\rho_i\rho_j) + \sigma_i\sigma_j\xi_i\xi_j - (\sigma_i + \sigma_j\rho_i\zeta_j)\xi_i - (\sigma_j + \sigma_i\rho_j\zeta_i)\xi_j\right]$$
$$\{\alpha_\pi^M\xi_j(\alpha_j + \alpha_i\zeta_i)\left[\xi_i(\alpha_i + \alpha_j\zeta_j)(\sigma_i + \sigma_j\rho_i) - \xi_i\sigma_i\sigma_j\xi_j + \xi_j(\alpha_j + \alpha_i\zeta_i)(\sigma_j + \sigma_i\rho_j)\right]$$
$$+\alpha_y^M(1-\zeta_i\zeta_j)\alpha_j\left[(\alpha_i\sigma_i + \alpha_i\sigma_j\rho_i + \alpha_j\sigma_j + \alpha_j\sigma_i\rho_j)(1-\zeta_i\zeta_j) - \alpha_i\sigma_i\sigma_j\xi_j(1-\zeta_i)\right.$$
$$\left. - \alpha_j\sigma_j\sigma_i\xi_i(1-\zeta_j)\right]\} > 0$$

Replacing equation (A5) for (i_t) and equation (A6) for ($g_{i,t}$)) and ($g_{j,t}$)) in equation (9), we obtain the following inflation rate in the country (i):

$$\pi_{i,t} = \left[\alpha_i^M\frac{\Delta_{10,i}}{\Delta_1} + \alpha_g^{Gi}\alpha_g^{Gj}(\delta_i\sigma_j + \sigma_i\delta_j)\frac{\Delta_{11,i}}{\Delta_1}\right](p_{j,t}^j - p_{i,t}^i) \qquad \text{(C2)}$$

$$\Delta_{10,i} = \left[(1-\zeta_i\zeta_j)(1-\rho_i\rho_j) + \sigma_i\sigma_j\xi_i\xi_j - (\sigma_i + \sigma_j\rho_i\zeta_j)\xi_i - (\sigma_j + \sigma_i\rho_j\zeta_i)\xi_j\right]$$
$$\{\alpha_g^{Gi}\alpha_g^{Gj}\left[\xi_i(\delta_i - \rho_i\delta_j) - \delta_i\sigma_j\xi_i\xi_j - (\delta_j - \rho_j\delta_i)\zeta_i\xi_j\right]$$
$$\left[(1-\zeta_i\zeta_j)(1-\rho_i\rho_j) + \sigma_i\sigma_j\xi_i\xi_j - (\sigma_i + \sigma_j\rho_i\zeta_j)\xi_i - (\sigma_j + \sigma_i\rho_j\zeta_i)\xi_j\right]$$
$$-\alpha_g^{Gj}\alpha_y^{Gi}\gamma_i^2\delta_j\zeta_i\xi_j(1-\zeta_i\zeta_j - \sigma_j\xi_j)$$
$$+\alpha_g^{Gi}\gamma_j^2\delta_i\xi_i\left[\alpha_y^{Gj}(1-\zeta_i\zeta_j - \sigma_i\xi_i) + \alpha_\pi^{Gj}\xi_j(\xi_j - \xi_i\sigma_i\xi_j + \xi_i\zeta_j\rho_i)\right]\}$$

$$\Delta_{11,i} = \left[(1-\zeta_i\zeta_j)(1-\rho_i\rho_j) + \sigma_i\sigma_j\xi_i\xi_j - (\sigma_i + \sigma_j\rho_i\zeta_j)\xi_i - (\sigma_j + \sigma_i\rho_j\zeta_i)\xi_j\right]$$
$$\{\alpha_\pi^M\alpha_j\xi_i\xi_j\left[\xi_i(\alpha_i + \alpha_j\zeta_j)(\sigma_i + \sigma_j\rho_i) - \xi_i\sigma_i\sigma_j\xi_j + \xi_j(\alpha_j + \alpha_i\zeta_i)(\sigma_j + \sigma_i\rho_j)\right]$$
$$+\alpha_y^M(\alpha_j\xi_i - \alpha_i\zeta_i\xi_j)\left[(\alpha_i\sigma_i + \alpha_i\sigma_j\rho_i + \alpha_j\sigma_j + \alpha_j\sigma_i\rho_j)(1-\zeta_i\zeta_j) - \alpha_i\sigma_i\sigma_j\xi_j(1-\zeta_i)\right.$$
$$\left. - \alpha_j\sigma_j\sigma_i\xi_i(1-\zeta_j)\right]\} > 0$$

$(\Delta_{8,i})$ increases if $(\alpha_\pi^{Gi} < \alpha_\pi^{Gj})$ and $(\alpha_y^{Gi} < \alpha_y^{Gj})$, but if $(\alpha_g^{Gi} > \alpha_g^{Gj})$

$(\Delta_{8,i})$ increases if $(\delta_i > \delta_j)$, $(\zeta_i \neq \zeta_j)$, $(\rho_i \neq \rho_j)$, and if $(\sigma_i < \sigma_j)$, $(\xi_i < \xi_j)$and $(\gamma_i < \gamma_j)$

$(\Delta_{9,i})$ increases if $(\sigma_i \neq \sigma_j)$, $(\rho_i \neq \rho_j)$, $(\zeta_i \neq \zeta_j)$, and if $(\xi_i < \xi_j)$ and $(\alpha_i < \alpha_j)$

$(\Delta_{10,i})$ increases if $(\alpha_\pi^{Gi} < \alpha_\pi^{Gj})$ and $(\alpha_y^{Gi} < \alpha_y^{Gj})$, but if $(\alpha_g^{Gi} > \alpha_g^{Gj})$

$(\Delta_{10,i})$ increases if $(\delta_i > \delta_j)$, $(\xi_i > \xi_j)$, and if $(\sigma_i < \sigma_j)$, $(\gamma_i < \gamma_j)$, $(\zeta_i < \zeta_j)$ and $(\rho_i < \rho_j)$

$(\Delta_{11,i})$ increases if $(\sigma_i \neq \sigma_j)$, $(\rho_i \neq \rho_j)$, $(\xi_i > \xi_j)$, but if $(\zeta_i < \zeta_j)$, and $(\alpha_i < \alpha_j)$.

